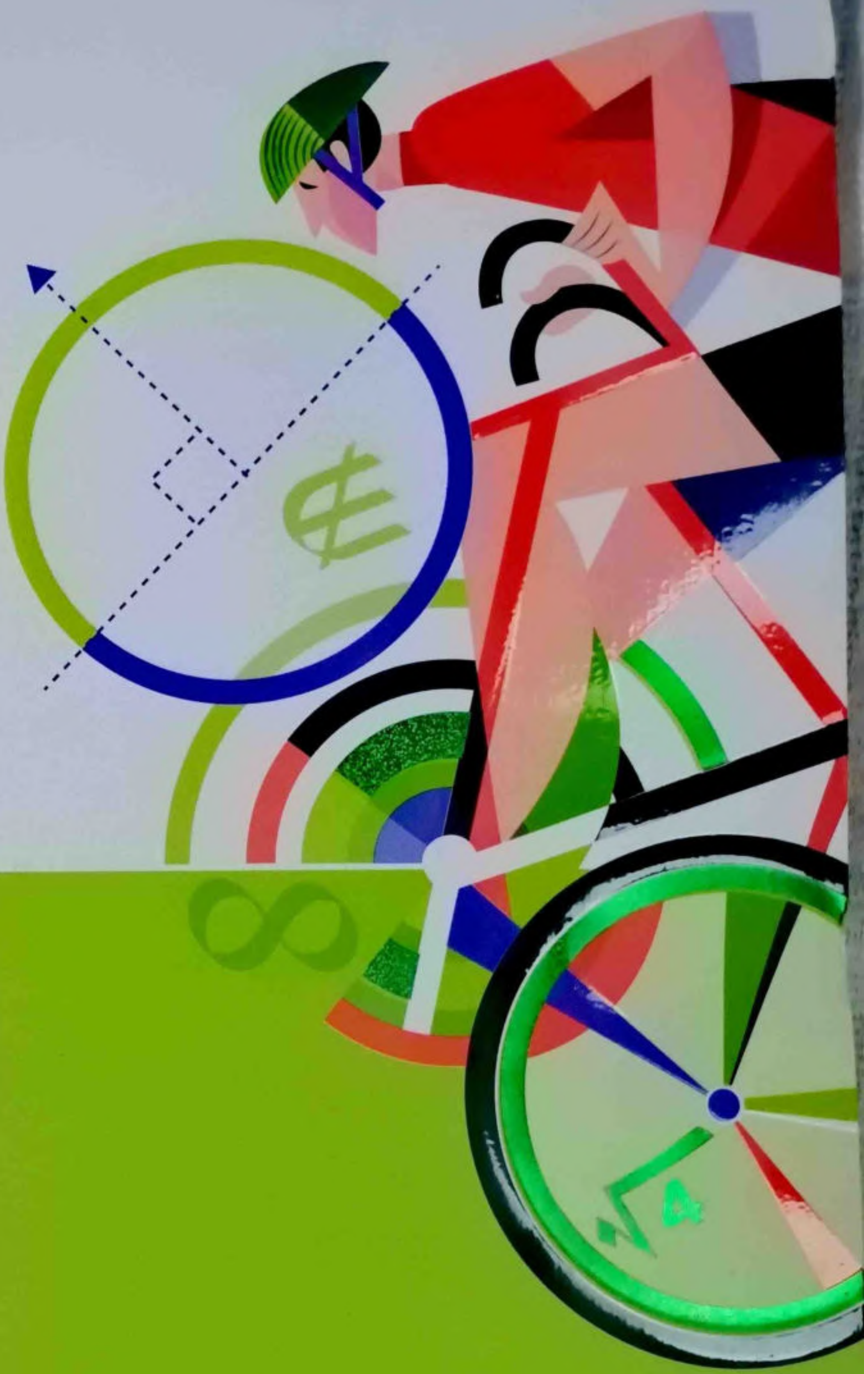
EL-MOASSER

By a group of supervisors

THE MAIN BOOK



3rd PREP. 2025

FIRST TERM

Maths

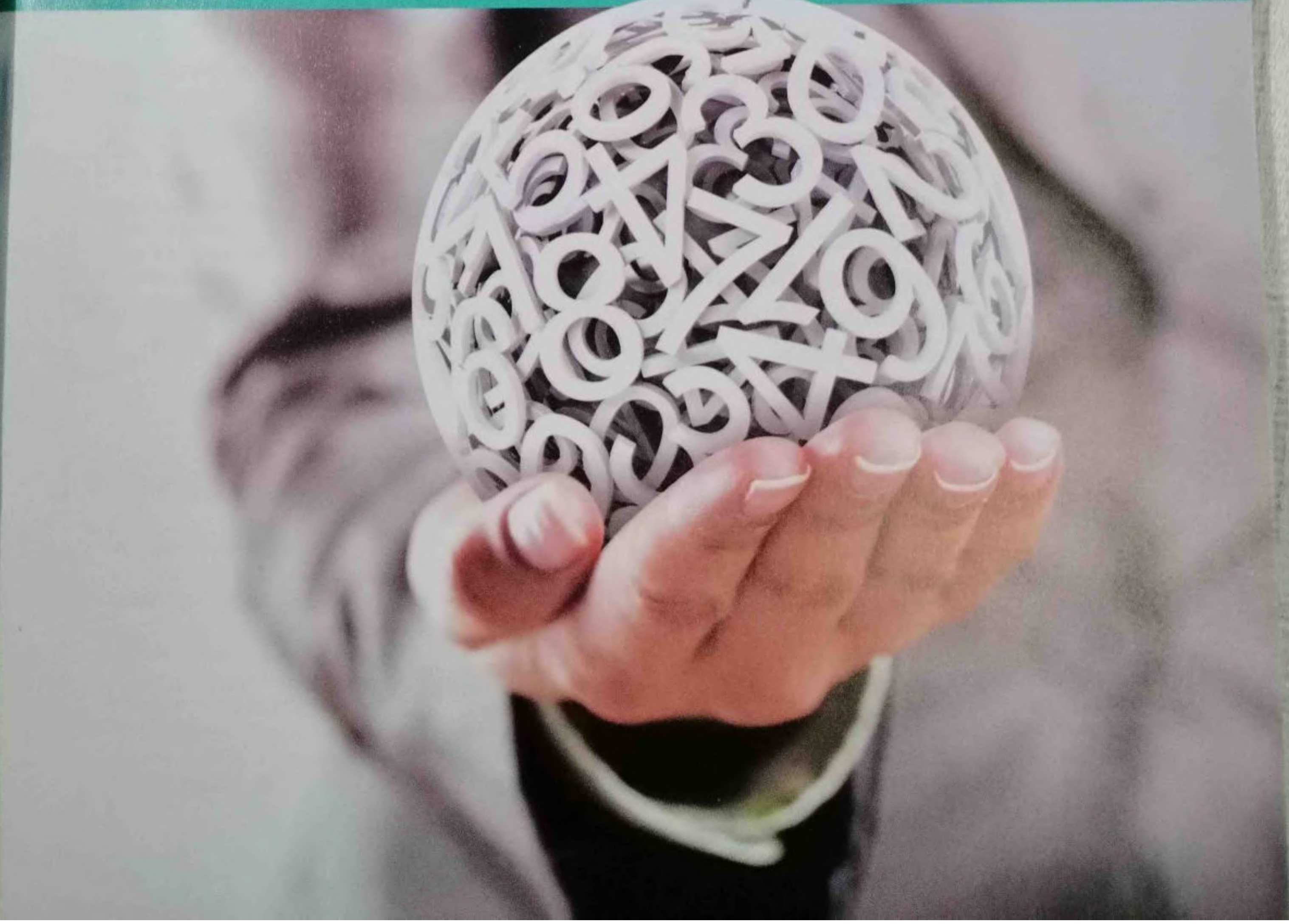
GPS

Interactive E-learning Application

Contents

First

Algebra and Statistics



UNIT ONE

Relations and functions

Lessons of the unit :

1. Cartesian product.
2. Relation - Function (Mapping).
3. The symbolic representation of the function - Polynomial functions.
4. The study of some polynomial functions.

Use your smart phone or tablet to scan the QR code and enjoy watching videos

Unit Objectives : By the end of this unit, student should be able to :

- recognize the concept of the Cartesian product of two finite sets.
- represent the Cartesian product of two finite sets by the arrow diagram and the graphical (Cartesian) diagram.
- recognize the concept of the Cartesian product of two infinite sets.
- find the Cartesian product of two intervals.
- recognize the concept of the relation from a set to another one.
- recognize whether the relation is a function or not.
- represent the function by the arrow diagram and the graphical (Cartesian) diagram.
- recognize the domain, the codomain and the range of the function.
- express the function symbolically.
- search the degree of the polynomial function.
- represent the linear function graphically.
- recognize the constant function and represent it graphically.
- represent graphically the quadratic function.
- find the vertex of the curve of the quadratic function.
- find the maximum or the minimum value of the quadratic function.
- find the equation of the axis of symmetry of the quadratic function.

Lesson 1 Cartesian product

In this lesson , we shall know the concept of the Cartesian product and how to find it and how to represent it graphically.

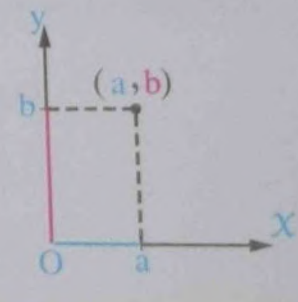
Before dealing with this subject , we shall remember together what we had studied about the ordered pair.

The ordered pair

(a , b) is called an ordered pair

- a is called the first projection
- b is called the second projection

and the ordered pair (a , b) could be represented by a point as shown in the opposite figure.

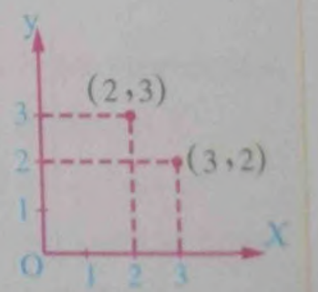


Remarks

- If $a \neq b$, then $(a , b) \neq (b , a)$

  For example: $(2 , 3) \neq (3 , 2)$

  and when representing them graphically as shown in the opposite figure , we find that they are represented by two different points.



- The ordered pair is not a set. i.e. $(a , b) \neq \{a , b\}$

- (a , a) is an ordered pair , while in the sets , we don't write $\{a , a\}$, but we write $\{a\}$ without repeating the element a
- There is an empty set of elements and denoted by the symbol $\varnothing$, but there is not an empty ordered pair.

The equality of two ordered pairs

If $(a , b) = (x , y)$, then $a = x$, $b = y$

For example:

- If $(a , b) = (3 , -4)$, then $a = 3$, $b = -4$
- If $(x , 2) = (-5 , y)$, then $x = -5$, $y = 2$

Example 1 **Choose the correct answer from the given ones :**

1 If $(3 , 8) = \left(3 , \sqrt{y}\right)$, then $\sqrt[3]{y} =$ ....

(a) -4 (b) 4 (c) 8 (d) 64

2 If $(32 , x + y) = (y^5 , 2)$, then $x =$ ....

(a) 0 (b) 2 (c) 4 (d) 5

3 If $(2^{x-1} , -3) = (1 , y)$, then $2x - y =$ ....

(a) -3 (b) -1 (c) 3 (d) 5

4 If $(x^2 - 1 , 4) = (48 , 2y)$, then $xy =$ ....

(a) -7 (b) 7 (c) 14 (d) ± 14

Solution

1 **(b) The reason :** $\because (3 , 8) = \left(3 , \sqrt{y}\right)$ $\therefore \sqrt{y} = 8$

$\therefore y = 8^2 = 64$ $\therefore \sqrt[3]{y} = \sqrt[3]{64} = 4$

2 **(a) The reason :** $\because (32 , x + y) = (y^5 , 2)$

$\therefore y^5 = 32$ $\therefore y = 2$ «because $2^5 = 32$»

, $x + y = 2$ substituting by $y = 2$ $\therefore x + 2 = 2$

$\therefore x = 0$

3 **(d) The reason :** $\because (2^{x-1} , -3) = (1 , y)$ $\therefore y = -3$

, $2^{x-1} = 1$, then $x - 1 = 0$ $\therefore x = 1$

$\therefore 2x - y = 2 \times 1 - (-3) = 2 + 3 = 5$

4 **(d) The reason :** $\because (x^2 - 1 , 4) = (48 , 2y)$ $\therefore x^2 - 1 = 48$

$\therefore x^2 = 49$

$\therefore x = \pm\sqrt{49} = \pm 7$, $2y = 4$ $\therefore y = \frac{4}{2} = 2$

$\therefore xy = \pm 7 \times 2 = \pm 14$

TRY yourself 1 ***Find the values of X and y in each of the following :***

1. $(X+1 , y^2) = (3 , 9)$
2. $(X^3 - 5 , 8) = (3 , 3y - 7)$
3. $(X^2 - 2 , 2y) = \left(y , \sqrt[3]{64}\right)$

The Cartesian product of two finite sets

For any two finite and non empty sets X and Y , we get :

The Cartesian product of the set X by the set Y and it is denoted by $\boxed{X \times Y}$ is the set of all ordered pairs whose first projection of each of them belongs to X and the second projection of each of them belongs to Y

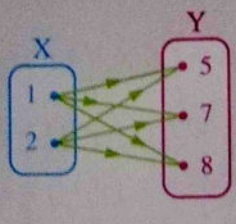
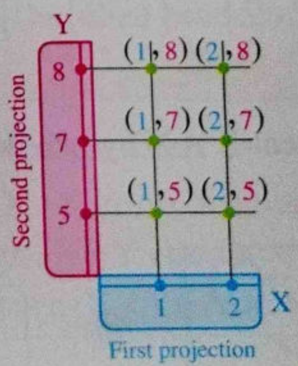
i.e. $X \times Y = \{(a , b) : a \in X , b \in Y\}$

For example :

1 If $X = \{1 , 2\}$, $Y = \{5 , 7 , 8\}$, then :

$$X \times Y = \{1 , 2\} \times \{5 , 7 , 8\}$$
$$= \{(1 , 5) , (1 , 7) , (1 , 8) , (2 , 5) , (2 , 7) , (2 , 8)\}$$

- We can represent $X \times Y$ by two ways as follows :

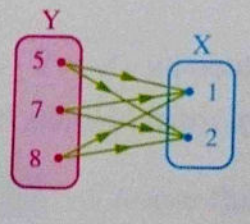
1st way : The arrow diagram	2nd way : The graphical (Cartesian) diagram
	
Where we draw an arrow going from each element representing the first projection (the elements of the set X) to each element representing the second projection (the elements of the set Y)	Where the elements of the set X are represented horizontally and the elements of the set Y are represented vertically and the points of intersection of the horizontal and vertical lines represent the Cartesian product of $X \times Y$

2 If $X = \{1, 2\}$, $Y = \{5, 7, 8\}$, then :

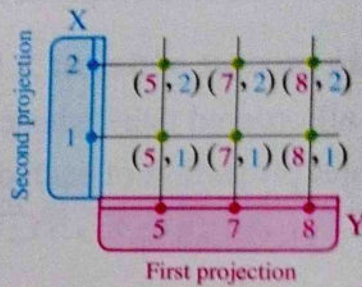
$$Y \times X = \{5, 7, 8\} \times \{1, 2\}$$

$$= \{(5, 1), (5, 2), (7, 1), (7, 2), (8, 1), (8, 2)\}$$

- Similarly , we can represent $Y \times X$ by two ways as follows :



The arrow diagram



The Cartesian diagram

The Cartesian product of a set by itself

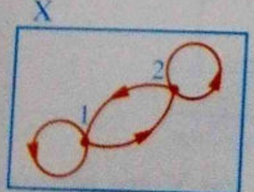
The Cartesian product of the set X by itself and we denote it by $\boxed{X \times X}$ or by $\boxed{X^2}$ (it is read X two) is the set of all ordered pairs whose first projections and second projections belong both to X

i.e. $X \times X = \{(a, b) : a \in X , b \in X\}$

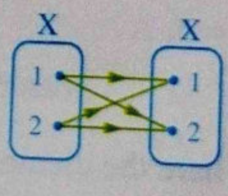
For example: If $X = \{1, 2\}$, then :

$$X \times X = \{1, 2\} \times \{1, 2\} = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$$

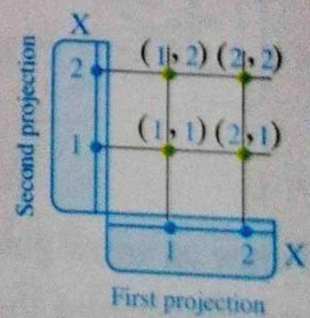
- We can represent $X \times X$ by two ways as follows :



or



The arrow diagram



The Cartesian diagram

Notice that : The figure (loop at 1) is called a loop to show that the arrow goes from the point and returns to the same point.

Remarks

- For any two finite and non empty sets X and Y , then $X \times Y \neq Y \times X$ where $X \neq Y$
- For any set X , then $X \times \varnothing = \varnothing \times X = \varnothing$ where $\varnothing$ is the empty set.
- If $(a , b) \in X \times Y$, then $a \in X$, $b \in Y$

For example: If $(3 , 5) \in X \times Y$, then $3 \in X$, $5 \in Y$

Example 2 If $X = \{2 , 3 , 4\}$ and $Y = \{a , b\}$, **find each of :**

1 $X \times Y$ 2 $Y \times X$ 3 $X \times X$ 4 Y^2

Solution

1 $X \times Y = \{(2 , a) , (2 , b) , (3 , a) , (3 , b) , (4 , a) , (4 , b)\}$

2 $Y \times X = \{(a , 2) , (a , 3) , (a , 4) , (b , 2) , (b , 3) , (b , 4)\}$

3 $X \times X = \{(2 , 2) , (2 , 3) , (2 , 4) , (3 , 2) , (3 , 3) , (3 , 4) , (4 , 2) , (4 , 3) , (4 , 4)\}$

4 $Y^2 = \{(a , a) , (a , b) , (b , a) , (b , b)\}$

TRY by yourself 2 If $X = \{3 , 4 , 5\}$ and $Y = \{5 , 6\}$, ***find each of the following :***

1 $Y \times X$ and represent it by an arrow diagram

2 X^2 and represent it by a Cartesian diagram

The number of the elements of the Cartesian product

If we denote the number of elements of the set X by n (X) and the number of elements of the set Y by n (Y) , then the number of elements of the Cartesian product $X \times Y$ is denoted by $n (X \times Y)$, and :

- $n (X \times Y) = n (Y \times X) = n (X) \times n (Y)$
- $n (X \times X) = n (X) \times n (X) = [n (X)]^2$
- $n (X \times \varnothing) = n (X) \times n (\varnothing)$
  $= 0$ [Because $n (\varnothing) = 0$]

Notice that :
If X , Y are two finite and non empty sets , $X \neq Y$, then $X \times Y \neq Y \times X$, but $n (X \times Y) = n (Y \times X)$

For example :

If $X = \{2 , -1 , 0\}$ and $Y = \{5 , -7\}$, then $n (X) = 3$, $n (Y) = 2$, then :

- $n (X \times Y) = 3 \times 2 = 6$
- $n (X^2) = 3^2 = 9$
- $n (Y \times X) = 2 \times 3 = 6$
- $n (Y^2) = 2^2 = 4$

Find the previous Cartesian products and verify the number of their elements.

Example 3 **Choose the correct answer from the given ones :**

1 If $X = \{0, 2\}$, $n(Y) = 5$, then $n(X \times Y) =$ ....

(a) 2 (b) 5 (c) 7 (d) 10

2 If $n(Y) = 4$, $n(X \times Y) = 8$, then $n(X) =$ ....

(a) 2 (b) 4 (c) 8 (d) 32

3 If $n(X^2) = 9$, $n(Y^2) = 16$, then $n(Y \times X) =$ ....

(a) 7 (b) 12 (c) 36 (d) 144

Solution

1 (d) **The reason :** $\because n(X) = 2$, $n(Y) = 5$

$\therefore n(X \times Y) = 2 \times 5 = 10$

2 (a) **The reason :** $n(X) = \frac{n(X \times Y)}{n(Y)} = \frac{8}{4} = 2$

3 (b) **The reason :** $\because n(X^2) = 9$ $\therefore n(X) = \sqrt{9} = 3$

, $\because n(Y^2) = 16$ $\therefore n(Y) = \sqrt{16} = 4$

$\therefore n(Y \times X) = 4 \times 3 = 12$

TRY by yourself 3 ***Choose the correct answer from the given ones :***

1 If $n(X) = 3$, $n(X \times Y) = 12$, then $n(Y) =$ ....

(a) 4 (b) 9 (c) 15 (d) 36

2 If $Y = \{-1, 0, 1\}$, $n(X \times Y) = 15$, then $n(Y^2) =$ ....

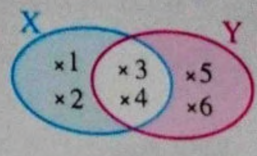
(a) 5 (b) 9 (c) 15 (d) 25

3 If $n(X^2) = 4$, $n(X \times Y) = 4$, then $n(Y^2) =$ ....

(a) 1 (b) 2 (c) 4 (d) 16

Remember the operations on sets

If $X = \{1, 2, 3, 4\}$, $Y = \{3, 4, 5, 6\}$, then :



- $X \cap Y$ = the set of elements which are common in X and Y = $\{3, 4\}$
- $X \cup Y$ = the set of all elements in X or Y without repeating = $\{1, 2, 3, 4, 5, 6\}$
- $X - Y$ = the set of elements which are in X and not in Y = $\{1, 2\}$
- $Y - X$ = the set of elements which are in Y and not in X = $\{5, 6\}$

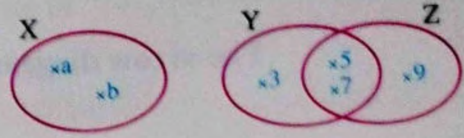
Example 4 If $X = \{a , b\}$, $Y = \{3 , 5 , 7\}$, $Z = \{5 , 7 , 9\}$

, **represent the sets X , Y and Z by Venn diagram , then find :**

1 $X \times (Y \cup Z)$, $(X \times Y) \cup (X \times Z)$

2 $X \times (Y \cap Z)$, $(X \times Y) \cap (X \times Z)$

3 $X \times (Z - Y)$, $(X \times Z) - (X \times Y)$



Solution

1 $\because Y \cup Z = \{3 , 5 , 7 , 9\}$

$\therefore X \times (Y \cup Z) = \{a , b\} \times \{3 , 5 , 7 , 9\}$

$= \{(a , 3) , (a , 5) , (a , 7) , (a , 9) , (b , 3) , (b , 5) , (b , 7) , (b , 9)\}$

, $X \times Y = \{a , b\} \times \{3 , 5 , 7\}$

$= \{(a , 3) , (a , 5) , (a , 7) , (b , 3) , (b , 5) , (b , 7)\}$ (1)

, $X \times Z = \{a , b\} \times \{5 , 7 , 9\}$

$= \{(a , 5) , (a , 7) , (a , 9) , (b , 5) , (b , 7) , (b , 9)\}$ (2)

From (1) and (2) :

$\therefore (X \times Y) \cup (X \times Z) =$

$\{(a , 3) , (a , 5) , (a , 7) , (a , 9) , (b , 3) , (b , 5) , (b , 7) , (b , 9)\}$

2 $\because Y \cap Z = \{5 , 7\}$

$\therefore X \times (Y \cap Z) = \{a , b\} \times \{5 , 7\}$

$= \{(a , 5) , (a , 7) , (b , 5) , (b , 7)\}$

From (1) and (2) :

$\therefore (X \times Y) \cap (X \times Z) = \{(a , 5) , (a , 7) , (b , 5) , (b , 7)\}$

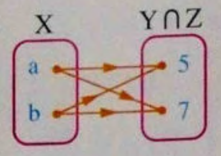
3 $\because Z - Y = \{9\}$

$\therefore X \times (Z - Y) = \{a , b\} \times \{9\} = \{(a , 9) , (b , 9)\}$

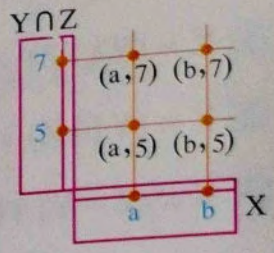
From (1) and (2) : $\therefore (X \times Z) - (X \times Y) = \{(a , 9) , (b , 9)\}$

! Remark

In the previous example , we can represent $X \times (Y \cap Z)$ by an arrow diagram and a Cartesian diagram as follows :



The arrow diagram



The Cartesian diagram

TRY by yourself 4

If $X = \{2 , 3\}$, $Y = \{1 , 3 , 5\}$, $Z = \{2\}$

, represent each of X , Y and Z by Venn diagram , then find :

1 $Z \times (X \cap Y)$ 2 $(Z \times X) \cup (Z \times Y)$

The Cartesian product of two infinite sets

- We know that if X is a finite set (having n elements) , then the Cartesian product $X \times X$ is also a finite set (having n^2 elements).

  For example: If $n(X) = 3$, then $n(X \times X) = 9$

- But if X is an infinite set , then $X \times X$ is an infinite set also

  As examples for that :

  $\mathbb{N} \times \mathbb{N} = \{(x , y) : x \in \mathbb{N} , y \in \mathbb{N}\}$, $\mathbb{Z} \times \mathbb{Z} = \{(x , y) : x \in \mathbb{Z} , y \in \mathbb{Z}\}$,

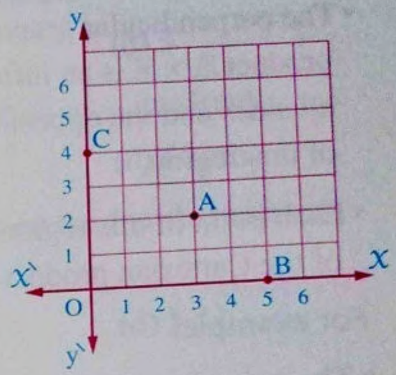
  $\mathbb{Q} \times \mathbb{Q} = \{(x , y) : x \in \mathbb{Q} , y \in \mathbb{Q}\}$, $\mathbb{R} \times \mathbb{R} = \{(x , y) : x \in \mathbb{R} , y \in \mathbb{R}\}$

- We know that if X is a finite set , we represent the Cartesian product $X \times X$ graphically by a finite number of points.

- But if X is an infinite set , then the Cartesian product $X \times X$ is represented graphically by an infinite number of points.

The following is the graphical representation of each of : $\mathbb{N} \times \mathbb{N}$, $\mathbb{Z} \times \mathbb{Z}$, $\mathbb{R} \times \mathbb{R}$:

First Representing the Cartesian product $\mathbb{N} \times \mathbb{N}$ $(\mathbb{N}^2)$

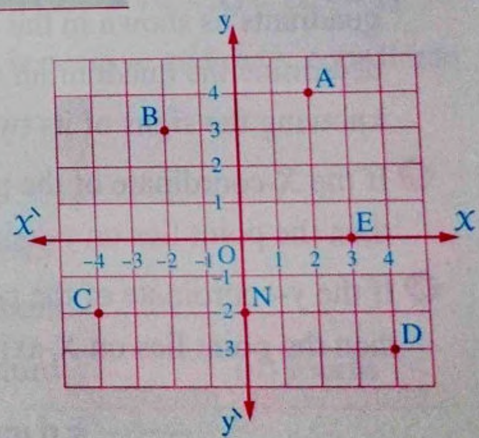


- Represent the natural numbers on two perpendicular straight lines, one of them $\overleftrightarrow{XX}$ is horizontal and the other $\overleftrightarrow{yy}$ is vertical, where they intersect at the point which represents the number zero on each of them i.e. O = (0 , 0)
- The opposite figure shows a small part of the perpendicular graphical net of the Cartesian product $\mathbb{N} \times \mathbb{N}$ which consists of the vertical and the horizontal straight lines that pass through the points which represent the natural numbers on each of $\overleftrightarrow{XX}$ and $\overleftrightarrow{yy}$
- And each point of the points of this net represents an ordered pair of the Cartesian product $\mathbb{N} \times \mathbb{N}$

For example :

- The point A represents the ordered pair (3 , 2)
- The point B represents the ordered pair (5 , 0)
- The point C represents the ordered pair (0 , 4)
- The point O represents the ordered pair (0 , 0)

Second Representing the Cartesian product $\mathbb{Z} \times \mathbb{Z}$ $(\mathbb{Z}^2)$

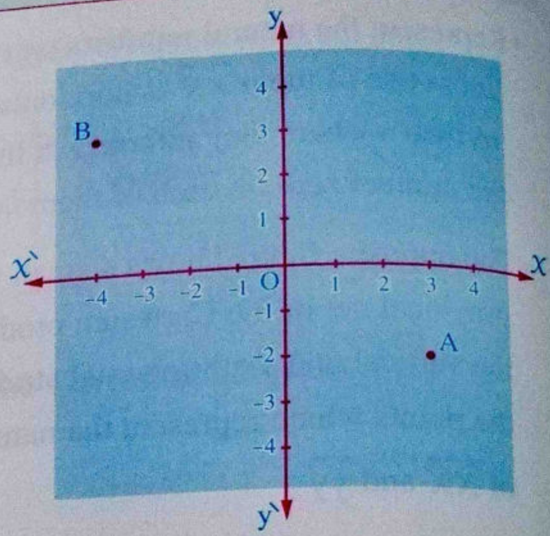


- Represent the integers on each of $\overleftrightarrow{XX}$ and $\overleftrightarrow{yy}$ which are intersecting at O (0 , 0)
- The opposite figure shows a small part of the perpendicular graphical net of the Cartesian product $\mathbb{Z} \times \mathbb{Z}$
- And each point of its points represents an ordered pair of the Cartesian product $\mathbb{Z} \times \mathbb{Z}$

For example:

- The point A represents the ordered pair (2 , 4)
- The point B represents the ordered pair (– 2 , 3)
- The point C represents the ordered pair (– 4 , – 2)
- The point D represents the ordered pair (4 , – 3)
- The point E represents the ordered pair (3 , 0)
- The point N represents the ordered pair (0 , – 2)

Third Representing the Cartesian product $\mathbb{R} \times \mathbb{R}$ $(\mathbb{R}^2)$



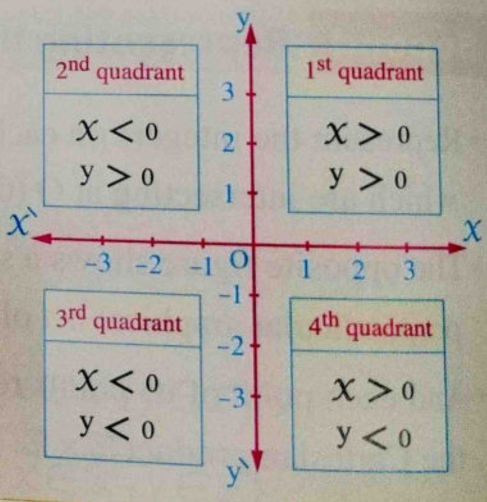
- The perpendicular graphical net of the Cartesian product $\mathbb{R} \times \mathbb{R}$ is an infinite extended surface from all sides and the opposite figure shows a small part of this region.
- Each point of this region represents an ordered pair of the Cartesian product $\mathbb{R} \times \mathbb{R}$

For example:

- The point A represents the ordered pair $(3 , -2)$
- The point B represents the ordered pair $(-4 , 3)$

! Remarks

1. The horizontal straight line $\overleftrightarrow{XX^{\backprime}}$ is called X-axis or the horizontal axis and the vertical straight line $\overleftrightarrow{yy^{\backprime}}$ is called y-axis or the vertical axis.
2. The point of intersection of the two axes $\overleftrightarrow{XX^{\backprime}}$ and $\overleftrightarrow{yy^{\backprime}}$ is called the origin point.
3. If the point A represents the ordered pair (X , y) in the Cartesian product $\mathbb{R} \times \mathbb{R}$, then :
   - The first projection X is called the X-coordinate of the point A
   - The second projection y is called the y-coordinate of the point A
4. The two axes $\overleftrightarrow{XX^{\backprime}}$ and $\overleftrightarrow{yy^{\backprime}}$ divide the plane into four quadrants as shown in the opposite figure and we can determine the quadrant in which any point lies by knowing the signs of its two coordinates.



5. If the X-coordinate of the point = 0 , then the point lies on y-axis.
6. If the y-coordinate of the point = 0 , then the point lies on X-axis.

Example 5 **Choose the correct answer from the given ones :**

1 The point $(4 , -3)$ lies on the .... quadrant.

(a) first (b) second (c) third (d) fourth

2 Which of the following points lies on the third quadrant ?

(a) $(2 , 5)$ (b) $(2 , -5)$ (c) $(-2 , 5)$ (d) $(-2 , -5)$

3 If the point (a , 3 – a) lies on the X-axis , then a = ...

(a) – 3 (b) 0 (c) 3 (d) 5

4 If b < 2 , then the point (b – 2 , 4) lies on the .... quadrant.

(a) first (b) second (c) third (d) fourth

5 If the point (X – 3 , 4 – X) where $X \in \mathbb{Z}$ lies on the fourth quadrant , then X = ...

(a) 2 (b) 3 (c) 4 (d) 5

Solution

1 **(d) The reason :** Because the X-coordinate is positive and the y-coordinate is negative.

2 **(d) The reason :** Because the X-coordinate and the y-coordinate of all the points on the third quadrant are negative.

3 **(c) The reason :** $\because$ (a , 3 – a) $\in \overleftrightarrow{XX}$

$\therefore$ 3 – a = 0 $\therefore$ a = 3

4 **(b) The reason :** $\because$ b < 2

$\therefore$ The X-coordinate of the point (b – 2 , 4) is negative and its y-coordinate is positive.

$\therefore$ (b – 2 , 4) lies on the second quadrant.

5 **(d) The reason :** Because at X = 5 , then (X – 3 , 4 – X) = (2 , – 1)

i.e. The X-coordinate is positive and the y-coordinate is negative.

TRY by yourself 5

Choose the correct answer from the given ones :

1 The point (– 2 , – 7) lies on the .... quadrant.

(a) first (b) second (c) third (d) fourth

2 If the point (b – 5 , b) lies on the y-axis , then b = ...

(a) – 5 (b) 0 (c) 1 (d) 5

3 If $\left(X - 2 , \sqrt{9}\right) = (-3 , y)$, then the point (y , X) lies on the .... quadrant.

(a) first (b) second (c) third (d) fourth

4 The point (X^2 , y^2) where $X \neq 0$, $y \neq 0$ lies on the .... quadrant.

(a) first (b) second (c) third (d) fourth

The Cartesian product of two intervals

We studied that the interval is a subset of the set of the real numbers ($\mathbb{R}$) and then the Cartesian product of two intervals is a subset of the Cartesian product $\mathbb{R} \times \mathbb{R}$ and we can explain that in the following example.

Example 6 If $X = [0, 3]$, $Y = [1, 3]$

, represent graphically using the perpendicular graphical net of the Cartesian product $\mathbb{R} \times \mathbb{R}$ the region which represents each of :

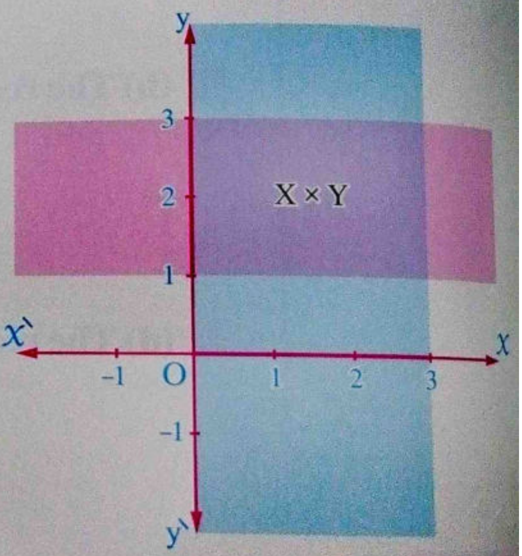
1 $X \times Y$ 2 $X \times X$ 3 $Y \times Y$

, then show , in each case , which of the following points belongs to the previous Cartesian products : $(2, 2)$, $(1, 0)$, $(0, 3)$

Solution

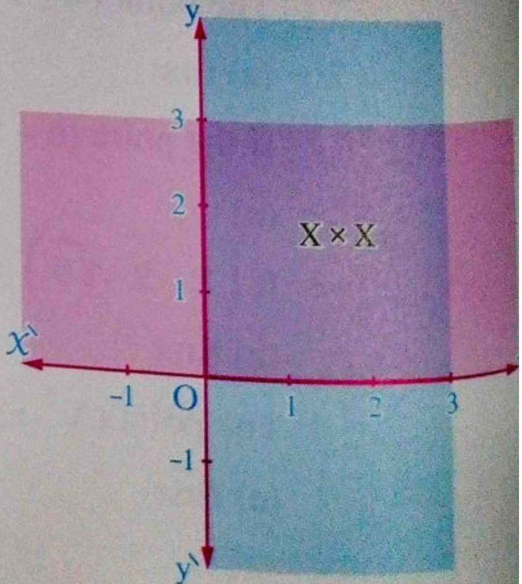
1 To represent $X \times Y$ graphically , do as follows :

- Represent the interval X on X-axis
- Represent the interval Y on y-axis
- The intersection region of the two colours represents $X \times Y$
- $(2, 2) \in X \times Y$ because it belongs to the region which represents $X \times Y$
- $(1, 0) \notin X \times Y$ because it lies outside the region which represents $X \times Y$
- $(0, 3) \in X \times Y$



2 To represent $X \times X$ graphically , do as follows :

- Represent the interval X one time on X-axis and another time on y-axis.
- The intersection region of the two colours represents $X \times X$
- $(2, 2) \in X \times X$, $(1, 0) \in X \times X$ and $(0, 3) \in X \times X$

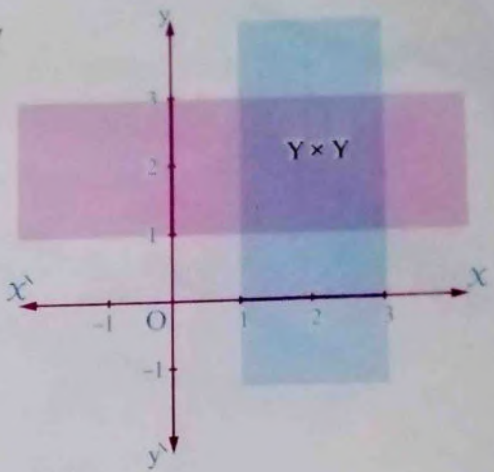


3 Similarly , you can represent Y × Y as shown in the opposite figure :

• (2 , 2) ∈ Y × Y

, (1 , 0) ∉ Y × Y

and (0 , 3) ∉ Y × Y



Lesson 2 Relation - Function (Mapping)

First The relation

The relation from set X to set Y is a connection that connects some or all the elements of set X with some or all the elements of set Y and it is denoted by "R"

- The relation R from X to Y is a set of ordered pairs whose first projection belongs to X and its second projection belongs to Y and the first projection is connected with the second projection by this relation.

  If $(a , b) \in R$ where $a \in X , b \in Y$

  So , we express this as "a R b"

- The relation R from set X to set Y is a subset of the Cartesian product $X \times Y$

  i.e. $R \subset X \times Y$

- The relation can be expressed by an arrow diagram or a Cartesian diagram (graphical).

Example 1 If $X = \{2, 5\}$, $Y = \{1, 4, 7\}$ and R is a relation from X to Y where "a R b" means "a < b" for every $a \in X$, $b \in Y$, state the relation R and represent it by an arrow diagram and by a Cartesian diagram.

Solution

$\because$ 2 is not less than 1 $\quad \therefore (2, 1) \notin R$

$\because 2 < 4 \quad \therefore (2, 4) \in R$

$\because 2 < 7 \quad \therefore (2, 7) \in R$

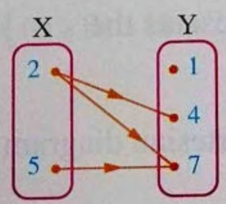
$\because$ 5 is not less than 1 $\quad \therefore (5, 1) \notin R$

$\because$ 5 is not less than 4 $\quad \therefore (5, 4) \notin R$

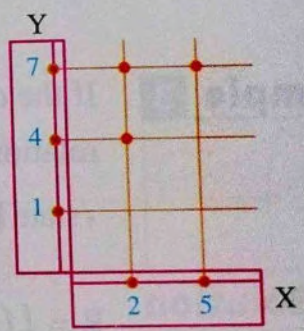
$\because 5 < 7 \quad \therefore (5, 7) \in R$

$\therefore$ The relation $R = \{(2, 4), (2, 7), (5, 7)\}$

The following figures represent the arrow diagram and the Cartesian diagram of this relation :



The arrow diagram



The Cartesian diagram

TRY by yourself 1

If $X = \{1, 2, 3\}$, $Y = \{3, 4, 5, 6\}$ and R is a relation from X to Y where "a R b" means "a + b = 6" for every $a \in X$ and $b \in Y$, state the relation R and represent it by an arrow diagram.

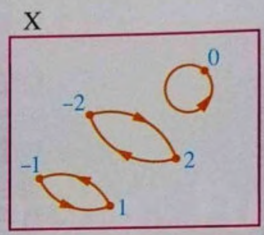
! Remark

If R is a relation from X to X , then : R is a relation on X and the relation $R \subset X \times X$

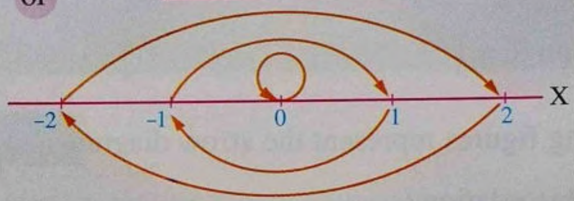
Example 2 If $X = \{-2, -1, 0, 1, 2\}$ and R is a relation on X where "a R b" means "a is the additive inverse of the number b" for every $a \in X$ and $b \in X$, state R, then represent it by an arrow diagram and a Cartesian diagram.

Solution $R = \{(-2, 2), (-1, 1), (0, 0), (1, -1), (2, -2)\}$

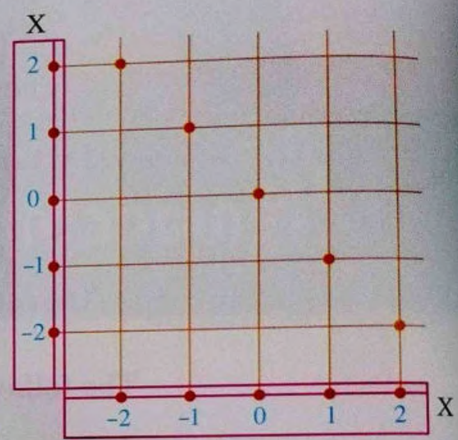
- **The arrow diagram :**



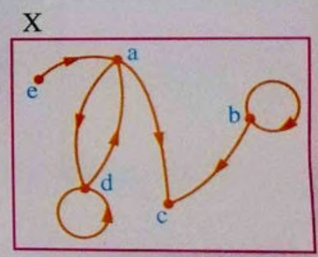
or



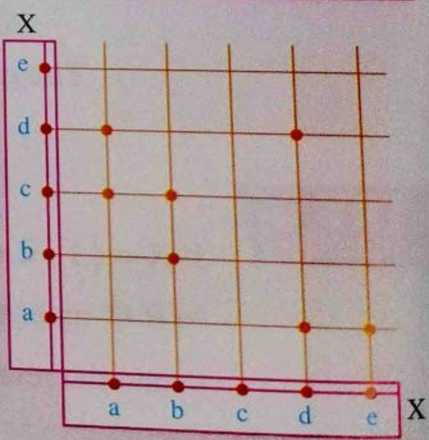
- **The Cartesian diagram :**



Example 3 If the opposite arrow diagram represents the relation R on X, state R, then represent it by a Cartesian diagram.



Solution $R = \{(a, c), (a, d), (b, b), (b, c), (d, d), (d, a), (e, a)\}$



TRY by yourself 2

If $X = \{1, 2, 4\}$ and R is a relation on X where "a R b" means "a is twice b" for every $a \in X$ and $b \in X$, state R and represent it by a Cartesian diagram.

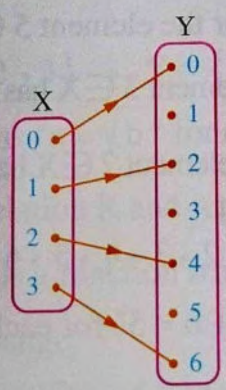
Second Function (Mapping)

A relation from X to Y is said to be a function (mapping) if one of the following cases is satisfied :

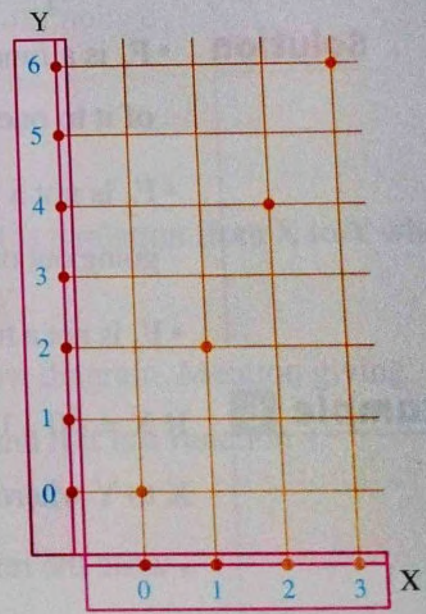
1. In the relation , **each element** of the set X appears **only once** as a first projection in one of the ordered pairs of **the relation**.
2. In the arrow diagram which represents the relation , **each element** of X has **one and only one arrow** going out of it to one element of Y
3. In the Cartesian diagram which represents the relation , **each vertical line** has **one and only one point** lying on it of the points which represent the relation.

Example 4 If $X = \{0 , 1 , 2 , 3\}$, $Y = \{0 , 1 , 2 , 3 , 4 , 5 , 6\}$ and R is a relation from X to Y where "a R b" means "$a = \frac{1}{2}$ b" for each $a \in X$, $b \in Y$, write R and represent it by an arrow diagram and a Cartesian diagram. Is R a function or not ? If R is a function write its range.

Solution $R = \{(0 , 0) , (1 , 2) , (2 , 4) , (3 , 6)\}$



The arrow diagram



The Cartesian diagram

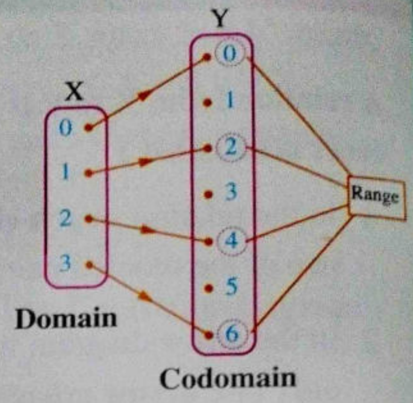
Each element of the set X has been connected with **one and only one element** of the elements of the set Y

So , the relation R is called **a function** , its range $= \{0 , 2 , 4 , 6\}$

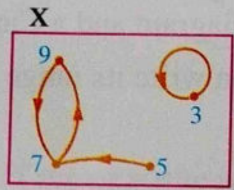
Notice that :

From the previouse example

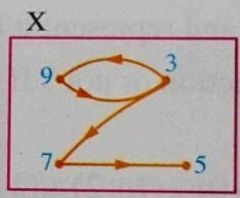
- The set $X = \{0, 1, 2, 3\}$ is called "the domain of the function".
- The set $Y = \{0, 1, 2, 3, 4, 5, 6\}$ is called "the codomain of the function".
- The set $\{0, 2, 4, 6\}$ is called "the range of the function" and it is a subset from the codomain of the function.



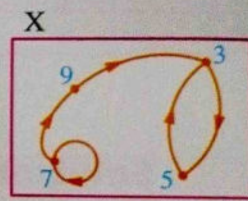
Example 5 If $X = \{3, 5, 7, 9\}$, show which of the following arrow diagrams represents a function on X (i.e. from X to X) and if it is a function, mention its range :



F_1



F_2



F_3

Solution

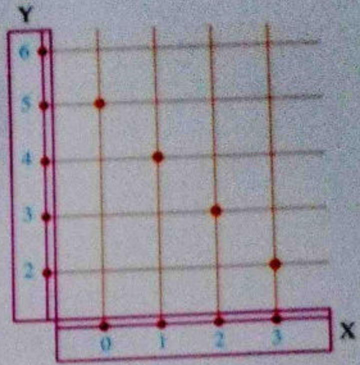
- F_1 is a function because each element of X has only one arrow going out of it to one element of X, the range of the function F_1 is $\{3, 7, 9\}$
- F_2 is not a function because for the element $5 \in X$ there are no arrows going out of it or because the element $3 \in X$ has two arrows going out of it.
- F_3 is not a function because the element $7 \in X$ has two arrows going out of it.

Example 6 If $X = \{0, 1, 2, 3\}$, $Y = \{2, 3, 4, 5, 6\}$ and R is a relation from X to Y where "a R b" means "$a + b = 5$" for each $a \in X$, $b \in Y$, write the relation R and represent it by a Cartesian diagram. Mention giving reasons if R is a function from X to Y or not. And if it is a function, find its range.

Solution

- $R = \{(0, 5), (1, 4), (2, 3), (3, 2)\}$
- R represents a function from X to Y because each element of X is connected with only one element of Y

The range of the function $= \{5, 4, 3, 2\}$

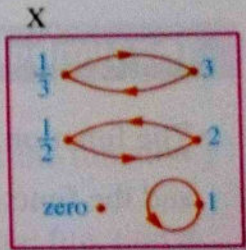


Example 7

If $X = \{3, 2, 1, \text{zero}, \frac{1}{2}, \frac{1}{3}\}$ and R is a relation on X where "a R b" means "a is the multiplicative inverse of b" for each $a \in X$, $b \in X$, write R and represent it by an arrow diagram and mention giving reasons if R represents a function or not.

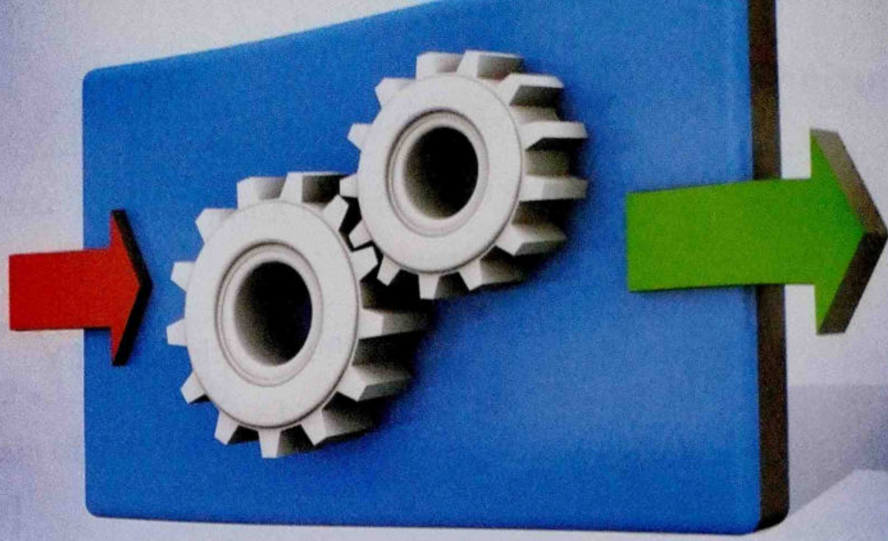
Solution

- $R = \{(3, \frac{1}{3}), (2, \frac{1}{2}), (1, 1), (\frac{1}{2}, 2), (\frac{1}{3}, 3)\}$
- R does not represent a function because the element zero $\in X$ is not connected with any element in X (There is no arrow going out from zero in the arrow diagram which represents the relation)



TRY by yourself 3

If $X = \{1, 2, 3\}$, $Y = \{1, 4, 6, 9\}$ and R is a relation from X to Y where "a R b" means "$a = \sqrt{b}$" for each $a \in X$, $b \in Y$, write the relation R and represent it by an arrow diagram. Mention giving reasons if R is a function from X to Y or not, and if it is a function, mention its range.



Lesson 3 The symbolic representation of the function - Polynomial functions

The symbolic representation of the function

- The function is usually denoted by one of the letters f or g or k or ...

and the function f from the set X to the set Y is written mathematically as :

$f : X \longrightarrow Y$ and is read as f is a function from X to Y

or $g : X \longrightarrow Y$ and is read as g is a function from X to Y and so on ...

- If the ordered pair (x , y) belongs to the function , then the element y is called the image of the element x by the function f and we express it by one of the following two forms :

$f : x \longmapsto y$ and it is read as f maps x to y

or $f : f(x) = y$ and it is read as f is a function where $f(x) = y$

For example:

If $f : X \longrightarrow Y$ where $f : x \longmapsto x^2$, then $f : 3 \longmapsto 9$

, also can be written in the form : $f(x) = x^2$, hence $f(3) = 9$

! Remark

The mathematical form $f(x) = x^2$ is called the rule of the function f, and it is used to find the image of any element of the domain by the function f

Remember that

If f is a function from the set X to the set Y i.e. $f : X \longrightarrow Y$, then :

1. X is called the **domain** of the function f
2. Y is called the **codomain** of the function f
3. The set of images of the elements of the set X by the function f is called the **range** of the function f which is a subset of the codomain Y

Example 1 If $X = \{-1 , 0 , 1\}$, $Y = \{0 , -1 , -2\}$ and the function $f : X \longrightarrow Y$ where $f(X) = X^2 - 1$, find the set of the function f and represent it by an arrow diagram , then write its range.

Solution

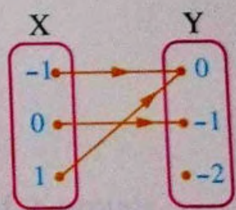
$\because f(X) = X^2 - 1$

$\therefore f(-1) = (-1)^2 - 1 = 0$ $\therefore (-1 , 0) \in$ the set of the function f

$, f(0) = (0)^2 - 1 = -1$ $\therefore (0 , -1) \in$ the set of the function f

$, f(1) = (1)^2 - 1 = 0$ $\therefore (1 , 0) \in$ the set of the function f

$\therefore$ The set of the function $f = \{(-1 , 0) , (0 , -1) , (1 , 0)\}$



The range of the function $f = \{0 , -1\}$

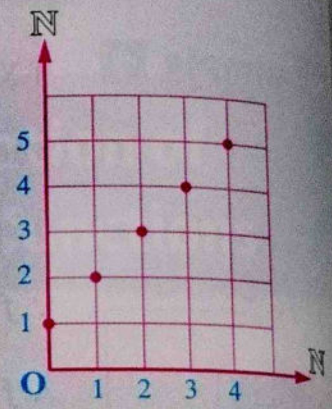
Remark

If f is a function from the set X to itself : i.e. $f : X \longrightarrow X$,
then we say «f is a function on X»

Example 2 If $f : \mathbb{N} \longrightarrow \mathbb{N}$ where $\mathbb{N}$ is the set of natural numbers and $f(X) = X + 1$ find $f(0)$, $f(1)$, $f(2)$, $f(3)$ and $f(4)$, then graph a part of the square net of the Cartesian product $\mathbb{N} \times \mathbb{N}$ and represent on it five elements of this function. What is the range of the function f ?

Solution $f(X) = X + 1$ for each $X \in \mathbb{N}$

means that the image of any natural number by the function f is "the number + 1"



$\therefore f(0) = 0 + 1 = 1$

, $f(1) = 2$

, $f(2) = 3$

, $f(3) = 4$

, $f(4) = 5$

$\therefore (0 , 1) , (1 , 2) , (2 , 3) , (3 , 4) , (4 , 5)$

are five elements of f

- The range of f is all the natural numbers except zero. (because there is no natural number added 1 gives zero)

i.e. The range of $f = \mathbb{N} - \{0\}$

TRY by yourself 1

If $X = \{2 , 4 , 6 , 8\}$

, $Y = \{1 , 2 , 3 , 4 , 5 , 6\}$

and the function $f : X \longrightarrow Y$ where $f(X) = \frac{1}{2} X$

, write the set of the function f and represent it by a Cartesian diagram , then find its range.

Polynomial functions

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = a_0 + a_1 X + a_2 X^2 + \cdots + a_n X^n$

where $a_0 , a_1 , a_2 , \cdots , a_n \in \mathbb{R}$, $n \in \mathbb{N}$ is called a polynomial function.

i.e. **The polynomial function is a function whose rule is a term or an algebraic expression in condition that the following should be identified :**

1. Each of the domain and the codomain of the function is the set of real numbers.
2. The power (the index) of the variable X in any of its terms is a natural number.

For example: The following functions are all polynomial functions :

- $f : f(X) = 2X + 5$
- $k : k(X) = 8$
- $g : g(X) = X^2 - 2X + 1$
- $n : n(X) = 1 + \sqrt{2}\,X - 9X^3$

Remark

If the domain or the codomain of a function is not the set of real numbers , then that function is not a polynomial function.

For example :

- $f : f(X) = \sqrt{X}$ is not a polynomial function because $f(X)$ doesn't exist in $\mathbb{R}$ if X equals a negative number.

  For example : $f(-1) \notin \mathbb{R}$ because $\sqrt{-1} \notin \mathbb{R}$

  , so the domain of the function f is not the set of real numbers.

- $h : h(X) = \frac{1}{X}$ is not a polynomial function

  because $h(X)$ doesn't exist in $\mathbb{R}$ if X equals zero. i.e. $h(0) \notin \mathbb{R}$

  , so the domain of the function h is not the set of real numbers.

! Remark

When we search if the function is a polynomial or not , we do not simplify its rule.

For example:

The function $f_1 : f_1(x) = x\left(x + \frac{1}{x}\right)$ doesn't represent a polynomial function because $f_1(0) \notin \mathbb{R}$ while the function $f_2 : f_2(x) = x^2 + 1$ represents a polynomial function.

And notice that: $x\left(x + \frac{1}{x}\right) = x^2 + 1$ for all real numbers except 0

TRY by yourself 2 ***Which of the functions defined by the following rules represents a polynomial function :***

1. $f_1(x) = x(x^2 - 3)$
2. $f_2(x) = x\left(\frac{2}{x} + 5\right)$
3. $f_3(x) = x^2 - \sqrt{x} + 1$
4. $f_4(x) = x^2 - (x^2 - 4)$

The degree of the polynomial function

The degree of the polynomial function is the highest power of the variable in the function rule.

For example:

- The function $f_1 : f_1(x) = 3x - \frac{1}{2}$ is of the first degree (a linear function)
- The function $f_2 : f_2(x) = \sqrt{5}\,x^2 - 3x + 4$ is of the second degree (a quadratic function)
- The function $f_3 : f_3(x) = x^3 - 5x^2 + 4$ is of the third degree (a cubic function)

! Remarks

- The function $f : f(x) = a$ where $a \in \mathbb{R} - \{0\}$ is a polynomial function of zero degree (a constant function) as $f : f(x) = 3$

  In the case of $a = 0$ **i.e.** When $f(x) = 0$, then the function f has no degree.
- When you want to determine the degree of the function you should simplify its rule to the simplest form before telling its degree.

Example 3 **Choose the correct answer from the given ones :**

1 The function $f : f(X) = X^2 (2 + X)^2$ is a polynomial function of the .... degree.

(a) first (b) second (c) third (d) fourth

2 The function $f : f(X) = X^2 - (X - 5)^2$ is a polynomial function of the .... degree.

(a) zero (b) first (c) second (d) fourth

3 The function $f : f(X) = X^4 - (X^2 + 1)(X^2 - 1)$ is a polynomial function of the .... degree.

(a) zero (b) first (c) second (d) fourth

4 If $f(X) = X^2 - X - 2$, then $f(-3) =$ ....

(a) -3 (b) 4 (c) 10 (d) 14

5 If $f(X) = X^2 - 2X + 5$, then $f(0) =$ ....

(a) 2 (b) 4 (c) 5 (d) 7

6 If $f(X) = X^2 - \sqrt{3}\, X$, then $f\left(-\sqrt{3}\right) =$ ....

(a) 0 (b) 3 (c) 6 (d) $2\sqrt{3}$

7 If $f(X) = X^3$, then $f(3) + f(-3) =$ ....

(a) 54 (b) 27 (c) 6 (d) 0

8 If $f(X) = aX - 6$, $f(2) = 0$, then a = ...

(a) -6 (b) -3 (c) 3 (d) 0

Solution

1 **(d) The reason :** $\because f(X) = X^2 (4 + 4X + X^2) = 4X^2 + 4X^3 + X^4$

$\therefore f$ is a function of the fourth degree.

2 **(b) The reason :** $\because f(X) = X^2 - (X^2 - 10X + 25) = X^2 - X^2 + 10X - 25$

$= 10X - 25$

$\therefore f$ is a function of the first degree.

3 **(a) The reason :** $\because f(X) = X^4 - (X^4 - 1) = X^4 - X^4 + 1 = 1$

$\therefore f$ is a function of the zero degree.

4 **(c) The reason :** Substituting by $X = -3$ at the function rule

$\therefore f(-3) = (-3)^2 - (-3) - 2 = 9 + 3 - 2 = 10$

5 (c) The reason : Substituting by $X = 0$ at the function rule

$$\therefore f(0) = 0^2 - 2(0) + 5 = 0 - 0 + 5 = 5$$

6 (c) The reason : Substituting by $X = -\sqrt{3}$ at the function rule

$$\therefore f\left(-\sqrt{3}\right) = \left(-\sqrt{3}\right)^2 - \left(\sqrt{3}\right)\left(-\sqrt{3}\right) = 3 + 3 = 6$$

7 (d) The reason : $\because f(3) = 3^3 = 27$, $f(-3) = (-3)^3 = -27$

$$\therefore f(3) + f(-3) = 27 + (-27) = 0$$

8 (c) The reason : $\because f(2) = 0$ $\quad \therefore a \times 2 - 6 = 0$

$\therefore 2a = 6$ $\quad \therefore a = 3$

TRY by yourself 3

Choose the correct answer from the given ones :

1 The function $f : f(X) = X(X^3 - 2)$ is a polynomial function of the .... degree.

(a) first (b) second (c) third (d) fourth

2 If $f(X) = 3 - 5X$, then $f(-2) =$ ....

(a) 1 (b) 5 (c) 7 (d) 13

3 If $f(X) = X^2 + X - 1$, then $f(1) + f(-1) =$ ....

(a) -2 (b) 0 (c) 2 (d) 3

4 If $f(X) = 4X + k$, $f(2) = 15$, then $k =$ ....

(a) 2 (b) 4 (c) 7 (d) 15

Example 4 If $f(X) = X^2 - 2X + 5$

, **prove that :** $f\left(2\sqrt{2} + 1\right) = 2f\left(1 - \sqrt{2}\right)$

Solution

$$\because f\left(2\sqrt{2} + 1\right) = \left(2\sqrt{2} + 1\right)^2 - 2\left(2\sqrt{2} + 1\right) + 5$$

$$= 8 + 1 + 4\sqrt{2} - 4\sqrt{2} - 2 + 5 = 12$$

$$, f\left(1 - \sqrt{2}\right) = \left(1 - \sqrt{2}\right)^2 - 2\left(1 - \sqrt{2}\right) + 5$$

$$= 1 + 2 - 2\sqrt{2} - 2 + 2\sqrt{2} + 5 = 6$$

From (1) and (2) : $\therefore f\left(2\sqrt{2} + 1\right) = 2f\left(1 - \sqrt{2}\right)$

Example 5 If $f(X) = 2X + b$ and $g(X) = X^2 + b$ and if $f(2) + g(-4) = 30$,

then find : $f(-2) - g(2)$

Solution

$\because f(2) = 2 \times 2 + b = 4 + b$, $g(-4) = (-4)^2 + b = 16 + b$

, $\because f(2) + g(-4) = 30$ $\qquad \therefore 4 + b + 16 + b = 30$

$\therefore 20 + 2b = 30$ $\qquad \therefore 2b = 30 - 20 = 10$

$\therefore b = \frac{10}{2} = 5$

$\boxed{\therefore f(X) = 2X + 5}$, $\boxed{g(X) = X^2 + 5}$

$\therefore f(-2) = 2 \times (-2) + 5 = 1$, $g(2) = 2^2 + 5 = 9$

$\therefore f(-2) - g(2) = 1 - 9 = -8$

TRY by yourself 4 If $f(X) = 2X + 5$ and $g(X) = X - 6$, *then prove that :* $f(2) + 3g(3) = 0$

Lesson **4**

The study of some polynomial functions

First | The linear function

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$, $a \in \mathbb{R} - \{0\}$, $b \in \mathbb{R}$ is called a linear function (it is a polynomial function of the first degree).

Examples of linear functions :

- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = X - 1$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 2X + 1$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 3X$

Notice that :

In each of the shown functions , the index of X is 1 , therefore each of them is a function of the first degree.

The graphical representation of the linear function

- The linear function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$, $a \in \mathbb{R} - \{0\}$, $b \in \mathbb{R}$ is represented graphically by a straight line intersecting :

  – The y-axis at the point $(0 , b)$ – The X-axis at the point $\left(\frac{-b}{a} , 0\right)$
- To represent a linear function , it is enough to find two ordered pairs belonging to the function.
- You can find a third ordered pair to check that the three points are on the same straight line.

Example 1 **Graph each of the following linear functions :**

1 $f : f(X) = 2X - 3$

2 $r : r(X) = -\frac{1}{2}X$

Solution 1 Determine three ordered pairs belonging to the function.

$\because f(x) = 2x - 3$

$\therefore f(-1) = 2(-1) - 3 = -5$ $\quad \therefore (-1, -5) \in f$

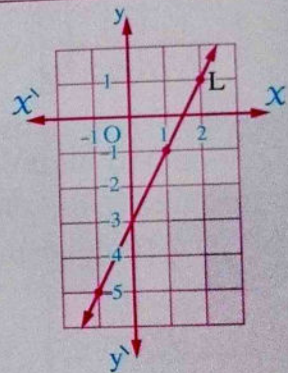
$, f(1) = 2 \times 1 - 3 = -1$ $\quad \therefore (1, -1) \in f$

and $f(2) = 2 \times 2 - 3 = 1$ $\quad \therefore (2, 1) \in f$

You can arrange these ordered pairs in the opposite table :

x	-1	1	2
$y = f(x)$	-5	-1	1

Locate these three points which represents the three ordered pairs in the Cartesian plane and draw the straight line L which passes through any two points of them.
Then check that the third point lies on the same straight line.
Then this straight line is the graphical representation of this function.



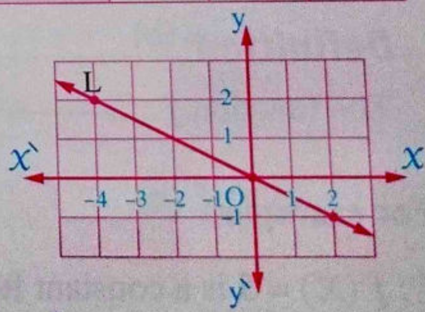
Notice that :

- The point of intersection with y-axis = $(0, b) = (0, -3)$
- The point of intersection with x-axis = $\left(-\frac{b}{a}, 0\right) = \left(\frac{3}{2}, 0\right)$

2 $\because r(x) = -\frac{1}{2}x$ $\quad \therefore$

x	0	2	-4
$y = r(x)$	0	-1	2

From the opposite graph notice that , the straight line L passes through the origin point O $(0, 0)$



Generally

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = ax$, $a \in \mathbb{R}^*$
is represented graphically by a straight line passing through the origin point $(0, 0)$

TRY by yourself 1 ***Represent graphically each of the following linear functions :***

1 $f : f(x) = 3x - 3$

2 $f : f(x) = 2x$

Example 2

1. If the point (a , – a) lies on the straight line representing the function $f : f(X) = X - 6$, find the value of a

2. If the straight line representing the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$ intersects the y-axis at (0 , 3) and $f(2) = 7$, find the value of each of a , b

Solution

1. $\because$ (a , – a) lies on the straight line representing the function f

$\therefore$ (a , – a) satisfies the function

$\therefore a - 6 = -a$ $\quad\therefore 2a = 6$ $\quad\therefore \boxed{a = 3}$

2. $\because$ The straight line intersects the y-axis at (0 , 3)

$\therefore$ (0 , 3) satisfies the function $\quad\therefore 3 = a \times 0 + b$

$\therefore \boxed{b = 3}$ $\quad , \because f(2) = 7$ $\quad\therefore 7 = 2a + 3$

$\therefore 2a = 4$ $\quad\therefore \boxed{a = 2}$

TRY by yourself 2

If the straight line representing the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 4X - a$ intersects the X-axis at (2 , b) , find the value of each of a , b

Second The constant function

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = b$, $b \in \mathbb{R}$ is called a constant function.

For example:

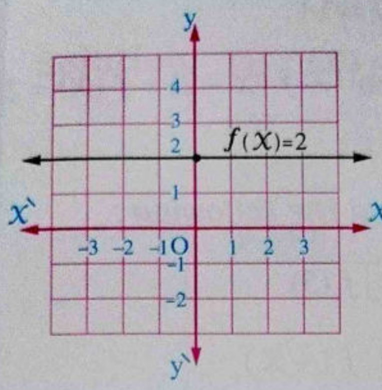
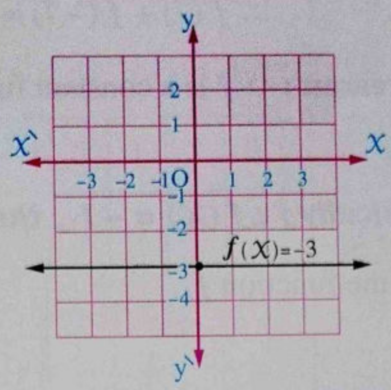
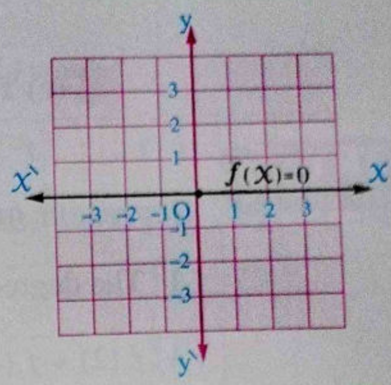
$f : f(X) = 5$ is a constant function where $f(1) = 5$, $f(0) = 5$, $f(-2) = 5$, ... and so on.

The graphical representation of the constant function

The constant function $f : f(X) = b$ (where $b \in \mathbb{R}$) is represented by a straight line parallel to X-axis and passing through the point (0 , b) **and this line is :**

- **above** X-axis if $b > 0$
- **below** X-axis if $b < 0$
- **coincident** with X-axis if $b = 0$

The following examples illustrate that :

$f : f(X) = 2$	$f : f(X) = -3$	$f : f(X) = 0$
		
The straight line is above X-axis and passes through $(0 , 2)$	The straight line is below X-axis and passes through $(0 , -3)$	The straight line is coincident with X-axis and passes through $(0 , 0)$

Example 3 **Choose the correct answer from the given ones :**

1 The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = -3$ is represented by a straight line intersecting y-axis at the point ....

(a) $(-3 , 0)$ (b) $(0 , -3)$ (c) $(3 , 0)$ (d) $(0 , 3)$

2 If $f(X) = 4$, then $f(2)$ .... $f(3)$

(a) $<$ (b) $>$ (c) $=$ (d) $\neq$

3 If $f(X) = 5$, then $2 f(3) =$ ....

(a) 6 (b) $f(6)$ (c) 10 (d) $3 f(2)$

4 If $f(X) = 7$, then $f(7) + f(-7) =$ ....

(a) -14 (b) -7 (c) 7 (d) 14

5 If $f(X) = 2$, then $f(X - 2) =$ ....

(a) -2 (b) 0 (c) 2 (d) 4

Solution

1 (b)

2 (c) **The reason :** $\because f$ is a constant function $\therefore f(2) = f(3) = 4$

3 (c) **The reason :** $\because f$ is a constant function $\therefore 2 f(3) = 2 \times 5 = 10$

4 (d) **The reason :** $\because f$ is a constant function

$\therefore f(7) + f(-7) = 7 + 7 = 14$

5 (c) **The reason :** $\because f$ is a constant function $\therefore f(x - 2) = f(x) = 2$

TRY by yourself 3

Represent graphically $f : f(x) = -1$, then find the following :

1 The degree of the function f

2 $f(5)$

3 $f(2) + f(-2)$

4 $f(-x)$

Third The quadratic function

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = a x^2 + b x + c$

where a , b and c are real numbers , $a \neq 0$

is called a quadratic function (it is a polynomial function of the second degree).

Examples of quadratic functions :

- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = x^2$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = x^2 - 2$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = 3 x^2 - 7 x + 2$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = 6 - x^2 + x$

Notice that :

In each of the shown functions , the highest index of x is 2 , therefore each of them is a function of the 2^{nd} degree.

The graphical representation of the quadratic function

We know that the domain of the quadratic function is the set of real numbers $\mathbb{R}$ which is an infinite set. So , to represent this function graphically , we should represent it on a certain interval by determining some of ordered pairs which belong to the function. Then we draw the curve (paved curve) passing through the points which represent these ordered pairs.

The following examples illustrate that :

Example 4 **Graph each of the following quadratic functions :**

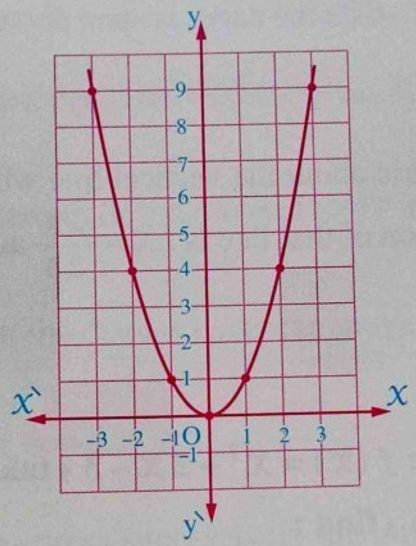
1 $f : f(x) = x^2$, taking $x \in [-3 , 3]$

2 $f : f(x) = -x^2$, taking $x \in [-3 , 3]$

Solution

1 $f(x) = x^2$

x	-3	-2	-1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9



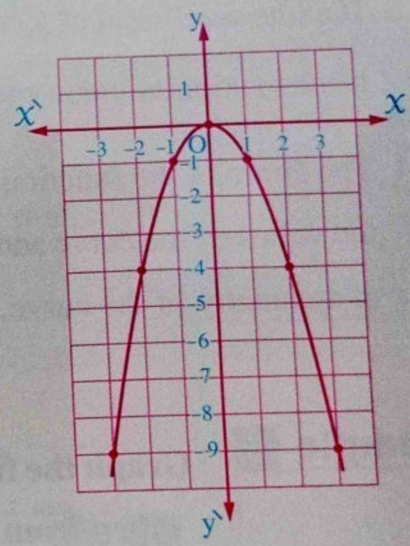
Notice that :

The coefficient of $x^2 > 0$

- The point (0 , 0) is the point of the vertex of the curve , it is considered as **a minimum value** point of the curve because the whole curve **lies up on it.**
- **The minimum** value of the function is zero which is the y-coordinate of the vertex of the curve.
- The curve is symmetric about y-axis
  i.e. The y-axis is the line of symmetry of the curve and its equation is $x = 0$

2 $f(x) = -x^2$

x	-3	-2	-1	0	1	2	3
$f(x)$	-9	-4	-1	0	-1	-4	-9



Notice that :

The coefficient of $x^2 < 0$

- The point (0 , 0) is the point of the vertex of the curve , it is considered as **a maximum value** point of the curve because the whole curve **lies below it.**
- **The maximum** value of the function is zero which is the y-coordinate of the vertex of the curve.
- The curve is symmetric about y-axis
  i.e. The y-axis is the line of symmetry of the curve and its equation is $x = 0$

Generally

The quadratic function $f : f(X) = aX^2 + bX + c$ where a , b and c are real numbers , $a \neq 0$ has the following properties :

1. The vertex of the curve $= \left(\frac{-b}{2a} , f\left(\frac{-b}{2a}\right)\right)$
2. If a (the coefficient of X^2) is positive , then the curve is open upwards and the function has a minimum value equals $f\left(\frac{-b}{2a}\right)$
3. If a (the coefficient of X^2) is negative , then the curve is open downwards and the function has a maximum value equals $f\left(\frac{-b}{2a}\right)$
4. The curve of the function is symmetric about the vertical line which passes through the vertex of the curve and the equation of that line is : $X = \frac{-b}{2a}$ and it is called the axis of symmetry of the curve.

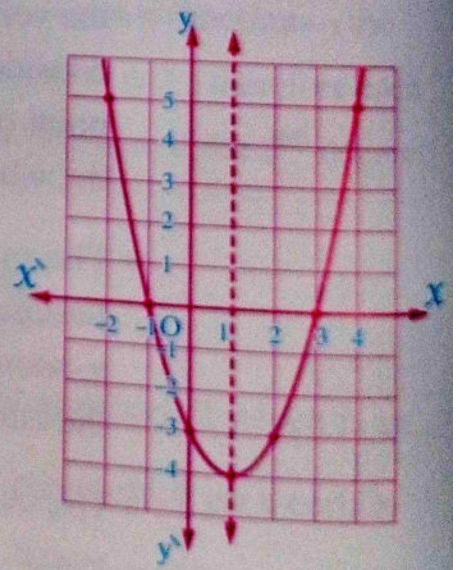
Example 5 **Graph the function $f : f(X) = X^2 - 2X - 3$, taking $X \in [-2 , 4]$, then from the graph , find :**

1. The point of the vertex of the curve.
2. The equation of the line of symmetry.
3. The maximum or minimum value of the function.

Solution

$f(X) = X^2 - 2X - 3$

X	-2	-1	0	1	2	3	4
$f(X)$	5	0	-3	-4	-3	0	5



From the graph , we deduce that :

1. The vertex of the curve is (1 , −4)
2. The equation of the line of symmetry is $X = 1$, it is a straight line parallel to y-axis and passing through the vertex of the curve.
3. The minimum value of the function = −4

! Remark

We can form the table used in graphing the function $f : f(x) = x^2 - 2x - 3$ where $x \in [-2, 4]$ by using the scientific calculator which supports (Table) as follows :

1. Turn the calculator on (Table) as follows : Press MODE SETUP , then choose TABLE
2. Input data : Write the rule of the previous function, press successively the following buttons : Start → ALPHA) x^2 − 2 ALPHA) − 3
3. Press the button = , then at the beginning of the interval START? write (−) 2 , then press =
4. At the end of the interval END? write the number 4 , then press =
5. To determine the length of the interval STEP? write 1 , then press =

	x	$F(x)$
1	−2	5
2	−1	0
3	0	−3
4	1	−4
5	2	−3
6	3	0
7	4	5

The table is formed in the display, you can move by using button REPLAY up or down.

- To exit the program, press successively the buttons : Start → MODE SETUP 1

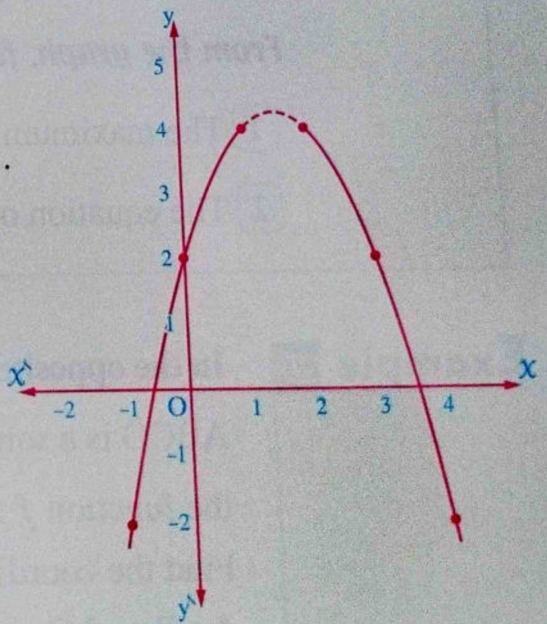
Example 6

Graph the function $f : f(x) = -x^2 + 3x + 2$, taking $x \in [-1, 4]$, then find :

1. The maximum value or minimum value of the function.
2. The equation of the line of symmetry.

Solution

x	−1	0	1	2	3	4
$f(x)$	−2	2	4	4	2	−2



When we represent these ordered pairs, we notice that the point of the vertex of the curve is not among these points which makes the drawing of the dotted part in the opposite figure is inaccurate, so the studying of the curve will be difficult, then we should find the vertex point of the curve algebraically as the following :

Finding the vertex point

At the point of the vertex of the curve of the quadratic function , it will be :

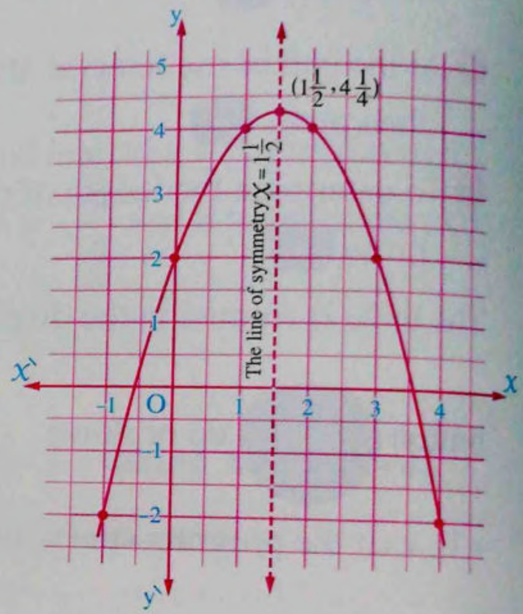
- The X-coordinate $= \frac{-b}{2a}$
- The y-coordinate $= f\left(\frac{-b}{2a}\right)$

where b is the coefficient of X , a is the coefficient of X^2

$\therefore X$ at the vertex of the curve $= \frac{-3}{2 \times -1} = \frac{-3}{-2} = 1\frac{1}{2}$

, $\because f\left(1\frac{1}{2}\right) = -\frac{9}{4} + \frac{9}{2} + 2 = 4\frac{1}{4}$

$\therefore$ The vertex of the curve is $\left(1\frac{1}{2}, 4\frac{1}{4}\right)$



From the vertex of the curve ,

we find that :

1. The maximum value $= 4\frac{1}{4}$
2. The equation of the line of symmetry is $X = 1\frac{1}{2}$

TRY by yourself 4

Graph the curve of the function $f : f(X) = X^2 + 2X - 3$ on the interval $[-4, 2]$

From the graph, find :

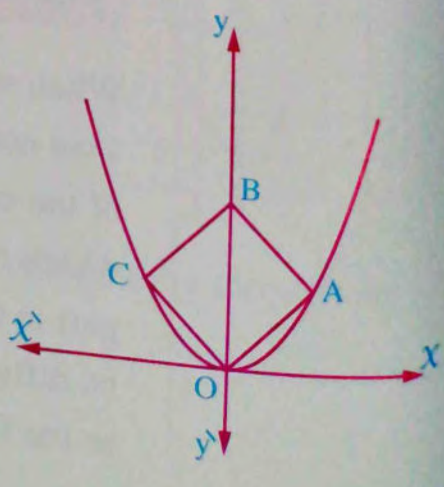
1. The maximum or minimum value of the function.
2. The equation of the line of symmetry.

Example 7

In the opposite figure :

ABCO is a square and the curve represents the function $f : f(X) = X^2$

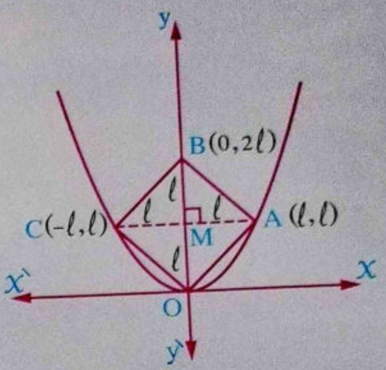
Find the coordinates of the points : A , B and C



Solution

Draw the square diagonal $\overline{AC}$ to intersect the another diagonal $\overline{BO}$ at the point M

$\because$ The two diagonals of the square are equal in length and bisect each other.



$\therefore$ MA = MB = MC = MO and let : MA = ℓ

$\therefore$ MA = MB = MC = MO = ℓ

$\therefore A(\ell, \ell), C(-\ell, \ell), B(0, 2\ell)$

$\because A(\ell, \ell) \in$ the function $f : f(X) = X^2$

By substituting in the rule of the function

$\therefore \ell = \ell^2 \qquad \therefore \ell^2 - \ell = 0 \qquad \therefore \ell(\ell - 1) = 0$

$\therefore \ell = 0$ (refused) $\qquad$ or $\ell - 1 = 0 \qquad \therefore \ell = 1$

$\therefore A(1, 1), B(0, 2)$ and $C(-1, 1)$

UNIT TWO

Ratio, proportion, direct variation and inverse variation

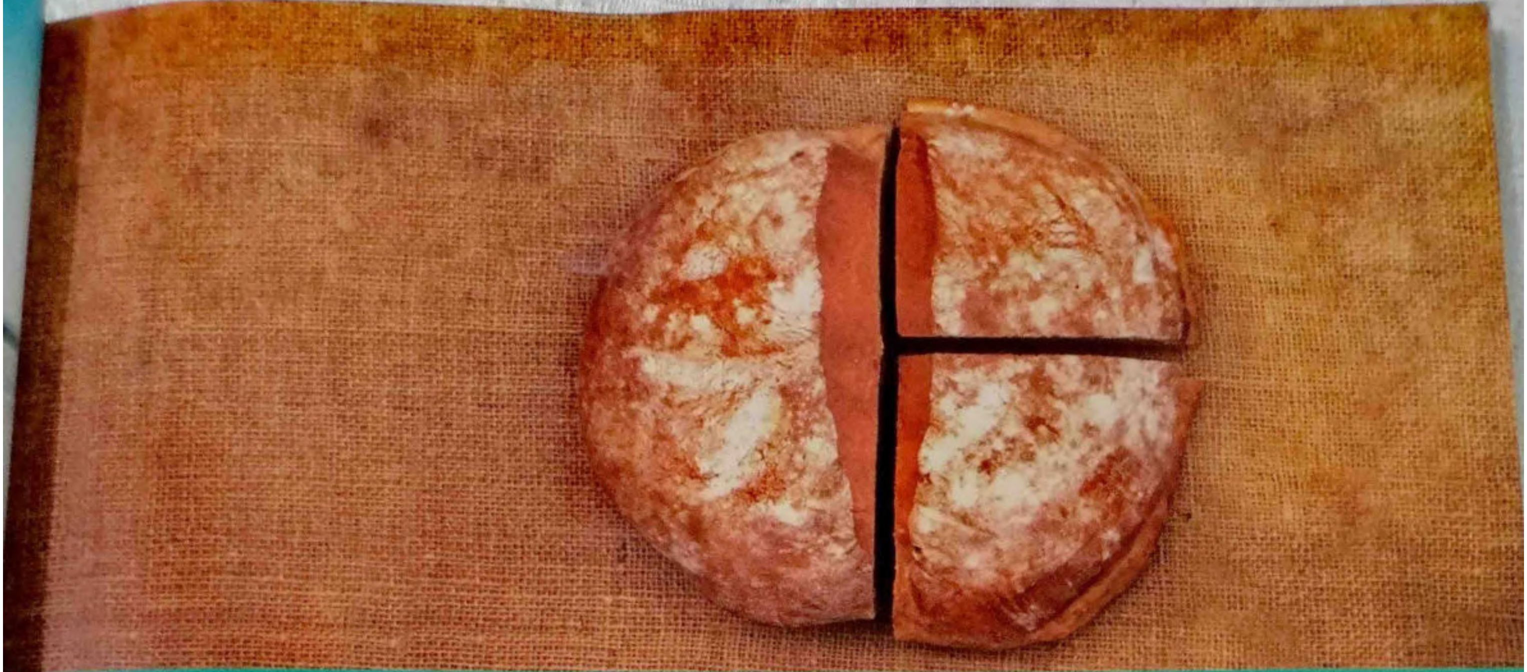
Use your smart phone or tablet to scan the **QR code** and enjoy watching videos

Lessons of the unit :

1. Ratio and proportion.
2. Follow properties of proportion.
3. Continued proportion.
4. Direct variation and inverse variation.

Unit Objectives : By the end of this unit, student should be able to :

- recognize the concept of the ratio.
- recognize the properties of the ratio.
- recognize the concept of the proportion.
- recognize the properties of the proportion.
- recognize the concept of the continued proportion.
- use the properties of the ratio and the proportion for solving a lot of problems.
- recognize the concept of the direct variation.
- recognize the concept of the inverse variation.
- differentiate between the direct variation and the inverse variation.
- solve real life problems on the direct variation and the inverse variation.
- appreciate the role of mathematics in solving a lot of real life problems.



Lesson **1**

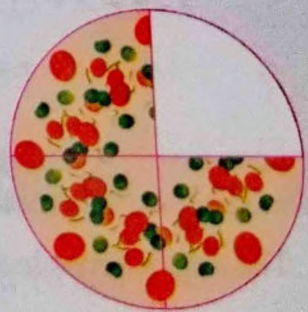
Ratio and proportion

First Ratio

We have studied in the primary stage that the ratio is one of methods of comparison between two quantities.

For example:

If a pie is divided into four equal parts and Hany ate one part only of it , then :



- The ratio of what Hany ate to the whole pie is 1 : 4

  and it may written as $\frac{1}{4}$
- The ratio of what was left of the pie to the whole pie is 3 : 4

  and it may written as $\frac{3}{4}$
- The ratio of what Hany ate to which was left of the pie is 1 : 3

  and it may written as $\frac{1}{3}$

Generally

If a and b are two real numbers , then :

The ratio between a and b is written as a : b or $\frac{a}{b}$

and is read as a to b where :

a is called the antecedent of the ratio , b is called the consequent of the ratio , a and b are called together the two terms of the ratio.

Properties of the ratio

Property 1

The value of the ratio does not change if each of its terms is multiplied or divided by the same non-zero real number.

i.e.

$a : b = ak : bk$, $k \in \mathbb{R}^*$

For example:

$1 : 2 = 1 \times 4 : 2 \times 4$

i.e. $1 : 2 = 4 : 8$

$a : b = \frac{a}{n} : \frac{b}{n}$, $n \in \mathbb{R}^*$

For example:

$4 : 6 = \frac{4}{2} : \frac{6}{2}$

i.e. $4 : 6 = 2 : 3$

Property 2

The value of the ratio ($\neq 1$) changes if we add or subtract (to or from) each of its two terms a non-zero real number.

i.e.

$a : b \neq a + k : b + k$, $k \in \mathbb{R}^*$ where $a \neq b$

For example:

$3 : 4 \neq 3 + 1 : 4 + 1$

i.e. $3 : 4 \neq 4 : 5$

$a : b \neq a - k : b - k$, $k \in \mathbb{R}^*$ where $a \neq b$

For example:

$5 : 8 \neq 5 - 3 : 8 - 3$

i.e. $5 : 8 \neq 2 : 5$

Second Proportion

The opposite table shows two sets of numbers.

The set (A)	2	4	7	3	6
The set (B)	8	16	28	12	24

If we look at these sets , we can notice that :

$\frac{2}{8} = \frac{4}{16} = \frac{7}{28} = \frac{3}{12} = \frac{6}{24}$ each of them equals $\frac{1}{4}$

In this case , we say that the numbers of set (A) are proportional to the corresponding numbers in the set (B)

The previous form which expresses the equality of two ratios or more is called proportion.

Definition of proportion

It is the equality of two ratios or more.

i.e.

If $\frac{a}{b} = \frac{c}{d}$, then the quantities a , b , c and d are proportional.

And vice versa : If a , b , c and d are proportional , then : $\frac{a}{b} = \frac{c}{d}$

- $\boxed{a}$ is called the **first** proportional.
- $\boxed{b}$ is called the **second** proportional.
- $\boxed{c}$ is called the **third** proportional.
- $\boxed{d}$ is called the **fourth** proportional.

$\boxed{\textbf{a and d}}$ are called **extremes** and $\boxed{\textbf{b and c}}$ are called **means**.

For example: The numbers 1 , 4 , 7 and 28 are proportional numbers , because $\frac{1}{4} = \frac{7}{28}$

And : $\boxed{1}$ is the first proportional, $\boxed{4}$ is the second proportional , $\boxed{7}$ is the third proportional , $\boxed{28}$ is the fourth proportional, $\boxed{\text{1 and 28}}$ are the extremes of this proportion and $\boxed{\text{4 and 7}}$ are the means.

WATCH VIDEO

Properties of proportion

Property 1

If $\frac{a}{b} = \frac{c}{d}$, then : $a \times d = b \times c$ (the product of the extremes = the product of the means)

The reason : If we multiply each ratio by b d , we get : $\frac{a}{b} \times b\,d = \frac{c}{d} \times b\,d$

i.e. $a \times d = b \times c$

Example 1

Choose the correct answer from the given ones :

1 The third proportional for the quantities 2 , 4 and 20 is ....

(a) 10 (b) 15 (c) 20 (d) 40

2 The fourth proportional for the numbers 4 , 12 and 16 is ....

(a) 24 (b) ± 24 (c) 48 (d) ± 48

3 If 2 , x , 4 and 6 are proportional , then x = ...

(a) 1 (b) 3 (c) 5 (d) 8

Solution

1 (a) **The reason :** Let the third proportional be x

$\therefore$ The quantities 2 , 4 , x and 20 are proportional

$\therefore \frac{2}{4} = \frac{x}{20}$ $\therefore 2 \times 20 = 4 \times x$

$\therefore 40 = 4x$ $\therefore x = 10$

2 **(c) The reason :** Let the fourth proportional be X

$\therefore$ The numbers 4 , 12 , 16 and X are proportional

$\therefore \frac{4}{12} = \frac{16}{X}$ $\quad \therefore 4X = 12 \times 16$ $\quad \therefore X = \frac{12 \times 16}{4} = 48$

3 **(b) The reason :** $\because$ 2 , X , 4 and 6 are proportional

$\therefore \frac{2}{X} = \frac{4}{6}$ $\quad \therefore 4X = 12$ $\quad \therefore X = 3$

TRY by yourself 1

If the quantities X , 23 , 15 and 69 are proportional , ***find the value of :*** X

Example 2

Find the number that will be added to each of the numbers : 1 , 13 , 7 and 31 to get proportional numbers.

Solution Let the number be X $\quad \therefore 1 + X , 13 + X , 7 + X , 31 + X$ are proportional.

$\therefore \frac{1 + X}{13 + X} = \frac{7 + X}{31 + X}$ $\quad \therefore (X + 1)(X + 31) = (X + 7)(X + 13)$

$\therefore \cancel{X^2} + 32X + 31 = \cancel{X^2} + 20X + 91$ $\quad \therefore 32X - 20X = 91 - 31$

$\therefore 12X = 60$ $\quad \therefore X = 5$ $\quad \therefore$ The required number = 5

Example 3

If $(2X + 5) : (3X - 3) = 5 : 4$, **find the value of :** X

Solution $\because \frac{2X + 5}{3X - 3} = \frac{5}{4}$ $\quad \therefore 4(2X + 5) = 5(3X - 3)$

$\therefore 8X + 20 = 15X - 15$ $\quad \therefore 20 + 15 = 15X - 8X$

$\therefore 35 = 7X$ $\quad \therefore X = \frac{35}{7} = 5$

Example 4

Find the number that if we add to the two terms of the ratio 17 : 22 , the result will be 6 : 7

Solution Let the required number be X $\quad \therefore \frac{17 + X}{22 + X} = \frac{6}{7}$

$\therefore 7(17 + X) = 6(22 + X)$ $\quad \therefore 119 + 7X = 132 + 6X$

$\therefore 7X - 6X = 132 - 119$ $\quad \therefore X$ (The required number) = 13

TRY by yourself 2

Find the real number that if we subtract from both terms of the ratio $\frac{5}{6}$, it will become $\frac{3}{2}$

Property 2

If $a \times d = b \times c$, then $\frac{a}{b} = \frac{c}{d}$

The reason : If we divide each side by b d , we get : $\frac{a \times d}{b\,d} = \frac{b \times c}{b\,d}$ **i.e.** $\frac{a}{b} = \frac{c}{d}$

Also we can deduce that :

- If $a \times d = b \times c$, then $\frac{a}{c} = \frac{b}{d}$
- If $a \times d = b \times c$, then $\frac{b}{a} = \frac{d}{c}$
- If $a \times d = b \times c$, then $\frac{c}{a} = \frac{d}{b}$

Example 5 In each of the following , find $\frac{x}{y}$ if :

1. $12\,x = 3\,y$
2. $4\,x - 3\,y = 0$

Solution

1. $\because 12\,x = 3\,y$ $\quad \therefore \frac{x}{y} = \frac{3}{12} = \frac{1}{4}$
2. $\because 4\,x - 3\,y = 0$ $\quad \therefore 4\,x = 3\,y$ $\quad \therefore \frac{x}{y} = \frac{3}{4}$

Example 6 If $(4\,x - 3\,y) : (2\,x + y) = \frac{4}{7}$, find in the simplest form the ratio $x : y$

Solution

$\because \frac{4\,x - 3\,y}{2\,x + y} = \frac{4}{7}$ $\quad \therefore 7\,(4\,x - 3\,y) = 4\,(2\,x + y)$

$\therefore 28\,x - 21\,y = 8\,x + 4\,y$ $\quad \therefore 28\,x - 8\,x = 21\,y + 4\,y$

$\therefore 20\,x = 25\,y$ $\quad \therefore \frac{x}{y} = \frac{25}{20}$ $\quad \therefore \frac{x}{y} = \frac{5}{4}$

Example 7 If $2\,x^2 - 6\,y^2 = x\,y$, find : $x : y$

Solution

$\because 2\,x^2 - 6\,y^2 = x\,y$ $\quad \therefore 2\,x^2 - x\,y - 6\,y^2 = 0$

$\therefore (2\,x + 3\,y)(x - 2\,y) = 0$ $\quad \therefore 2\,x + 3\,y = 0$

, then $2\,x = -3\,y$ $\quad \therefore \boxed{\frac{x}{y} = -\frac{3}{2}}$

or $x - 2\,y = 0$, then $x = 2\,y$ $\quad \therefore \boxed{\frac{x}{y} = \frac{2}{1}}$

i.e. $\frac{x}{y} = -\frac{3}{2}$ or $\frac{x}{y} = \frac{2}{1}$

TRY by yourself 3

1. If $2a - 5b = 0$, ***find :*** $\frac{a}{b}$
2. If $\frac{x + 2y}{4x - 3y} = \frac{7}{6}$, ***then prove that :*** $\frac{x}{y} = \frac{3}{2}$
3. If $4a^2 - 9b^2 = 0$, ***find :*** $a : b$

Property 3

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

i.e. $\frac{\text{The antecedent of the first ratio}}{\text{The antecedent of the second ratio}} = \frac{\text{The consequent of the first ratio}}{\text{The consequent of the second ratio}}$

The reason : If we multiply each ratio by $\frac{b}{c}$, we get : $\frac{a}{\cancel{b}} \times \frac{\cancel{b}}{c} = \frac{\cancel{c}}{d} \times \frac{b}{\cancel{c}}$

i.e. $\frac{a}{c} = \frac{b}{d}$

For example: If $\frac{a}{4} = \frac{b}{3}$, then $\frac{a}{b} = \frac{4}{3}$ and $\frac{b}{a} = \frac{3}{4}$

Property 4

If $\frac{a}{b} = \frac{c}{d}$, then $a = cm$ and $b = dm$ (where m is a constant $\neq 0$)

For example: If $\frac{a}{b} = \frac{3}{4}$, then : $a = 3m$, $b = 4m$ (where m is a constant $\neq 0$)

Example 8 If $a : b = 3 : 5$, find the ratio $20a - 7b : 15a + b$

Solution $\because \frac{a}{b} = \frac{3}{5}$ $\quad \therefore a = 3m$, $b = 5m$ (where $m \neq 0$)

Substituting by a and b in terms of m :

$$\therefore \frac{20a - 7b}{15a + b} = \frac{60m - 35m}{45m + 5m} = \frac{25m}{50m} = \frac{1}{2}$$

Another solution :

By dividing the terms of the ratio $\frac{20a - 7b}{15a + b}$ by b

, then substituting by the value $\frac{a}{b} = \frac{3}{5}$

$$\therefore \frac{20a - 7b}{15a + b} = \frac{20\left(\frac{a}{b}\right) - 7}{15\left(\frac{a}{b}\right) + 1} = \frac{20 \times \frac{3}{5} - 7}{15 \times \frac{3}{5} + 1} = \frac{12 - 7}{9 + 1} = \frac{5}{10} = \frac{1}{2}$$

Example 9 If $\frac{a}{b} = \frac{2}{3}$ and $\frac{x}{y} = \frac{3}{5}$, **prove that :**

$(7\,a\,x + 4\,b\,y)$, $(11\,a\,y + b\,x)$, 12 and 14 are proportional quantities.

Solution $\because \frac{a}{b} = \frac{2}{3}$ $\quad \therefore a = 2\,m$, $b = 3\,m$ (where $m \neq 0$)

$\because \frac{x}{y} = \frac{3}{5}$ $\quad \therefore x = 3\,k$, $y = 5\,k$ (where $k \neq 0$)

[**Notice that :** We used two different constants m and k]

Substituting by a , b , x and y

$$\therefore \frac{7\,a\,x + 4\,b\,y}{11\,a\,y + b\,x} = \frac{7 \times 2\,m \times 3\,k + 4 \times 3\,m \times 5\,k}{11 \times 2\,m \times 5\,k + 3\,m \times 3\,k}$$

$$= \frac{42\,m\,k + 60\,m\,k}{110\,m\,k + 9\,m\,k} = \frac{102\,m\,k}{119\,m\,k} = \frac{6}{7}$$

$$\because \frac{12}{14} = \frac{6}{7}$$

$\therefore (7\,a\,x + 4\,b\,y)$, $(11\,a\,y + b\,x)$, 12 and 14 are proportional quantities.

TRY by yourself 4

If $\frac{x}{y} = \frac{2}{5}$, ***prove that :*** $(2\,x + y)$, $(x + 2\,y)$, 12 and 16 are proportional quantities.

Example 10 The ratio between two real numbers is 4 : 7

If we subtract 16 from each of them , then the ratio between the two obtained numbers is 2 : 5 Find the two numbers.

Solution Let the two numbers be a and b

$\therefore \frac{a}{b} = \frac{4}{7}$ $\qquad \therefore a = 4\,m$, $b = 7\,m$ (where $m \neq 0$)

$\therefore \frac{4\,m - 16}{7\,m - 16} = \frac{2}{5}$ $\qquad \therefore 14\,m - 32 = 20\,m - 80$

$\therefore 80 - 32 = 20\,m - 14\,m$ $\qquad \therefore 48 = 6\,m$ $\qquad \therefore m = \frac{48}{6} = 8$

$\therefore a = 4 \times 8 = 32$, $b = 7 \times 8 = 56$ **i.e.** The two numbers are 32 and 56

TRY by yourself 5

The ratio between two integers is 2 : 5 If 2 is subtracted from the first integer and 1 is added to the second , then the ratio becomes 1 : 4

Find the two integers.

Lesson 2 Follow properties of proportion

In this lesson , we will study the property (5) from properties of proportion , before studying this property , we will study an important remark in proportion to help us solving problems.

Important remark

* If a , b , c and d are proportional quantities and we assume that : $\frac{a}{b} = \frac{c}{d} = m$, then

$a = bm$, $c = dm$

For example:

If $\frac{a}{b} = \frac{c}{d} = \frac{3}{4}$, then $a = \frac{3}{4}\, b$, $c = \frac{3}{4}\, d$

* **Generally**

If a , b , c , d , e , f , ... are proportional quantities and we assume that :

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots = m$, then $a = bm$, $c = dm$, $e = fm$, ...

Example 1 If a , b , c and d are proportional quantities , **prove that :**

1 $\frac{2a + 3c}{7a - 5c} = \frac{2b + 3d}{7b - 5d}$

2 $\frac{a + c}{b + d} = \frac{a^2 + c^2}{ab + cd}$

Solution

1 Let $\frac{a}{b} = \frac{c}{d} = m$ $\therefore a = bm$, $c = dm$

L.H.S. $= \frac{2bm + 3dm}{7bm - 5dm} = \frac{m(2b + 3d)}{m(7b - 5d)} = \frac{2b + 3d}{7b - 5d} =$ R.H.S.

2 Let $\frac{a}{b} = \frac{c}{d} = m$ $\quad \therefore a = bm \ , \ c = dm$

$$\because \frac{a+c}{b+d} = \frac{bm+dm}{b+d} = \frac{m(b+d)}{(b+d)} = m \qquad (1)$$

$$, \because \frac{a^2+c^2}{ab+cd} = \frac{(bm)^2+(dm)^2}{bm \times b + dm \times d} = \frac{b^2m^2+d^2m^2}{b^2m+d^2m} = \frac{m^2(b^2+d^2)}{m(b^2+d^2)} = m \qquad (2)$$

From (1) and (2) we deduce that : $\frac{a+c}{b+d} = \frac{a^2+c^2}{ab+cd}$

Example 2 If a , b , c , d , e and f are positive proportional quantities ,

prove that : $\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \frac{a}{b}$

Solution Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = m$ $\quad \therefore a = bm \ , \ c = dm \ , \ e = fm$

$$\therefore \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \sqrt{\frac{(bm)^2+(dm)^2+(fm)^2}{b^2+d^2+f^2}} = \sqrt{\frac{b^2m^2+d^2m^2+f^2m^2}{b^2+d^2+f^2}}$$

$$= \sqrt{\frac{m^2(b^2+d^2+f^2)}{(b^2+d^2+f^2)}} = \sqrt{m^2} = m$$

$, \because \frac{a}{b} = m$ $\qquad \therefore \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \frac{a}{b}$

TRY by yourself 1

If $\frac{a}{b} = \frac{c}{d}$, ***prove that :*** $\frac{5a-2c}{5b-2d} = \frac{4a+3c}{4b+3d}$

Property 5

We know that : $\frac{9}{15} = \frac{6}{10} = \frac{3}{5}$

- If we add the antecedents and consequents of the 1st and the 2nd ratios, we get the ratio $\frac{9+6}{15+10} = \frac{15}{25} = \frac{3}{5}$ which is one of the given ratios.
- Also if we add the antecedents and consequents of the 2nd and the 3rd ratios, we get the ratio $\frac{6+3}{10+5} = \frac{9}{15}$ = one of the given ratios.
- If we add the antecedents and consequents of the three given ratios, we get the ratio $\frac{9+6+3}{15+10+5} = \frac{18}{30} = \frac{3}{5}$ = one of the given ratios.

- Since the ratio does not change if we multiply its two terms by a non-zero real number , then if we multiply the two terms of the first ratio by any number as 2 and multiply the two terms of the second ratio by any other number as (-4) , then the previous proportion stays true.

i.e. $\frac{18}{30} = \frac{-24}{-40} = \frac{3}{5}$

- If we add the antecedents and consequents of the first and the second ratios , we get the ratio $\frac{18-24}{30-40} = \frac{-6}{-10} = \frac{3}{5}$ = one of the given ratios.
- If we add the antecedents and consequents of the three ratios, we get the ratio $\frac{18-24+3}{30-40+5} = \frac{-3}{-5} = \frac{3}{5}$ = one of the given ratios.

From the previous points , we can say that :
If we have some equal ratios , then we can obtain many other ratios , each of them equals any of the initial ratios. This will happen by adding the antecedents and consequents of all the ratios or some of them directly or after multiplying the two terms of each ratio by a non-zero real number.

i.e.

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots$ and m_1 , m_2 , m_3 , $\cdots$ are non-zero real numbers

, then $\frac{m_1 a + m_2 c + m_3 e + \cdots}{m_1 b + m_2 d + m_3 f + \cdots}$ = one of the given ratios.

Example 3 If $\frac{a}{4} = \frac{b}{5} = \frac{c}{3}$,

find : $\frac{a-b+c}{a+b-c}$

Solution Multiplying the two terms of the 2nd ratio by (-1) , then add the antecedents and the consequents of the three ratios :

$\therefore \frac{a-b+c}{4-5+3} = \frac{a-b+c}{2}$ = one of the given ratios. (1)

Multiplying the two terms of the 3rd ratio by (-1) , then add the antecedents and the consequents of the three ratios :

$\therefore \frac{a+b-c}{4+5-3} = \frac{a+b-c}{6}$ = one of the given ratios. (2)

From (1) and (2) : $\therefore \frac{a-b+c}{2} = \frac{a+b-c}{6}$

$\therefore \frac{a-b+c}{a+b-c} = \frac{2}{6} = \frac{1}{3}$

Another solution :

Let : $\frac{a}{4} = \frac{b}{5} = \frac{c}{3} = m$ $\therefore a = 4m$, $b = 5m$, $c = 3m$

$$\therefore \frac{a - b + c}{a + b - c} = \frac{4m - 5m + 3m}{4m + 5m - 3m} = \frac{2m}{6m} = \frac{1}{3}$$

Example 4 If $\frac{x + y}{l + m} = \frac{y + z}{m + n} = \frac{z + x}{n + l}$, prove that : $\frac{x}{l} = \frac{y - x}{m - l}$

Solution Multiplying the two terms of the 2nd ratio by (– 1) and adding the antecedents and the consquents of the three ratios :

$$\therefore \frac{x + y - y - z + z + x}{l + m - m - n + n + l} = \frac{2x}{2l} = \frac{x}{l} = \text{one of the given ratios} \quad (1)$$

Multiplying the two terms of the 3rd ratio by (– 1) and adding the antecedents and consequents of the 2nd and 3rd ratios

$$\therefore \frac{y + z - z - x}{m + n - n - l} = \frac{y - x}{m - l} = \text{one of the given ratios} \quad (2)$$

From (1) and (2) : $\therefore \frac{x}{l} = \frac{y - x}{m - l}$

Example 5 If $\frac{a + 4b}{x + 2y} = \frac{4b + 7c}{2y + 5z} = \frac{7c + a}{5z + x}$,

prove that : $\frac{a}{2b} = \frac{x}{y}$

Solution Multiplying the two terms of the 2nd ratio by (– 1) , then add the antecedents and the consequents of the three ratios :

$$\therefore \frac{a + 4b - 4b - 7c + 7c + a}{x + 2y - 2y - 5z + 5z + x} = \frac{2a}{2x} = \frac{a}{x} = \text{one of the given ratios.} \quad (1)$$

Multiplying the two terms of the 3rd ratio by (– 1) , then add the antecedents and the consequents of the three ratios :

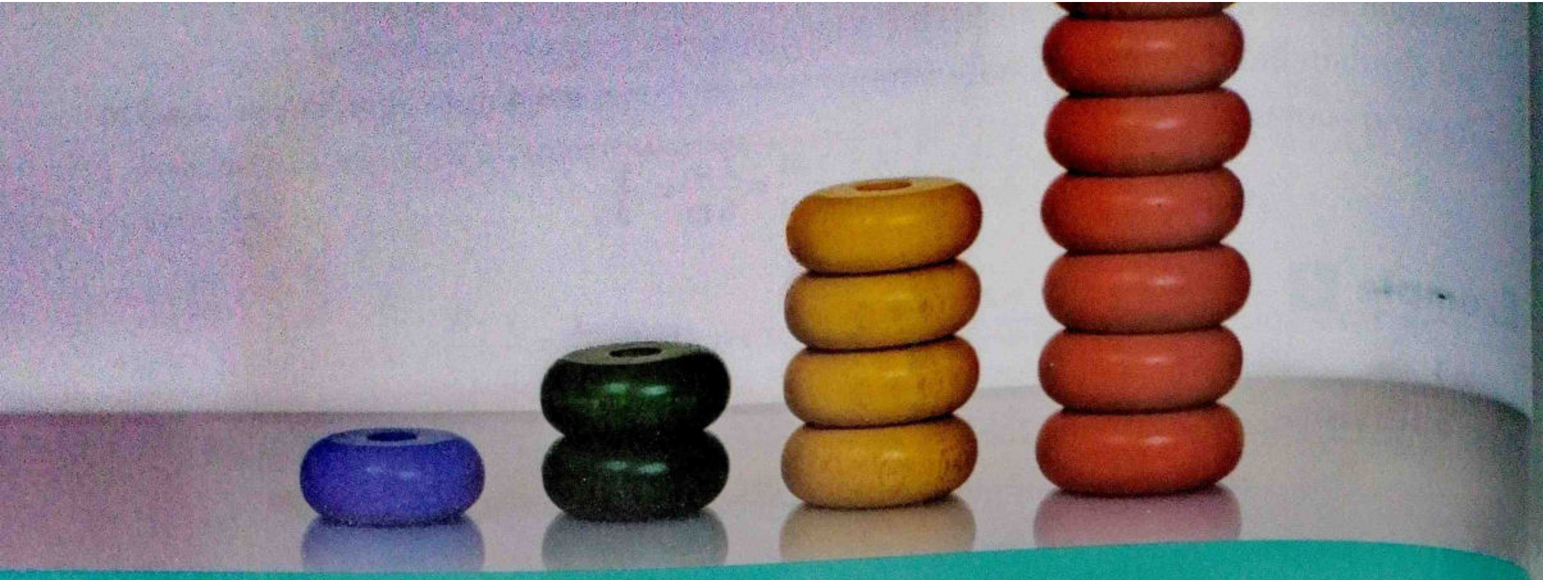
$$\therefore \frac{a + 4b + 4b + 7c - 7c - a}{x + 2y + 2y + 5z - 5z - x} = \frac{8b}{4y} = \frac{2b}{y} = \text{one of the given ratios.} \quad (2)$$

From (1) and (2) : $\therefore \frac{a}{x} = \frac{2b}{y}$ $\therefore \frac{a}{2b} = \frac{x}{y}$

TRY by yourself 2

If $\frac{x}{a - 2b} = \frac{y}{b - 2c} = \frac{z}{c - 2a}$,

prove that : $\frac{x + 2y - z}{3a - 5c} = \frac{y + 2z}{b - 4a}$



Lesson 3 Continued proportion

Definition

The quantities a , b and c are said to be in continued proportion if $\boxed{\frac{a}{b} = \frac{b}{c}}$ or $\boxed{b^2 = a\,c}$

WATCH VIDEO

In this proportion , **a** is called the **first proportional** , **c** is called the **third proportional** and **b** is called the **middle proportional (proportional mean)**.

For example:

The numbers 4 , 6 and 9 form a continued proportion because : $\frac{4}{6} = \frac{6}{9}$ or because : $(6)^2 = 4 \times 9$ where 6 is the middle proportional , 4 is the first proportional and 9 is the third proportional.

Notice that :

1. If a , b and c are in continued proportion , then : $b^2 = a\,c$ i.e. $b = \pm\sqrt{ac}$ and the two quantities a and c should be either both positive or both negative.
2. For any two positive numbers or any two negative numbers x and y , there are two middle proportional $\left(\sqrt{x\,y} \text{ and } -\sqrt{x\,y}\right)$

Example 1

Choose the correct answer from the given ones :

1. The middle proportional between 5 and 20 is ....
   (a) -10 (b) 10 (c) ± 10 (d) 100
2. The middle proportional between 3 and $\frac{1}{3}$ is ....
   (a) ± 1 (b) 9 (c) $\frac{1}{9}$ (d) ± 9
3. The middle proportional between $3\,x^3$ and $27\,x$ is ....
   (a) $9\,x^2$ (b) $\pm 9\,x^2$ (c) $9\,x^4$ (d) $\pm 9\,x^4$

4 The first proportional of 12 and 18 is ....

(a) 8 (b) ± 8 (c) 12 (d) 27

5 The third proportional of -6 and 12 is ....

(a) -24 (b) 6 (c) 18 (d) 72

Solution

1 **(c) The reason :** The middle proportional $= \pm\sqrt{5 \times 20} = \pm\sqrt{100} = \pm 10$

2 **(a) The reason :** The middle proportional $= \pm\sqrt{3 \times \frac{1}{3}} = \pm\sqrt{1} = \pm 1$

3 **(b) The reason :** The middle proportional $= \pm\sqrt{3x^3 \times 27x} = \pm\sqrt{81x^4}$
$= \pm 9x^2$

4 **(a) The reason :** Let the first proportional be a

$\therefore \frac{a}{12} = \frac{12}{18}$ $\therefore a = \frac{12 \times 12}{18} = 8$

5 **(a) The reason :** Let the third proportional be c

$\therefore \frac{-6}{12} = \frac{12}{c}$ $\therefore c = \frac{12 \times 12}{-6} = -24$

TRY by yourself 1

1 ***Find the middle proportional between 32 and 18***

2 ***Find the first proportional of 8 and 16***

Remark

If a , b and c are in continued proportion and we assume that : $\frac{a}{b} = \frac{b}{c} = m$

, then $\frac{b}{c} = m$ $\therefore b = cm$ (1)

, $\because \frac{a}{b} = m$ $\therefore a = bm$

Substituting for b from (1) : $\therefore a = (cm)\, m$ $\therefore a = cm^2$

i.e.

If $\frac{a}{b} = \frac{b}{c} = m$, then $\begin{cases} b = cm \\ a = cm^2 \end{cases}$

Example 2

If a , b and c are in continued proportion ,

prove that : $\frac{4a^2 - 3b^2}{4b^2 - 3c^2} = \frac{a}{c}$

$\therefore b = cm \quad , \quad a = cm^2$

Solution Let $\frac{a}{b} = \frac{b}{c} = m$

$\therefore \frac{4 a^2 - 3 b^2}{4 b^2 - 3 c^2} = \frac{4 (cm^2)^2 - 3 (cm)^2}{4 (cm)^2 - 3 c^2} = \frac{4 c^2 m^4 - 3 c^2 m^2}{4 c^2 m^2 - 3 c^2} = \frac{c^2 m^2 (4 m^2 - 3)}{c^2 (4 m^2 - 3)} = m^2$ (1)

$, \because \frac{a}{c} = \frac{cm^2}{c} = m^2$ (2)

From (1) and (2) , we deduce that : $\frac{4 a^2 - 3 b^2}{4 b^2 - 3 c^2} = \frac{a}{c}$

Another solution :

$\because \frac{a}{b} = \frac{b}{c}$ $\quad \therefore b^2 = ac$

$\therefore \text{L.H.S.} = \frac{4 a^2 - 3 ac}{4 ac - 3 c^2} = \frac{a (4 a - 3 c)}{c (4 a - 3 c)} = \frac{a}{c} = \text{R.H.S.}$

Example 3 If b is the middle proportional between a and c , **prove that :**

1 $\frac{a - b}{a} = \frac{a - c}{a + b}$

2 $ab - c^2 = (b - c)(a + b + c)$

Solution $\because$ b is the middle proportional between a and c

$\therefore$ a , b and c are in continued proportion

Let $\frac{a}{b} = \frac{b}{c} = m$

$\therefore b = cm \quad , \quad a = cm^2$

1 $\because \frac{a - b}{a} = \frac{cm^2 - cm}{cm^2} = \frac{cm (m - 1)}{cm^2} = \frac{m - 1}{m}$ (1)

$, \frac{a - c}{a + b} = \frac{cm^2 - c}{cm^2 + cm} = \frac{c (m^2 - 1)}{cm (m + 1)} = \frac{c (m - 1)(m + 1)}{cm (m + 1)} = \frac{m - 1}{m}$ (2)

From (1) and (2) , we deduce that :

$\frac{a - b}{a} = \frac{a - c}{a + b}$

2 $\because ab - c^2 = cm^2 \times cm - c^2 = c^2 m^3 - c^2 = c^2 (m^3 - 1)$ (1)

$, (b - c)(a + b + c) = (cm - c)(cm^2 + cm + c)$

$= c (m - 1) \times c (m^2 + m + 1)$

$= c^2 (m - 1)(m^2 + m + 1) = c^2 (m^3 - 1)$ (2)

From (1) and (2) , we deduce that : $ab - c^2 = (b - c)(a + b + c)$

TRY by yourself 2

If a , b and c are in continued proportion , ***prove that :*** $\frac{3 c^2 - 4 b^2}{3 b^2 - 4 a^2} = \frac{c^2}{b^2}$

Generalizing the definition of the continued proportion

The quantities a , b , c , d , ... are in continued proportion if : $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \cdots$

For example:

The numbers 16 , 24 , 36 and 54 are in continued proportion because : $\frac{16}{24} = \frac{24}{36} = \frac{36}{54}$ $\left(\text{each ratio} = \frac{2}{3}\right)$

! Remark

If a , b , c and d are in continued proportion and we assume that : $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$, then :

$\frac{c}{d} = m$ $\quad \therefore c = dm \quad (1)$

, $\frac{b}{c} = m$ $\quad \therefore b = cm$

Substituting for c from (1) : $\therefore b = (dm)\,m$ $\quad \therefore b = dm^2 \quad (2)$

, $\frac{a}{b} = m$ $\quad \therefore a = bm$

Substituting for b from (2) : $\therefore a = (dm^2)\,m$ $\quad \therefore a = dm^3$

i.e.

If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$, then $\boxed{c = dm}$, $\boxed{b = dm^2}$ and $\boxed{a = dm^3}$

Example 4

If a , b , c and d are in continued proportion

, **prove that :** $\frac{a+d}{b-c+d} = \frac{a-c}{b-c}$

Solution

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$ $\quad \therefore c = dm$, $b = dm^2$, $a = dm^3$

$$\therefore \frac{a+d}{b-c+d} = \frac{dm^3+d}{dm^2-dm+d} = \frac{d\,(m^3+1)}{d\,(m^2-m+1)}$$

$$= \frac{(m+1)\,(m^2-m+1)}{m^2-m+1} = m+1 \quad (1)$$

$$, \frac{a-c}{b-c} = \frac{dm^3-dm}{dm^2-dm} = \frac{dm\,(m^2-1)}{dm\,(m-1)} = \frac{(m-1)\,(m+1)}{(m-1)} = m+1 \quad (2)$$

From (1) and (2) , we deduce that : $\frac{a+d}{b-c+d} = \frac{a-c}{b-c}$

TRY by yourself 3

If a , b , c and d are in continued proportion , ***prove that :*** $\frac{a+2b}{b+2c} = \frac{c+a}{d+b}$

Example 5 If the quantities a , 2 b , 3 c and 4 d are in continued proportion , **prove that :** (2 b – 3 c) is the middle proportional between (a – 2 b) and (3 c – 4 d)

Solution Let $\frac{a}{2b} = \frac{2b}{3c} = \frac{3c}{4d} = m$ $\therefore$ 3c = 4 dm , 2b = 4 dm^2 , a = 4 dm^3

Proving that : (2b – 3c) is the middle proportional between (a – 2b) and (3c – 4d)

means proving that : $(2b - 3c)^2 = (a - 2b)(3c - 4d)$

$\because (2b - 3c)^2 = (4\,dm^2 - 4\,dm)^2$

$= \left(4\,dm\,(m - 1)\right)^2 = 16\,d^2\,m^2\,(m - 1)^2$ (1)

, $(a - 2b)(3c - 4d) = (4\,dm^3 - 4\,dm^2)(4\,dm - 4d)$

$= 4\,dm^2\,(m - 1) \times 4d\,(m - 1) = 16\,d^2m^2\,(m - 1)^2$ (2)

From (1) and (2) , we deduce that : $(2b - 3c)^2 = (a - 2b)(3c - 4d)$

$\therefore$ (2b – 3c) is the middle proportional between (a – 2b) and (3c – 4d)

Another solution :

$\because$ a , 2b , 3c and 4d are in continued proportion.

$\therefore \frac{a}{2b} = \frac{2b}{3c} = \frac{3c}{4d}$

Subtracting the terms of the 2nd ratio from the terms of the 1st ratio

$\therefore \frac{a - 2b}{2b - 3c}$ = one of the given ratios. (1)

Subtracting the terms of the 3rd ratio from the terms of the 2nd ratio

$\therefore \frac{2b - 3c}{3c - 4d}$ = one of the given ratios. (2)

From (1) and (2) , we deduce that : $\frac{a - 2b}{2b - 3c} = \frac{2b - 3c}{3c - 4d}$

$\therefore$ (2b – 3c) is the middle proportional between (a – 2b) and (3c – 4d)

Lesson **4**

Direct variation and inverse variation

WATCH VIDEO

First The direct variation

Definition

It is said that y varies directly as x and it is written $y \propto x$ if $\boxed{y = m\,x}$

i.e. $\frac{y}{x} = m$, where m is a constant $\neq 0$

, the relation : $y = m\,x$ is represented graphically by a straight line passing through the origin point $(0\ ,0)$

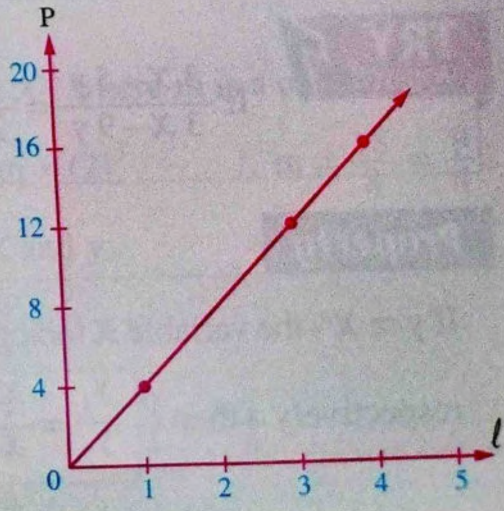
For example:

The perimeter of the square (P) is varying directly with its side length (ℓ) and it is written as $P \propto \ell$

Because : $P = 4\,\ell$ or $\frac{P}{\ell} = 4$

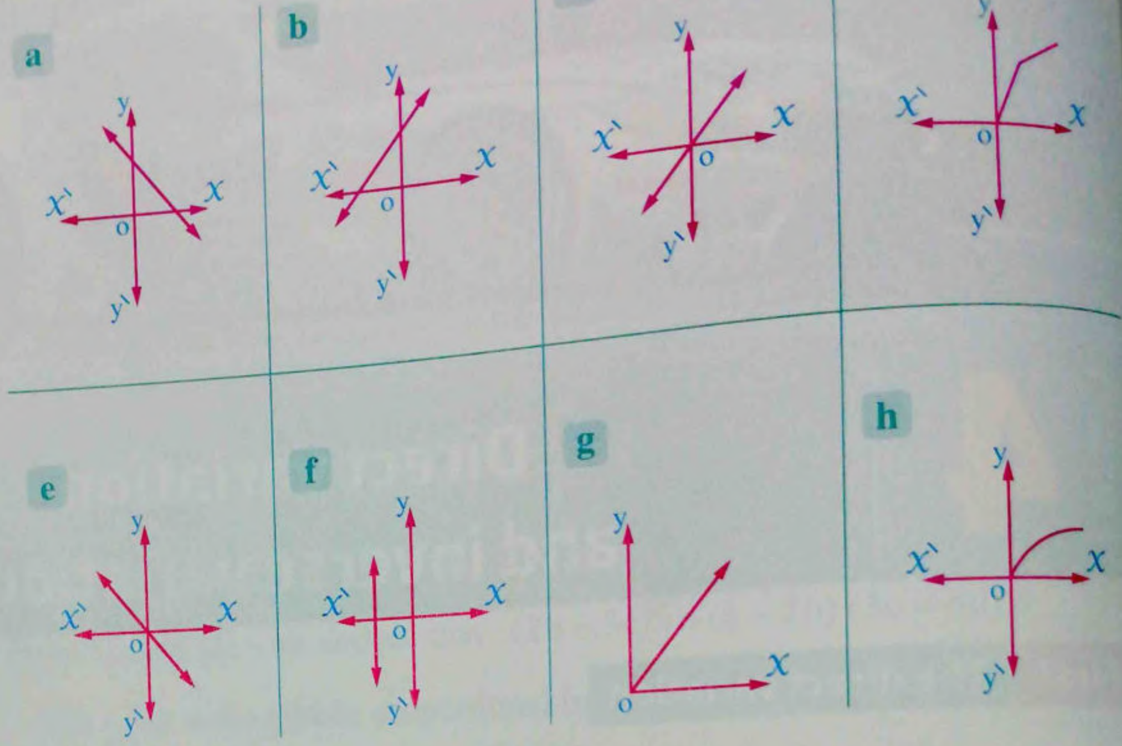
and the following table shows some values of ℓ and the values of P corresponding to them.

Side length (ℓ)	1	3	4
The perimeter (P)	4	12	16



and the opposite figure represents graphically the relation between P and ℓ

Example 1 Show which of the following graphs represents a [illegible] between x and y :



Solution The graphs which represent a direct variation between x and y are :

c , e and g because in each of them , the straight line passes through the origin point.

Example 2 If $a^2 + 4b^2 = 4ab$, **prove that :** $a \propto b$

Solution To prove that $a \propto b$ we prove that $a = m\,b$ where m is a constant $\neq 0$

$\because a^2 + 4b^2 = 4ab$ $\quad\therefore a^2 - 4ab + 4b^2 = 0$

$\therefore (a - 2b)^2 = 0$ $\quad\therefore a - 2b = 0$

$\therefore \boxed{a = 2b}$ $\quad\therefore a \propto b$

TRY by yourself 1

If $\dfrac{3x - 5y}{3x - 9y} = \dfrac{1}{2}$ for every values of $x \in \mathbb{R}_+$, $y \in \mathbb{R}_+$, ***prove that :*** $x \propto y$

Property

If $y \propto x$, the variable x took the two values x_1 and x_2 and y took the two values y_1 and y_2 respectively , then : $\boxed{\dfrac{y_1}{y_2} = \dfrac{x_1}{x_2}}$

The reason : $\because y \propto x$ then $y = m\,x$ where m is a constant $\neq 0$

at $x = x_1$, $y = y_1$ then $y_1 = m\,x_1$ (1)

, at $x = x_2$, $y = y_2$ then $y_2 = m\,x_2$ (2)

Dividing (1) by (2) : $\therefore \frac{y_1}{y_2} = \frac{m\,x_1}{m\,x_2}$ $\therefore \frac{y_1}{y_2} = \frac{x_1}{x_2}$

Example 3

If $y \propto x$ and $y = 20$ when $x = 7$

, then find the value of y when $x = 14$

WATCH VIDEO

Solution

$\because y \propto x$ $\therefore \frac{y_1}{y_2} = \frac{x_1}{x_2}$

where $y_1 = 20$, $x_1 = 7$, $y_2 = ?$, $x_2 = 14$

$\therefore \frac{20}{y_2} = \frac{7}{14}$ $\therefore y_2 = \frac{20 \times 14}{7} = 40$

Another solution :

$\because y \propto x$ $\therefore y = m\,x$ (m is a constant $\neq 0$)

$\because y = 20$ as $x = 7$ $\therefore 20 = m \times 7$

$\therefore m = \frac{20}{7}$ $\therefore y = \frac{20}{7}\,x$

, when $x = 14$ $\therefore y = \frac{20}{7} \times 14$ $\therefore y = 40$

Example 4

If x and y are two variables where y varies directly as the multiplicative inverse of $\frac{1}{x^3}$, $y = 18$ when $x = 2$

, find the relation between x and y , then find the values of y when $x \in \{0 , 1 , 4\}$

Solution

$\because y \propto$ the multiplicative inverse of $\frac{1}{x^3}$

$\therefore y \propto x^3$ $\therefore y = m\,x^3$ where m is a constant $\neq 0$

$\because y = 18$ as $x = 2$ $\therefore 18 = m \times (2)^3$ $\therefore m = \frac{18}{8} = \frac{9}{4}$

$\therefore y = \frac{9}{4}\,x^3$ This is the relation between x and y

as $x = 0$ $\therefore y = \frac{9}{4} \times 0 = 0$

as $x = 1$ $\therefore y = \frac{9}{4} \times 1 = \frac{9}{4} = 2\frac{1}{4}$

as $x = 4$ $\therefore y = \frac{9}{4} \times 64 = 144$

Example 5 If (V) denotes the volume of a right circular cone , its height is constant and if (V) varies directly as the square of radius length of the base of the cone (r) and the volume of the cone was 477 cm^3 , when the radius length of its base = 15 cm.

Find the volume of the cone when the base radius length = 10 cm.

Solution $\because V \propto r^2$ $\qquad \therefore \frac{V_1}{V_2} = \frac{r_1^2}{r_2^2}$ $\qquad \therefore \frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^2$

where $V_1 = 477 \text{ cm}^3$, $r_1 = 15$ cm. , $V_2 = ?$, $r_2 = 10$ cm.

$\therefore \frac{477}{V_2} = \left(\frac{15}{10}\right)^2 = \frac{9}{4}$ $\qquad \therefore V_2 = \frac{477 \times 4}{9} = 212 \text{ cm}^3$

TRY by yourself 2 If $x \propto y$ and y = 2 when $x = 40$, find the value of x when y = 3

WATCH VIDEO

Second The inverse variation

Definition

It is said that y varies inversely as x and it is written $y \propto \frac{1}{x}$ if $\boxed{y = \frac{m}{x}}$

i.e. $x y = m$, where m is a constant $\neq 0$

For example:

The uniform velocity (v) varies inversely as time (t) when the covered distance (d) is constant

Because : $v = \frac{d}{t}$ or $vt = d$

, in this case we say that the velocity varies directly as the multiplicative inverse of time and it is written as : $v \propto \frac{1}{t}$

Example 6 If $a^2 b^4 - 10 ab^2 = -25$, **prove that :** a varies inversely as b^2

Solution To prove that a varies inversely as b^2 we prove that : $ab^2 = m$ where $m \neq 0$

$\because a^2 b^4 - 10 ab^2 = -25$ $\qquad \therefore a^2 b^4 - 10 ab^2 + 25 = 0$

$\therefore (ab^2 - 5)^2 = 0$ $\qquad \therefore ab^2 - 5 = 0$

$\therefore ab^2 = 5$ $\qquad \therefore$ a varies inversely as b^2

TRY yourself 3 If $a^2 b^2 + 49 = 14\,ab$, ***prove that*** : $a \propto \frac{1}{b}$

Property

If $y \propto \frac{1}{X}$, the variable X took the two values X_1 and X_2 and as a result for that y took the two values y_1 and y_2 respectively, then : $\boxed{\frac{y_1}{y_2} = \frac{X_2}{X_1}}$

The reason : $\because y \propto \frac{1}{X}$, then $y = \frac{m}{X}$ where m is a constant $\neq 0$

at $X = X_1$, $y = y_1$, then $y_1 = \frac{m}{X_1}$ (1)

, at $X = X_2$, $y = y_2$, then $y_2 = \frac{m}{X_2}$ (2)

Dividing (1) by (2) :

$\therefore \frac{y_1}{y_2} = \frac{m}{X_1} \div \frac{m}{X_2} = \frac{\cancel{m}}{X_1} \times \frac{X_2}{\cancel{m}} = \frac{X_2}{X_1}$

Example 7 If the length of a rectangle (ℓ) varies inversely as its width (w), when the area is constant and $\ell = 12$ cm. as w = 8 cm., **find :** ℓ when w = 3 cm.

Solution

$\because \ell \propto \frac{1}{w}$

$\therefore \frac{\ell_1}{\ell_2} = \frac{w_2}{w_1}$, where $\ell_1 = 12$ cm., $w_1 = 8$ cm., $\ell_2 = ?$, $w_2 = 3$ cm.

$\therefore \frac{12}{\ell_2} = \frac{3}{8}$ $\therefore \ell_2 = \frac{8 \times 12}{3} = 32$ cm.

Another solution :

$\because \ell \propto \frac{1}{w}$

$\therefore \ell\, w = m$, where m is a constant $\neq 0$

$\because \ell = 12$ cm. as w = 8 cm. $\therefore m = 12 \times 8 = 96$ $\therefore \ell\, w = 96$

When w = 3 cm. $\therefore 3\,\ell = 96$ $\therefore \ell = \frac{96}{3} = 32$ cm.

Example 8 If y varies inversely as x and $y = 6$ as $x = 2.5$, find the relatio
x and y , then find the value of y if $x = 5$

Solution

$\because y \propto \frac{1}{x}$ $\therefore x y = m$, where m is a con

$\because y = 6$ as $x = 2.5$ $\therefore m = 6 \times 2.5 = 15$

$\therefore$ The relation between x and y is $\boxed{x y = 15}$

, at $x = 5$ $\therefore 5 y = 15$ $\therefore$ y

Example 9 If $y = 1 + b$ where b varies inversely as x^2 and $y = 17$ as $x =$
, find the relation between x and y , then find the value of y w

Solution

$\because b \propto \frac{1}{x^2}$ $\therefore b = \frac{m}{x^2}$, where m is a constant $\neq 0$ $\therefore$ y

, $\because y = 17$ as $x = \frac{1}{2}$ $\therefore 17 = 1 + \frac{m}{\left(\frac{1}{2}\right)^2}$ $\therefore$ 1

Subtracting 1 from both sides : $\therefore 16 = \frac{m}{\frac{1}{4}}$

$\therefore m = 16 \times \frac{1}{4} = 4$ $\therefore$ $\boxed{y = 1 + \frac{4}{x^2}}$

at $x = 2$: $\therefore y = 1 + \frac{4}{2^2} = 1 + \frac{4}{4} = 2$

TRY by yourself 4

If y varies inversely as x and $y = 2$ as $x = 6$, calculate the value of y

UNIT THREE

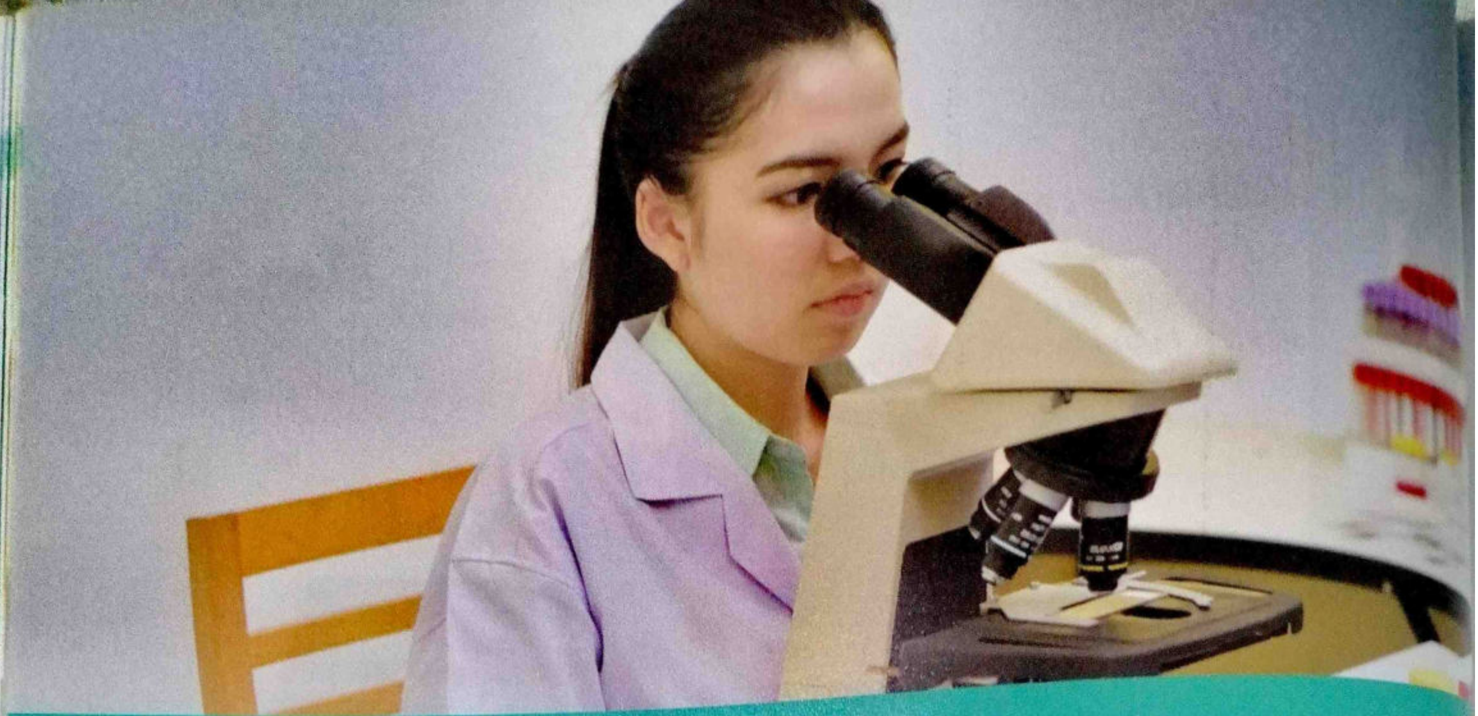
Statistics

Lessons of the unit :

1. Collecting data.
2. Dispersion.

Unit Objectives : By the end of this unit, student should be able to :

- recognize the different resources of collecting data.
- recognize the methods of collecting data , and the advantages and the disadvantages of each method.
- recognize the concept of the sample.
- recognize the methods of selection of samples.
- recognize the types of the samples.
- choose the best method to select a sample for studying a certain phenomenon.
- use the calculator and the computer for generating random numbers used in the samples.
- recognize the dispersion measurements.
- recognize the advantages and the disadvantages of the range as one of the dispersion measurements.
- calculate the range of a set of individuals.
- calculate the standard deviation of a set of individuals.
- calculate the standard deviation of a simple frequency distribution.
- calculate the standard deviation of a frequency distribution of sets.
- use the calculator to calculate the standard deviation.



Lesson 1 Collecting data

- The statistical investigator collects , classifies , represents and analyses data in purpose of deducing some results on which he depends in making the suitable decisions.
- The more data is accurate , the more the decisions will be true and reliable.
- Collecting data in such scientific methods will lead to get accurate outcomes when doing operations of statistical inference and proper decision making.
- Collecting statistical data demands knowing the resources of collecting it and determining the methods of collecting it.

Resources of collecting data is classified into

1 Primary resources (field resources) :

These are the resources from which we get data directly.

2 Secondary resources (historical resources) :

These are the resources from which we get data that previously collected and registered by some authorities , formal organisations or persons.

There are some examples for each resource with representing the advantages and the disadvantages of each one :

	1 Primary resources	2 Secondary resources
Examples :	• Personal interview. • Questionnaires (survey). • Observing and measuring.	• Central agency for public mobilization and statistics. • Mass-media and internet. • Documents of data of employees in a company.
Advantages :	Accuracy.	Saves time , effort and money.
Disadvantages :	It needs more time, effort and money besides it requires more investigators in large societies.	It is less accurate.

Methods of collecting data

- The method of collecting data depends on the aim of collecting these data and it also depends on the size of the statistical society under study.
- The statistical society is defined as all individuals which have general common characters.

For example:

- The workers in a factory represent a statistical society , whose individual is the worker.
- The pupils of a school represent a statistical society , whose individual is the pupil.

We will show two methods of collecting data :

1 Method of mass population :

It is based on collecting the data related to the phenomenon under study from all individuals of the statistical society.

2 Method of samples :

It is based on collecting data related to the phenomenon under study from a representative sample of the society , and applying the research on it , then generalizing the results on the whole society.

There are some examples for each method with representing the advantages and the disadvantages of each one :

	1 Method of mass population	2 Method of samples
Examples :	• Elections. • Census. • Setting up a data base of all employees in an organization.	• A sample of a patient's blood to make some clinical check up. • A sample of some products of a factory to find out if it matches the standard specifications.
Advantages :	• Accuracy. • Inclusiveness. • Neutrality. • Representing all the society individuals.	• Saving time , effort and money. • It is the only method for collecting data about large unlimited societies such as the search on contents of the desert sand. • It is the only method for collecting data about some limited societies in which mass population method leads to a great loss in it such as checking a sample of a patient's blood because of checking the whole blood of the patient leads to death.
Disadvantages :	• Sometimes it needs long time , great effort and a great cost.	• The results sometimes are not accurate specially if the sample doesn't represent the statistical society authentically , in this case the sample is called a biased sample.

In the following , we will explain the concept of the sample and its types and how we select it

The concept of the sample

It is a small part from a large society that looks like the society and represents it well.

How can we select the sample ?

The types of samples according to the method of selecting it

- **The biased selection**
  not a random sample or deliberate sample
- **The randomly selection**
  random sample
  - **Simple** random sample (in case of the homogeneous societies)
  - **Layer** random sample (in case of the heterogeneous societies)

At the following , we explain each type in details :

First The biased selection (samples are not randomly selected)

- It means that we select the sample in a way to satisfy the objectives of the research. This is called **the deliberate sample**.

For example:

If we want to know how the students understood a lesson in algebra , we must analyze the outcomes of the test by considering the outcomes of a group of students studying the same topic without the other students , this is not a random selection.

- The biased selection is not representing the statistical society.

Second Random selection (random samples)

It means to select a sample such that every member of the population has an equal chance of having selected.

The following are the most important types of the random samples which are :

1 Simple random sample.

2 Layer random sample.

1 Simple random sample

- It is used for the homogeneous societies which are not naturally divided into groups or classes.
- It is selected by two ways according to the number of individuals of statistical society as the following.

A The first method : If the size of the society is small :

- **This method will be carried out as follows :**

1 Each individual of the society takes a number , this number is written on a card such that all cards are identical. i.e. There is no difference in colour or size.

2 Each card is folded well such that the number does not appear , then they are put in a box and mixed well.

3 We select the sample by drawing one card from the box blindly , then we turned well the cards and select the next card , and so on till we reach the required number of the sample.

This method is suitable if , for example , we select a sample of 10 workers from a factory that has 50 workers.

B The second method : If the size of the society is large :

In this method , every individual of the society has a number , then we select the sample using the property of the random number in the scientific calculator as in the opposite picture.

- We press the following keys respectively from the left :

ON → SHIFT → Ran# · → =

then a decimal will appear on the display in the field from 0.000 to 0.999

- If we get a 1-decimal digit , add two zeroes to make it a part of 1000

  For example: (0.2 → 0.200)

- If we get a 2-decimal digit , add one zero to make it a part of 1000

For example: (0.64 → 0.640) and so on.

- Take the number neglecting the decimal point , then the individual who has this number is selected as a member of the sample , then repeat pressing on [=] to get more numbers.
- We will ignore the numbers which are greater than the number of society under study.
- And we ignore the repeated numbers which we selected before.
- The percentage 10% of the number of the society is suitable for holding the survey.

This method is more suitable for selecting a sample of 25 students from a school that has 900 students.

2 Layer random sample

- It is used in the statistical societies which are heterogeneous or made up of qualitative sets that are different in characteristics.
- In this case , we cannot select the sample by the simple random sample method because the sample will not represent the society well because it will not represent all the classes of the society.

Therefore we have to follow the following steps :

1. We divide the society into homogeneous sets according to the characteristics forming it , each set is called **a layer**.
2. We find the number of individuals of each layer , then we find its ratio referring to the total number of the society.
3. To form a sample , we select from each layer a certain number of individuals such that the ratio that represents each layer in the sample is the same ratio of the layer in the whole society , and this by using the following law :

The number of individuals of the layer in the sample

$$= \frac{\text{the total number of individuals in the layer}}{\text{the total number of individuals in the society}} \times \text{the number of individuals of the sample}$$

«approximating the result to the nearest unit»

For example:

When we want to study the educational level of the students of a school of 500 students (boys and girls) and if the ratio between the number of boys to the number of girls is 1 : 4 and we want to select a sample formed from 50 students , we should select 10 students from boys and 40 students from girls , for the sample representing all the society well.

Example 1 A factory has 300 workers. The people in charge of the monthly magazine of this factory want to develop this magazine by doing a survey of a sample representing 10% of the total number of the workers in this factory. Show how the selection of this sample can be carried out using the calculator.

Solution The number of workers in the factory = 300 workers.

$\therefore$ The number of the random sample $= \frac{10}{100} \times 300 = 30$ workers.

Then we want to select 30 workers to hold this survey.

The selection operation can be carried out as follows :

1. Each worker in the factory is given a number from 1 to 300
2. Use the calculator to select 30 numbers randomly , such that these numbers are included between 0 and 301 and the number that is above 300 should be ignored.

For example:

By pressing the keys 

successively from left to right.

- If we get the decimal 0.049 , then the number of the selected person is 49
- If we get the decimal 0.132 , then the number of the selected person is 132
- If we get the decimal 0.12 , then the number of the selected person is 120
- If we get the decimal 0.453 , it must be ignored because 453 is above 300 and so on till we get 30 numbers.
- Assuming that the calculator gave us the shown numbers in the opposite table , then the workers who carry these numbers are the selected sample to carry out this survey.

49	132	120	141	249	272
254	256	4	213	74	198
131	2	156	47	172	13
8	3	85	82	9	38
41	14	34	279	118	103

Example 2

A factory produced 200 TV sets from the type A , 300 TV sets from the type B and 500 TV sets from the type C , if we want to select a layer sample formed from 50 TV sets such that it represents all the types to examine them.

Calculate the number of TV sets which should be selected from each kind.

Solution

- The total number of TV sets = 200 + 300 + 500 = 1000 TV sets.
- The number of TV sets of the type A in the sample = $\frac{200}{1000} \times 50 = 10$ TV sets.
- The number of TV sets of the type B in the sample = $\frac{300}{1000} \times 50 = 15$ TV sets.
- The number of TV sets of the type C in the sample = $\frac{500}{1000} \times 50 = 25$ TV sets.

TRY by yourself

A school has 300 male students and 500 female students wanted to do a survey on a sample of 24 male and female students representing each layer according to its size. Calculate the number of students of each layer in the sample.

Lesson 2

Dispersion

- You studied before some of statistical measures which were known as "**measures of central tendency**" as the mean , the median and the mode.
- And we know that each of them describe the frequency distributions and the statistical data by identifying one numerical value , where the left values centralize about it.
- But in some cases the measures of central tendency are not enough to describe clearly the data.

To explain that , let's study the following case :

Two sets of 5 students each , an exam of maximum mark 50 marks is given for each sets , the marks of the students were as follows :

The set A : 29 , 26 , 35 , 35 , 35

The set B : 8 , 35 , 49 , 35 , 33

At calculating the mean , the median and the mode of the marks of the students in each set alone , we find the shown results in the following table :

	mean	median	mode
Set A	32	35	35
Set B	32	35	35

Remember that

- The mean = $\frac{\text{the sum of values}}{\text{the number of this values}}$
- The median of a set of values is the value which lies at the middle of the set of values after ordering them.
- The mode of a set of values is the most common value in the set.

- In the previous case , the two sets are different , and in spite of that , we found that they have the same mean , median and mode , which don't mean that these sets are necessarily homogeneous.
- Therefore , the measures of central tendency only are unable to describe all the characteristics a set of frequency distributions and statistical data.

  So we need besides the measures of central tendency that depends on determining one value that the other data centralize around it, another kind of measures which depends on determining a degree of convergence or divergence of data.

For example:

In the previous example , the marks of the set A are convergent because their values are included between 26 and 35 marks while the marks of the set B are divergent because their values are included between 8 and 49 marks.

i.e. The marks of the set B are more divergent than the marks of the set A

- These new measures are called the measures of dispersion. We will study each of the range and the standard deviation.

Dispersion of a set of values

It means the divergence or the differences among its values.

- The dispersion is small if the difference among the values is little while the dispersion is great if the difference among the values is great , the dispersion is zero if all the values are equal.

i.e. The dispersion of a set of values is a measure of the degree to which these values spread out and that expresses how much the sets are homogeneous.

Dispersion measurements

1 The range (the simplest measure of dispersion) :

It is the difference between the greatest value and the smallest value in the set.

The range = the greatest value – the smallest value

For example:

- If the values of set A are 60 , 58 , 62 , 61 and 59

  $\therefore$ The range = 62 – 58 = 4
- If the values of set B are 72 , 78 , 46 , 65 and 39

  $\therefore$ The range = 78 – 39 = 39

So the set B **is more divergent than the set A**

The advantages of range :

- It is an easy and simple method that gives a quick idea about the divergence or convergence of the values.
- It is considered as the simplest and the easiest method to measure dispersion.

The disadvantages of range :

- It does not reflect the influence of all values because its measure depends on the greatest and smallest values only , therefore it does not give a full idea of the dispersion of the set of values.
- It is influenced greatly by the outlier.

  For example:

  - The range of the set of values : 21 , 22 , 61 , 24 and 26 is (61 – 21 = 40)
  - While if we ignore the value 61 from the set , then the range becomes (26 – 21 = 5)

  i.e. The range equals $\frac{1}{8}$ the previous range , therefore the range is an approximated measure and we cannot depend on it.

2 Standard deviation :

It is the most important , common and accurate measure of dispersion. We can calculate it by calculating the positive square root of the average of squares of deviations of the values from their mean. It is denoted by σ and it is read as (sigma).

First Calculating the standard deviation of a set of values :

$$\text{The standard deviation } \sigma = \sqrt{\frac{\Sigma (X - \overline{X})^2}{n}}$$

Where :

X denotes a value of the values ,

$\overline{X}$ denotes the mean of the values and it is read as X bar ,

n denotes the number of values ,

Σ denotes the summation operation.

Example 1 Calculate the standard deviation of the values : 8 , 9 , 7 , 6 and 5

Solution

1 We find the mean of the values $(\overline{X}) = \frac{\Sigma X}{n} = \frac{8+9+7+6+5}{5} = 7$

2 We form the opposite table :

X	$X - \overline{X}$	$(X - \overline{X})^2$
8	$8 - 7 = 1$	1
9	$9 - 7 = 2$	4
7	$7 - 7 = 0$	0
6	$6 - 7 = -1$	1
5	$5 - 7 = -2$	4
Total		**10**

3 We calculate the standard deviation as follows :

$$\text{The standard deviation } (\sigma) = \sqrt{\frac{\Sigma (X - \overline{X})^2}{n}} = \sqrt{\frac{10}{5}} = \sqrt{2} \simeq 1.41$$

TRY by yourself 1

If 25 , 24 , 25 , 30 , 28 and 30 represent the marks of one of the pupils in examination of algebra in different months , ***find :***

1 The mean.

2 The standard deviation.

Second Calculating the standard deviation of a frequency distribution :

For any frequency distribution : The standard deviation $\sigma = \sqrt{\frac{\Sigma (X - \overline{X})^2 k}{\Sigma k}}$

Where :

X represents the value or the centre of the set ,

k represents the frequence of the value or the set ,

Σk is the sum of frequences and $\overline{X}$ (the mean) $= \frac{\Sigma (X \times k)}{\Sigma k}$

A Calculating the standard deviation of a simple frequency distribution :

Example 2 The following table shows the distribution of ages of 20 persons in years :

The age	15	20	22	23	25	30	Total
Number of persons	2	3	5	5	1	4	20

Find the standard deviation of the ages.

Solution

1 We find the mean of the ages $(\overline{X})$ by using the following table :

The age (X)	Number of persons (k)	$X \times k$
15	2	30
20	3	60
22	5	110
23	5	115
25	1	25
30	4	120
Total	**20**	**460**

The mean $(\overline{X}) = \frac{\Sigma (X \times k)}{\Sigma k} = \frac{460}{20} = 23$ years.

2 We form the following table :

x	k	$x - \overline{x}$	$(x - \overline{x})^2$	$(x - \overline{x})^2 \times k$
15	2	$15 - 23 = -8$	64	128
20	3	$20 - 23 = -3$	9	27
22	5	$22 - 23 = -1$	1	5
23	5	$23 - 23 = 0$	0	0
25	1	$25 - 23 = 2$	4	4
30	4	$30 - 23 = 7$	49	196
Total	20			360

3 We calculate the standard deviation as follows :

$$\text{Standard deviation } (\sigma) = \sqrt{\frac{\sum (x - \overline{x})^2 \times k}{\sum k}} = \sqrt{\frac{360}{20}} = \sqrt{18} \simeq 4.24 \text{ years}$$

TRY yourself 2

The following frequency distribution shows the number of days of absentees in a class :

Number of absence days	0	1	2	3	4	Total
Number of pupils	5	7	7	5	6	30

Calculate the mean and the standard deviation for the number of days of absence.

B Calculating the standard deviation of a frequency distribution of sets :

Example 3 **The following is the frequency distribution of weekly incentives of 100 workers in a factory :**

Incentives in pounds	35 –	45 –	55 –	65 –	75 –	85 –
Number of workers	10	14	20	28	20	8

Find the standard deviation of this distribution.

Solution

1 We find the mean ($\overline{X}$) by using the following table :

Remember that

The centre of the set $= \dfrac{\text{lower limit} + \text{upper limit}}{2}$

Sets	Centres of sets (X)	Frequence (k)	$X \times k$
35 –	40	10	400
45 –	50	14	700
55 –	60	20	1200
65 –	70	28	1960
75 –	80	20	1600
85 –	90	8	720
Total		**100**	**6580**

$\therefore$ The mean $(\overline{X}) = \dfrac{\sum (X \times k)}{\sum k} = \dfrac{6580}{100} = 65.8$ pounds.

2 We form the following table :

X	k	$X - \overline{X}$	$(X - \overline{X})^2$	$(X - \overline{X})^2 \times k$
40	10	$40 - 65.8 = -25.8$	665.64	6656.4
50	14	$50 - 65.8 = -15.8$	249.64	3494.96
60	20	$60 - 65.8 = -5.8$	33.64	672.8
70	28	$70 - 65.8 = 4.2$	17.64	493.92
80	20	$80 - 65.8 = 14.2$	201.64	4032.8
90	8	$90 - 65.8 = 24.2$	585.64	4685.12
Total	**100**			20036

3 We calculate the standard deviation as follows :

Standard deviation $(\sigma) = \sqrt{\dfrac{\sum (X - \overline{X})^2 \times k}{\sum k}} = \sqrt{\dfrac{20036}{100}} \simeq 14.15$ pounds.

Remarks

- The standard deviation is influenced by all values not by the two terminal values only (the smallest and the greatest value) as the range , therefore it represents the dispersion better than the range.
- The standard deviation has the same measuring units of the original data.
- The values which are more homogeneous have less dispersion and their standard deviation is small.
- If the standard deviation equals zero that means the all values are equal , it is the perfect homogeneous case (the vanished dispersion)

TRY by yourself 3 ***For the following frequency distribution , calculate :***

1 The mean. 2 The standard deviation.

Sets	1 –	3 –	5 –	7 –	9 – 11
Frequency	7	3	5	3	2

Using the calculator to calculate the standard deviation :

- We can use the calculator CASIO ($f X - 82$ ES , $f X - 85$ ES , $f X - 500$ ES , $f X - 95$ ES Plus , $f X - 991$ ES Plus) to calculate the standard deviation.
- The following steps show how to solve the previous example (example 3) using the calculator :
- We will use the calculator ($f X - 95$ ES Plus)

Step (1)

Before inserting the data of the previous example, we should set the calculator system by pressing the following keys from left :

Then the screen will appear as in the opposite figure.

Step (2)

- We insert the values (X) in the case of simple frequency distribution or the centres of sets (X) in the case of frequency distribution of sets in the first column (X)
- With respect to the previous example :

We insert the centres of sets :

40 , 50 , 60 , 70 , 80 and 90 by pressing the following keys from left as follows :

→ 4 0 = 5 0 = 6 0 =

7 0 = 8 0 = 9 0 =

Then the screen will appear as in the opposite figure.

Step (3)

Use the key REPLAY to move to the second column (FREQ), then insert frequencies

10 , 14 , 20 , 28 , 20 and 8 by pressing the following keys from left as follows :

1 0 = 1 4 = 2 0 = 2 8 = 2 0 = 8 = AC

Thus we insert the data of the previous example on the calculator.

Step (4)

For finding the value of the standard deviation , we press the following keys from left :

SHIFT 1 4 (VAR) 3 (σx) =

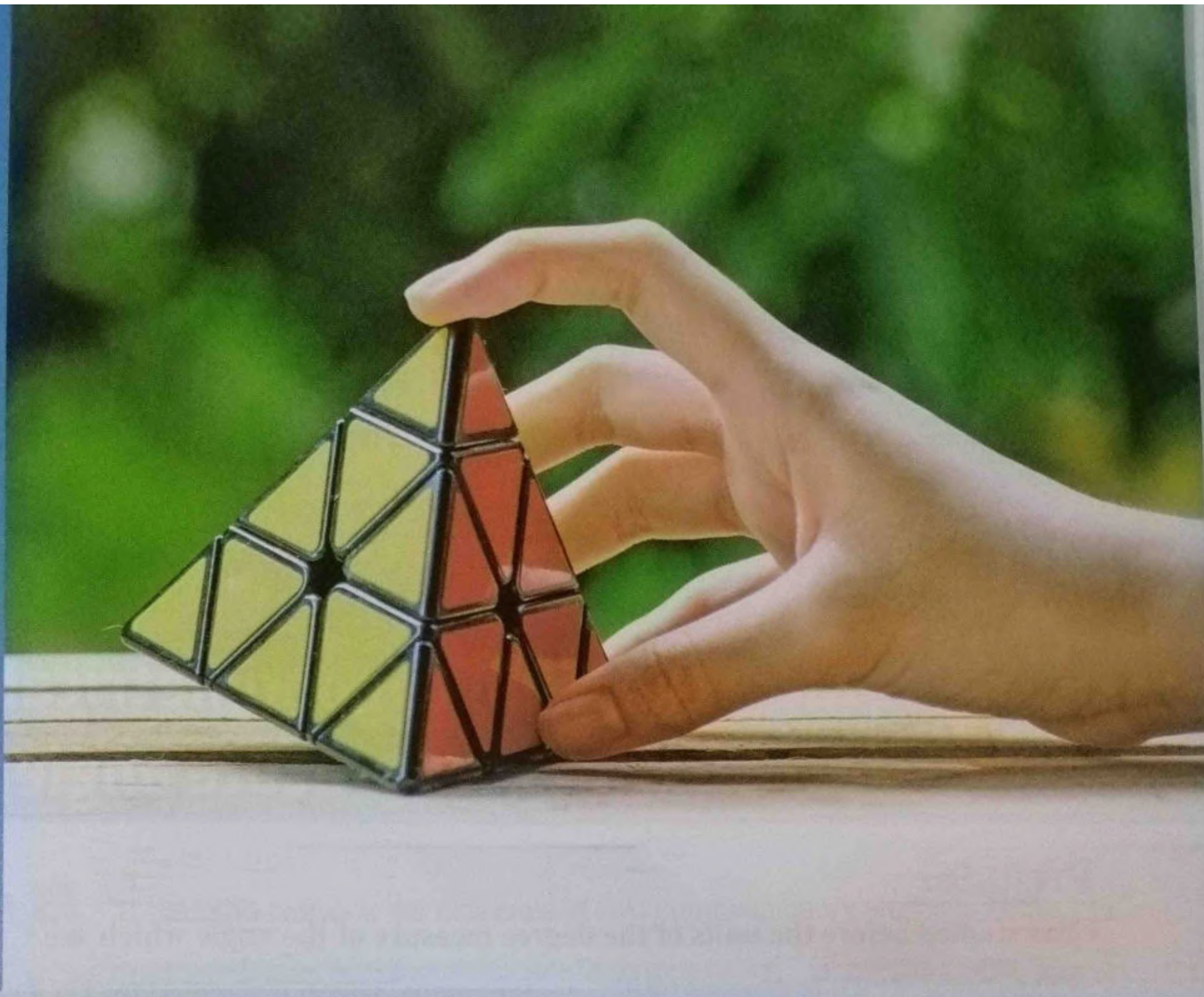
Then the screen will appear as in the opposite figure.

$\therefore$ Standard deviation $\sigma \simeq 14.15$

Second

Trigonometry and Geometry

UNIT FOUR



Trigonometry

Lessons of the unit :

1. The main trigonometrical ratios of the acute angle.
2. The main trigonometrical ratios of some angles.

Use your smart phone or tablet to scan the **QR code** and enjoy watching videos

Unit Objectives : By the end of this unit, student should be able to :

- recognize the main trigonometrical ratios of the acute angle.
- recognize the main trigonometrical ratios of the angles of measures 30° , 60° and 45°
- find the main trigonometrical ratios of a given angle.
- find the measure of an angle if one of its trigonometrical ratios is given.
- use the calculator to find the main trigonometrical ratios.

Enriching information :

- Trigonometry is one of mathematics branches and it is one of the general geometry branches , it concerneds studying the relations between the sides and angles of the triangle and the trigonometric ratios as the sine and cosine of the angle.
- Ancient Egyptians were the first to use the trigonometric theorems and rules in building pyramids and temples.
- Trigonometry has many applications in surveying roads and manufactoring motors , TV sets , football playgrounds , calculating geographic distances and astronomy discovering.

Lesson 1 The main trigonometrical ratios of the acute angle

Prelude

• **You studied before the units of the degree measure of the angle which are :**

The degree which is denoted by 1° , the minute which is denoted by $1'$ and the second which is denoted by $1''$

For example:

The angle whose measure is 22 degrees , 36 minutes and 48 seconds is written as $22^\circ\ 36'\ 48''$

The relation between the degrees, the minutes and the seconds

• $1^\circ = 60'$

i.e. $1^\circ = 60 \times 60 = 3600''$

• $1' = 60''$

Example 1

1. Write in degrees : $22^\circ\ 36'\ 48''$
2. Write in degrees , minutes and seconds : 45.18°

Solution

1. Convert the minutes into degrees , as the following :

$36' = \frac{36}{60} = 0.6^\circ$

Convert the seconds into degrees , as the following :

$48'' = \frac{48}{3600} = 0.01\dot{3}^\circ$

i.e. $22^\circ\ 36'\ 48'' = 22^\circ + 0.6^\circ + 0.01\dot{3}^\circ = 22.61\dot{3}^\circ$

Remember that

$0.00\dot{3}$ is read as the recurring decimal 0.003

Another solution by using the scientific calculator :

Press the keys in sequence from left as follows :

[2] [2] [°'"] [3] [6] [°'"] [4] [8] [°'"] [=] [°'"]

Then the result will be 22.61333333

2 Convert $0.18°$ into minutes as the following : $0.18 \times 60 = 10.8'$

Convert $0.8'$ into seconds as the following : $0.8 \times 60 = 48''$

i.e. $45.18° = 45°\ 10'\ 48''$

Another solution by using the scientific calculator :

Press the keys in sequence from left as follows :

[4] [5] [.] [1] [8] [=] [°'"]

Then the result will be $45°\ 10'\ 48''$

Example 2 If the ratio between the measures of two complementary angles is 7 : 9 , find the degree measure of each of them.

Solution Let the measures of the two angles be :

$7\,X$ and $9\,X$

$\therefore 7\,X + 9\,X = 90°$

$\therefore 16\,X = 90°$

$\therefore X = \frac{90°}{16} = 5.625°$

$\therefore$ The measure of the first angle

$= 5.625° \times 7 = 39.375°$

$= 39°\ 22'\ 30''$

, the measure of the second angle $= 5.625° \times 9 = 50.625° = 50°\ 37'\ 30''$

Remember that

- The sum of measures of two complementary angles = 90°
- The sum of measures of two supplementary angles = 180°
- The sum of measures of the interior angles of any triangle = 180°

TRY by yourself 1

If the ratio between the measures of two supplementary angles is 5 : 11 , find the degree measure of each of them.

The main trigonometrical ratios of the acute angle

The trigonometrical ratio of the acute angle

It is the ratio between two side lengths of the right-angled triangle that contains this angle.

There are three main trigonometrical ratios of the acute angle and they are :

1 **The sine of the angle :**

abbreviated (**sin**) and equals $\dfrac{\text{the length of the opposite side to the angle}}{\text{the length of the hypotenuse}}$

2 **The cosine of the angle :**

abbreviated (**cos**) and equals $\dfrac{\text{the length of the adjacent side to the angle}}{\text{the length of the hypotenuse}}$

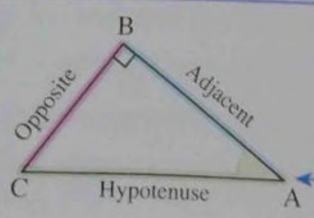
3 **The tangent of the angle :**

abbreviated (**tan**) and equals $\dfrac{\text{the length of the opposite side to the angle}}{\text{the length of the adjacent side to the angle}}$

i.e.

If Δ ABC is a right-angled triangle at B , then :

According to angle A

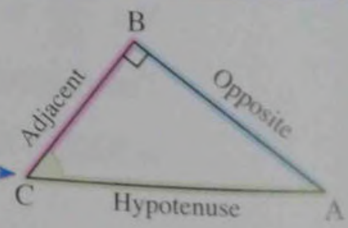


1 $\sin A = \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{BC}{AC}$

2 $\cos A = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{AB}{AC}$

3 $\tan A = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{BC}{AB}$

According to angle C



1 $\sin C = \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{AB}{AC}$

2 $\cos C = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{BC}{AC}$

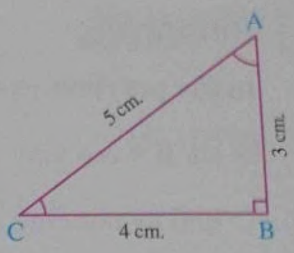
3 $\tan C = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{AB}{BC}$

For example:

In the opposite figure :

If Δ ABC is a right-angled triangle at B ,

AB = 3 cm. , BC = 4 cm. and AC = 5 cm. , then :



1. $\sin A = \frac{4}{5}$
2. $\cos A = \frac{3}{5}$
3. $\tan A = \frac{4}{3}$

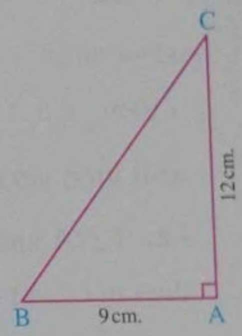
1. $\sin C = \frac{3}{5}$
2. $\cos C = \frac{4}{5}$
3. $\tan C = \frac{3}{4}$

Example 3

In the opposite figure :

Δ ABC is right-angled at A where

AB = 9 cm. and AC = 12 cm.



1. **Find each of :** sin B , cos B , tan B , sin C , cos C and tan C
2. **Prove that :** sin B cos C + cos B sin C = 1

Solution

$\because$ In Δ ABC : m ($\angle$ A) = 90°

$\therefore (BC)^2 = (AB)^2 + (AC)^2$ (Pythagoras' theorem)

$\therefore (BC)^2 = 81 + 144 = 225$ $\qquad \therefore$ BC = 15 cm.

1. $\sin B = \frac{AC}{BC} = \frac{12}{15} = \frac{4}{5}$,

   $\cos B = \frac{AB}{BC} = \frac{9}{15} = \frac{3}{5}$,

   $\tan B = \frac{AC}{AB} = \frac{12}{9} = \frac{4}{3}$,

   $\sin C = \frac{AB}{BC} = \frac{9}{15} = \frac{3}{5}$,

   $\cos C = \frac{AC}{BC} = \frac{12}{15} = \frac{4}{5}$,

   $\tan C = \frac{AB}{AC} = \frac{9}{12} = \frac{3}{4}$

2. $\sin B \cos C + \cos B \sin C = \frac{4}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{3}{5} = \frac{16}{25} + \frac{9}{25} = \frac{25}{25} = 1$

Remember Pythagoras' theorem :

If ABC is a right-angled triangle at B , then :

- $(AC)^2 = (AB)^2 + (BC)^2$
- $(AB)^2 = (AC)^2 - (BC)^2$
- $(BC)^2 = (AC)^2 - (AB)^2$

TRY by yourself 2

XYZ is a right-angled triangle at Y , XY = 4 cm. and XZ = 5 cm.

1. ***Find the value of :*** 2 sin X cos X
2. ***Prove that :*** sin X cos Z + cos X sin Z = 1

Remarks

In the previous example, note that :

❶ $\sin B = \cos C = \frac{4}{5}$, $\sin C = \cos B = \frac{3}{5}$

and by noticing : $m(\angle B) + m(\angle C) = 90°$ "Complementary angles"

We can deduce that :

The **sine** of any acute angle equals the **cosine** of its complementary angle

i.e. If $m(\angle A) + m(\angle B) = 90°$

, then $\boxed{\sin A = \cos B}$ $\boxed{\sin B = \cos A}$

and vice versa

i.e. If $\angle A$ and $\angle B$ are acute angles and $\sin A = \cos B$

then $m(\angle A) + m(\angle B) = 90°$

❷ $\frac{\sin B}{\cos B} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$, $\tan B = \frac{4}{3}$ $\therefore$ $\boxed{\tan B = \frac{\sin B}{\cos B}}$

, $\frac{\sin C}{\cos C} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$, $\tan C = \frac{3}{4}$ $\therefore$ $\boxed{\tan C = \frac{\sin C}{\cos C}}$

Generally : The tangent of the angle = $\dfrac{\text{The sine of the angle}}{\text{The cosine of the angle}}$

Example 4

Choose the correct answer from the given ones :

1 If $\sin 30° = \cos \theta$ where θ is the measure of an acute angle , then $\theta =$ ....

(a) 15° (b) 30° (c) 60° (d) 90°

2 If x and y are the measures of two complementary angles and $\cos x = \frac{4}{5}$, then $\sin y =$ ....

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

3 In Δ ABC , if m ($\angle$ A) = 60° and sin B = cos B , then m ($\angle$ C) = ...

(a) 30° (b) 75° (c) 90° (d) 105°

4 If Δ ABC is right-angled at B , then sin A + 2 cos C = ...

(a) 2 sin C (b) 3 sin A (c) 2 sin A (d) 3 cos A

Solution

1 (c) **The reason :** $\because \sin 30° = \cos \theta$ $\therefore 30° + \theta = 90°$

$\therefore \theta = 60°$

2 (b) **The reason :** $\because$ X and y are the measures of two complementary angles

$\therefore \sin y = \cos X$ $\therefore \sin y = \frac{4}{5}$

3 (b) **The reason :** $\because \sin B = \cos B$ $\therefore m(\angle B) = 45°$

$\therefore m(\angle C) = 180° - (45° + 60°) = 75°$

4 (b) **The reason :** $\because m(\angle B) = 90°$ $\therefore m(\angle A) + m(\angle C) = 90°$

$\therefore \sin A = \cos C$

$\therefore \sin A + 2\cos C = \sin A + 2\sin A = 3\sin A$

TRY yourself 3

Choose the correct answer from the given ones :

1 If m ($\angle$ A) = 75° , sin B = cos A where B is an acute angle , then m ($\angle$ B) = ...

(a) 15° (b) 45° (c) 75° (d) 105°

2 In Δ ABC , if m ($\angle$ B) = 90° , then cos A + sin C = ...

(a) 2 cos C (b) 2 cos A (c) 2 sin A (d) tan A

Example 5

ABC is a triangle in which : AB = AC = 10 cm. , BC = 12 cm. , $\overrightarrow{AD}$ is drawn perpendicular to $\overline{BC}$ to cut it at D

1 **Find the value of :** sin B + cos C

2 **Find the value of :** tan ($\angle$ CAD)

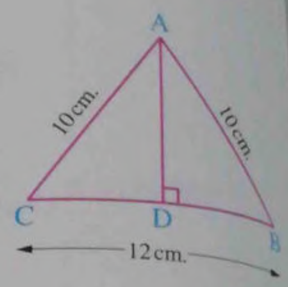
3 **Show that :** sin C + cos C > 1 **and find the value of :** $\sin^2 C + \cos^2 C$ **and deduce that :** $\sin^2 C + \cos^2 C < \sin C + \cos C$

Solution

$\because \overline{AD} \perp \overline{BC}$ and $AB = AC$

$\therefore$ D is the midpoint of $\overline{BC}$

$\therefore BD = DC = 6$ cm.



In Δ ADB :

$\because m(\angle ADB) = 90°$

$\therefore (AD)^2 = (AB)^2 - (BD)^2$ (Pythagoras' theorem)

$\therefore (AD)^2 = 100 - 36 = 64$ $\qquad \therefore AD = 8$ cm.

1 $\because \sin B = \frac{AD}{AB} = \frac{8}{10} = \frac{4}{5}$, $\cos C = \frac{CD}{AC} = \frac{6}{10} = \frac{3}{5}$

$\therefore \sin B + \cos C = \frac{4}{5} + \frac{3}{5} = \frac{7}{5}$

2 $\tan(\angle CAD) = \frac{CD}{AD} = \frac{6}{8} = \frac{3}{4}$

3 $\because \sin C = \frac{AD}{AC} = \frac{8}{10} = \frac{4}{5}$, $\cos C = \frac{3}{5}$

$\therefore \sin C + \cos C = \frac{4}{5} + \frac{3}{5} = \frac{7}{5}$ $\qquad \therefore \sin C + \cos C > 1$

, $\sin^2 C + \cos^2 C = \left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 = \frac{16}{25} + \frac{9}{25} = 1$

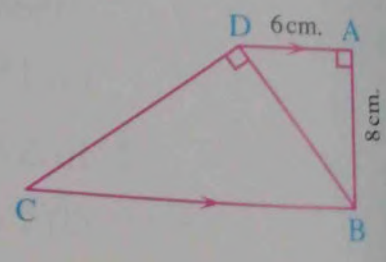
$\therefore \sin^2 C + \cos^2 C < \sin C + \cos C$

Example 6 **In the opposite figure :**

ABCD is a quadrilateral in which :

$m(\angle A) = m(\angle BDC) = 90°$

, $\overline{AD} \,//\, \overline{BC}$, $AD = 6$ cm. and $AB = 8$ cm.



Find the length of $\overline{DC}$

Solution

In Δ ABD :

$\because m(\angle A) = 90°$

$\therefore (DB)^2 = (AB)^2 + (AD)^2 = 64 + 36 = 100$

$\therefore DB = 10$ cm.

, $\because \overline{AD} \parallel \overline{BC}$ and $\overleftrightarrow{BD}$ is a transversal

$\therefore$ m ($\angle$ ADB) = m ($\angle$ DBC) "Alternate angles"

$\therefore$ tan ($\angle$ ADB) = tan ($\angle$ DBC)

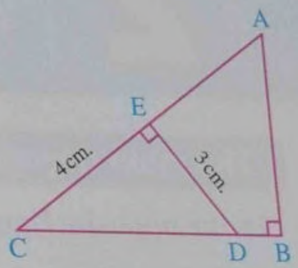
$\therefore \frac{AB}{AD} = \frac{DC}{BD}$ $\qquad\qquad \therefore \frac{8}{6} = \frac{DC}{10}$

$\therefore DC = \frac{10 \times 8}{6} = 13\frac{1}{3}$ cm. (The req.)

Notice that : ***Also , you can solve this example by using the similarity.***

TRY by yourself 4

In the opposite figure :



ABC is a triangle in which :

m ($\angle$ B) = 90° , D $\in \overline{BC}$, E $\in \overline{AC}$

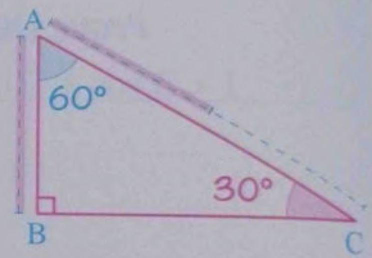
where $\overline{DE} \perp \overline{AC}$, DE = 3 cm. and EC = 4 cm.

Prove that : sin A cos C + sin C cos ($\angle$ EDC) = 1

Lesson 2 The main trigonometrical ratios of some angles

The main trigonometrical ratios of the angles measuring 30° and 60°

In the opposite figure :



ABC is a right-angled triangle at B in which : m ($\angle$ A) = 60° and m ($\angle$ C) = 30° and it is called "thirty and sixty triangle".

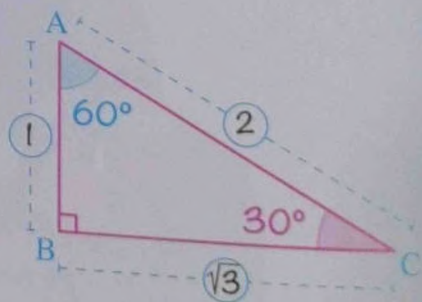
And in it , the length of the side opposite to the angle of measure 30° equals half the length of the hypotenuse.

i.e. $AB = \frac{1}{2} AC$

Assume that : The length of $\overline{AB} = \ell$ length unit , then the length of $\overline{AC} = 2\,\ell$ length unit.

By applying Pythagoras' theorem to find the length of $\overline{BC}$ **, we find that :**

$$BC = \sqrt{(AC)^2 - (AB)^2} = \sqrt{4\ell^2 - \ell^2} = \sqrt{3\,\ell^2} = \sqrt{3}\,\ell \text{ length unit.}$$



i.e. $AB : AC : BC = \ell : 2\ell : \sqrt{3}\,\ell = 1 : 2 : \sqrt{3}$

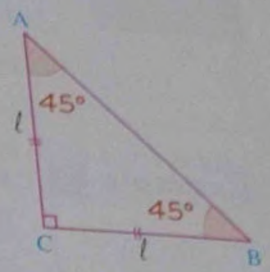
And from Δ ABC , we can find the main trigonometrical ratios of the angles measuring 30° and 60° as follows :

30° $\sin 30° = \frac{AB}{AC} = \frac{1}{2}$	$\cos 30° = \frac{BC}{AC} = \frac{\sqrt{3}}{2}$	$\tan 30° = \frac{AB}{BC} = \frac{1}{\sqrt{3}}$
60° $\sin 60° = \frac{BC}{AC} = \frac{\sqrt{3}}{2}$	$\cos 60° = \frac{AB}{AC} = \frac{1}{2}$	$\tan 60° = \frac{BC}{AB} = \sqrt{3}$

The main trigonometrical ratios of the angle measuring 45°

In the opposite figure :

ABC is an isosceles triangle where $AC = BC = \ell$ length unit and $m(\angle C) = 90°$ $\therefore m(\angle A) = m(\angle B) = 45°$

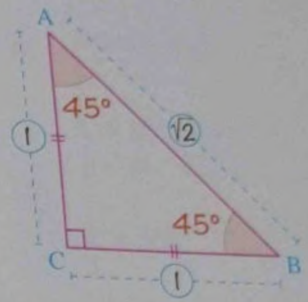


By applying Pythagoras' theorem to find the length of $\overline{AB}$

, we find that : $AB = \sqrt{(AC)^2 + (BC)^2}$

$$= \sqrt{\ell^2 + \ell^2} = \sqrt{2\ell^2} = \sqrt{2}\,\ell \text{ length unit.}$$

i.e. $AC : BC : AB = \ell : \ell : \sqrt{2}\,\ell = 1 : 1 : \sqrt{2}$



From ΔABC , we can find the main trigonometrical ratios of the angle measuring 45° as follows :

45°		
$\sin 45° = \frac{1}{\sqrt{2}}$	$\cos 45° = \frac{1}{\sqrt{2}}$	$\tan 45° = 1$

* **And the following table summarizes the main trigonometrical ratios of the angles whose measures are 30° , 60° and 45° :**

The measure of the angle / The trigonometrical ratio	30°	60°	45°
sin	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
tan	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1

Example 1 **Find the value of :** $\sin 30° \cos 60° + \cos^2 30° + 5 \tan 45° - 10 \cos^2 45°$

Solution The expression $= \frac{1}{2} \times \frac{1}{2} + \left(\frac{\sqrt{3}}{2}\right)^2 + 5 \times 1 - 10 \times \left(\frac{1}{\sqrt{2}}\right)^2$

$$= \frac{1}{4} + \frac{3}{4} + 5 - \frac{10}{2} = 1 + 5 - 5 = 1$$

Example 2 **Prove that :** $\sin^2 60° + \sin^2 45° + \sin^2 30° = \cos^2 30° + \frac{1}{3}\tan^2 60° - \cos^2 60°$

Solution L.H.S. $= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{2} + \frac{1}{4} = \frac{3}{2}$

R.H.S. $= \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{3}\left(\sqrt{3}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} + 1 - \frac{1}{4} = \frac{3}{2}$

$\therefore$ The two sides are equal.

TRY by yourself 1

1 *Find the value of :* (1) $\cos 60° + \sin 30° - \tan 45°$ (2) $\sin^2 30° + \sin^2 60°$

2 *Prove that :* $2 \sin 30° + 4 \cos 60° = \tan^2 60°$

Example 3 Find the value of X which satisfies :

1 $X \sin 30° \cos^2 45° = \cos^2 30°$

2 $2 \sin X = \tan^2 60° - 2 \tan 45°$ where X is the measure of an acute angle.

Solution

1 $\because X \sin 30° \cos^2 45° = \cos^2 30°$ $\therefore X \times \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$

$\therefore \frac{1}{4} X = \frac{3}{4}$ $\therefore X = 3$

2 $\because 2 \sin X = \tan^2 60° - 2 \tan 45°$ $\therefore 2 \sin X = \left(\sqrt{3}\right)^2 - 2 \times 1 = 3 - 2 = 1$

$\therefore \sin X = \frac{1}{2}$ $\therefore X = 30°$

TRY by yourself 2

Find the value of X which satisfies :

1 $X \cos 30° = \tan 60°$

2 $\tan X = \frac{2 \tan 30°}{1 - \tan^2 30°}$ where X is the measure of an acute angle.

Example 4 Choose the correct answer from the given ones :

1 If $\cos 4X = \frac{1}{2}$ where X is the measure of an acute angle , then $X =$ ....

(a) 15° (b) 30° (c) 45° (d) 60°

2 If $\tan (X + 10°) = \sqrt{3}$ where $(X + 10°)$ is the measure of an acute angle , then $X =$ ....

(a) 20° (b) 40° (c) 50° (d) 70°

3 If $\sin X = \frac{1}{2}$ where X is the measure of an acute angle , then $\sin 2X =$ ....

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

4 If $\cos(X + 15°) = \frac{1}{2}$ where $(X + 15°)$ is the measure of an acute angle , then $\sin(75° - X) =$ ....

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

5 If $4\cos 60° \sin 30° = \tan X$ where X is the measure of an acute angle , then $X =$ ....

(a) 30° (b) 45° (c) 60° (d) 90°

Solution

1 **(a) The reason :** $\because \cos 4X = \frac{1}{2}$ $\therefore 4X = 60°$

$\therefore X = \frac{60°}{4} = 15°$

2 **(c) The reason :** $\because \tan(X + 10°) = \sqrt{3}$ $\therefore X + 10° = 60°$

$\therefore X = 60° - 10° = 50°$

3 **(c) The reason :** $\because \sin X = \frac{1}{2}$ $\therefore X = 30°$

$\therefore \sin 2X = \sin 60° = \frac{\sqrt{3}}{2}$

4 **(a) The reason :** $\because \cos(X + 15°) = \frac{1}{2}$ $\therefore X + 15° = 60°$

$\therefore X = 60° - 15° = 45°$

$\therefore \sin(75° - X) = \sin(75° - 45°) = \sin 30° = \frac{1}{2}$

5 **(b) The reason :** $\because 4\cos 60° \sin 30° = 4 \times \frac{1}{2} \times \frac{1}{2} = 1$

$\therefore \tan X = 1$ $\therefore X = 45°$

TRY by yourself 3

Choose the correct answer from the given ones :

1 $2\cos^2 30° - 1 =$ ....

(a) cos 60° (b) sin 60° (c) 2 sin 30° (d) tan 60°

2 If $\tan(X + 15°) = 1$ where $(X + 15°)$ is the measure of an acute angle , then $X =$ ....

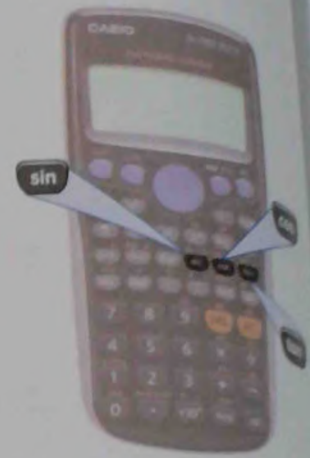
(a) 15° (b) 30° (c) 45° (d) 60°

3 If $\left(\cos X , \frac{1}{2}\right) = \left(\frac{1}{2} , \sin y\right)$ where X and y are the measures of two acute angles , then $X + y =$ ....

(a) 30° (b) 60° (c) 90° (d) 120°

Using the calculator

First Finding the main trigonometrical ratios of a given angle



In the calculator , there are three keys : [sin] , [cos] , [tan]

1. The key [sin] means sine.
2. The key [cos] means cosine.
3. The key [tan] means tangent.

By using these keys we can find the main trigonometrical ratios of any angle if its measure is known.

Example 5 **By using the calculator , find the value of each of the following approximated to the nearest four decimals :**

1. $\sin 36^\circ$
2. $\cos 72^\circ\ 35'$
3. $\tan 50^\circ\ 46'\ 25''$

Solution Use the keys of the calculator as the following sequence from left :

1. [sin] [3] [6] [=]

$\therefore \sin 36^\circ \simeq 0.5878$

2. [cos] [7] [2] [0,,,] [3] [5] [0,,,] [=]

$\therefore \cos 72^\circ\ 35' \simeq 0.2993$

3. [tan] [5] [0] [0,,,] [4] [6] [0,,,] [2] [5] [0,,,] [=]

$\therefore \tan 50^\circ\ 46'\ 25'' \simeq 1.2250$

TRY by yourself 4 ***By using the calculator, find the value of each of the following approximated to the nearest three decimals :***

1. $\sin 35^\circ\ 12'$
2. $\tan 58^\circ\ 24'$

Second: Finding the measure of an angle if one of its trigonometrical ratios is given

If $\sin A = 0.6218$, then A is the measure of the angle whose sine is 0.6218

To find the measure of this angle , we can use the calculator as the following sequence from left :

SHIFT sin . 6 2 1 8 = °''' Then $A \simeq 38^\circ\ 26'\ 52''$

Example 6 **Find A in each of the following , where A is the measure of an acute angle :**

1 $\sin A = 0.8$ 2 $\cos A = 0.7152$ 3 $\tan A = 1.5156$

Solution Use the keys of the calculator as the following sequence from left :

1 SHIFT sin . 8 = °'''

$\therefore A \simeq 53^\circ\ 7'\ 48''$

2 SHIFT cos . 7 1 5 2 = °'''

$\therefore A \simeq 44^\circ\ 20'\ 25''$

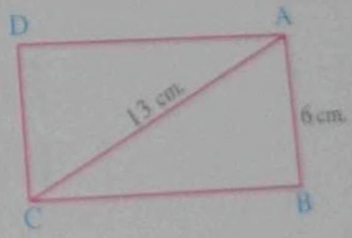
3 SHIFT tan 1 . 5 1 5 6 = °'''

$\therefore A \simeq 56^\circ\ 34'\ 59''$

TRY yourself 5 *Using the calculator , find A in each of the following where A is the measure of an acute angle :*

1 $\sin A = 0.3945$ 2 $\cos A = 0.3824$

Example 7 **In the opposite figure :**



ABCD is a rectangle in which :

AB = 6 cm. and AC = 13 cm. **Find :**

1 m ($\angle$ ACB)

2 The area of the rectangle ABCD to the nearest one decimal digit.

Solution

$\because$ ABCD is a rectangle.

$\therefore m(\angle B) = 90^\circ$

In ΔABC :

$\sin(\angle ACB) = \frac{AB}{AC} = \frac{6}{13}$

And by using the calculator :

$\therefore m(\angle ACB) \simeq 27^\circ\ 29'\ 11''$ (First req.)

$, \because \cos(\angle ACB) = \frac{BC}{AC}$

$\therefore \cos 27^\circ\ 29'\ 11'' = \frac{BC}{13}$

$\therefore BC = 13 \times \cos 27^\circ\ 29'\ 11''$

$\therefore$ The area of the rectangle ABCD = AB × BC

$= 6 \times 13 \times \cos 27^\circ\ 29'\ 11'' \simeq 69.2 \text{ cm}^2.$

(Second req.)

Notice that :

Also , you can find the length of $\overline{BC}$ by using Pythagoras' theorem in ΔABC

TRY by yourself 6

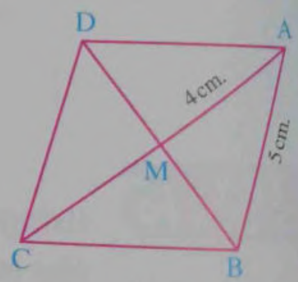
In the opposite figure :

ABCD is a rhombus , whose diagonals intersect at M

If AB = 5 cm. and AM = 4 cm.

, ***find :***

1. $m(\angle BAD)$
2. The area of the rhombus ABCD



UNIT FIVE

Analytical geometry

Use your smart phone or tablet to scan the **QR code** and enjoy watching videos

Lessons of the unit :

1. Distance between two points.
2. The two coordinates of the midpoint of a line segment.
3. The slope of the straight line.
4. The equation of the straight line given its slope and the intercepted part of y-axis.

Unit Objectives : By the end of this unit, student should be able to :

- find the distance between two points in the coordinates plane.
- find the two coordinates of the midpoint of a line segment.
- recognize the slope of the straight line.
- find the slope of the straight line given the measure of the positive angle which this straight line makes with the positive direction of the x-axis.
- recognize the relation between the two slopes of two parallel straight lines.
- recognize the relation between the two slopes of two perpendicular straight lines.
- find the slope of the straight line and the length of the intercepted part from y-axis given the equation of the straight line.
- find the equation of the straight line given its slope and the length of the intercepted part from y-axis.
- use the slope of the straight line for solving some life problems.

Lesson 1 Distance between two points

Let M (x_1 , y_1) and N (x_2 , y_2) be two points in the same coordinates plane.

From the geometry of the figure we find that :

$NL = NB - LB = y_2 - y_1$

Generally $NL = |y_2 - y_1|$

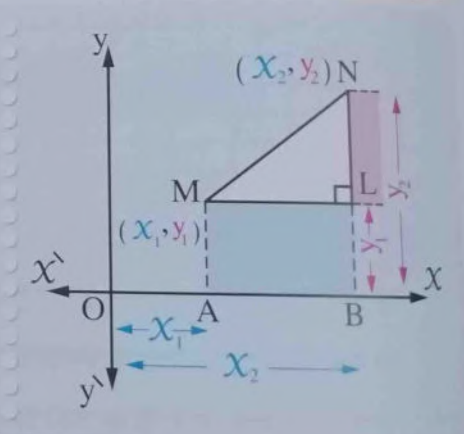
Similarly $LM = BO - AO = x_2 - x_1$

Generally $LM = |x_2 - x_1|$

$\because$ Δ NLM is right-angled at L

$\therefore (MN)^2 = (LM)^2 + (NL)^2$

$\therefore (MN)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$



$\therefore MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

i.e.

The distance between the two points M and N equals $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

and we know that :

$(x_2 - x_1)^2 = (x_1 - x_2)^2$, and similarly : $(y_2 - y_1)^2 = (y_1 - y_2)^2$, therefore :

The distance between the two points M and N equals also $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Generally :

The distance between two points =

$\sqrt{\text{square of the difference between } x\text{-coordinates} + \text{square of the difference between } y\text{-coordinates}}$

For example : If A (3 , 6) and B (– 1 , 4) , then

the length of $\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1-3)^2 + (4-6)^2} = \sqrt{(-4)^2 + (-2)^2}$

$= \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$ length unit.

you can find the length of $\overline{AB}$ as follows : the length of $\overline{AB}$

$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(3-(-1))^2 + (6-4)^2} = \sqrt{4^2 + 2^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$ length unit.

Example 1 **Choose the correct answer from the given ones :**

1 The distance between the two points (6 , 0) and (0 , 8) equals .... length unit.

(a) 12 (b) 10 (c) 8 (d) 6

2 The distance between the point A $(\sqrt{2}, 4)$ and the origin point equals .... length unit.

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $3\sqrt{2}$ (d) $4\sqrt{2}$

3 The distance between the point (– 7 , – 3) and y-axis equals .... length unit.

(a) – 7 (b) – 3 (c) 7 (d) 3

4 ABCD is a rectangle in which A (– 1 , – 3) and C (2 , 1) , then the length of $\overline{BD}$ = ... length unit.

(a) 25 (b) 5 (c) $\sqrt{7}$ (d) $\sqrt{5}$

Solution

1 (b) **The reason :** The required distance $= \sqrt{(0-6)^2 + (8-0)^2}$

$= \sqrt{(-6)^2 + (8)^2} = \sqrt{36 + 64}$

$= \sqrt{100} = 10$ length unit.

2 (c) **The reason :** The distance between any point (x , y) and the origin point (0 , 0) equals $\sqrt{x^2 + y^2}$

∴ The required distance $= \sqrt{\left(\sqrt{2}\right)^2 + (4)^2}$

$= \sqrt{2 + 16} = \sqrt{18} = \sqrt{9 \times 2}$

$= 3\sqrt{2}$ length unit.

3 (c) **The reason :** The distance between the point (– 7 , – 3) and $\overleftrightarrow{yy'}$ equals | – 7 | because the distance is a positive number.

∴ The required distance = 7 length unit.

4 **(b) The reason :** The length of $\overline{BD}$ = the length of $\overline{AC}$ because the rectangle diagonals are equal in length.

$\therefore$ The length of $\overline{BD} = \sqrt{(2+1)^2 + (1+3)^2}$

$= \sqrt{3^2 + 4^2} = \sqrt{9 + 16}$

$= \sqrt{25} = 5$ length unit.

Example 2 If the distance between the two points $(a , 5)$ and $(3a - 1 , 1)$ equals 5 length units , **find the value of :** a

Solution $\because \sqrt{(3a - 1 - a)^2 + (1 - 5)^2} = 5$

$\therefore \sqrt{(2a - 1)^2 + (-4)^2} = 5$ "Squaring the two sides"

$\therefore (2a - 1)^2 + 16 = 25$

$\therefore (2a - 1)^2 = 9$ "Taking the square root of the two sides"

$\therefore 2a - 1 = \pm 3$

$\therefore 2a - 1 = 3$ thus , $2a = 4$ $\therefore \boxed{a = 2}$

or $2a - 1 = -3$ thus , $2a = -2$ $\therefore \boxed{a = -1}$

TRY by yourself 1

If A $(2 , 5)$ and B $(-1 , 1)$, ***find the length of :*** $\overline{AB}$

Example 3 If ABC is a triangle where A $(0 , 0)$, B $(3 , 4)$ and C $(-4 , 3)$, find the perimeter of Δ ABC

Solution $\because$ The perimeter of $\Delta ABC = AB + BC + CA$

, $AB = \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16} = \sqrt{25} = 5$ length unit.

, $BC = \sqrt{(-4 - 3)^2 + (3 - 4)^2}$

$= \sqrt{(-7)^2 + (-1)^2}$

$= \sqrt{49 + 1} = \sqrt{50} = 5\sqrt{2}$ length unit.

, $CA = \sqrt{(-4)^2 + (3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$ length unit.

$\therefore$ The perimeter of $\Delta ABC = 5 + 5\sqrt{2} + 5 = \left(10 + 5\sqrt{2}\right)$ length unit.

Example 4 **Prove that :** ΔABC is an equilateral triangle where : A (6 , 0) , B (2 , 0) and C $\left(4 , 2\sqrt{3}\right)$, then find its area.

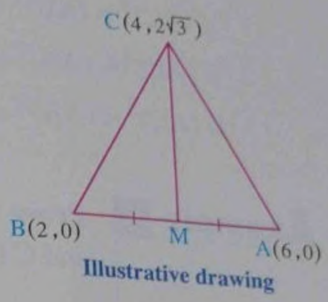
Solution $\because AB = \sqrt{(6-2)^2 + (0-0)^2} = \sqrt{16+0} = \sqrt{16} = 4$ length unit.

, $BC = \sqrt{(2-4)^2 + (0-2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$ length unit.

and $AC = \sqrt{(6-4)^2 + (0-2\sqrt{3})^2}$

$= \sqrt{4+12} = \sqrt{16} = 4$ length unit.

$\therefore AB = BC = AC$ $\qquad \therefore \Delta ABC$ is equilateral



Illustrative drawing

Let M be the midpoint of the base $\overline{AB}$

$\therefore \overline{CM} \perp \overline{AB}$

$\therefore$ By using Pythagoras' theorem , we find that :

$\therefore$ The height $MC = \sqrt{(AC)^2 - (AM)^2} = \sqrt{4^2 - 2^2} = \sqrt{12} = 2\sqrt{3}$ length unit

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times AB \times MC = \frac{1}{2} \times 4 \times 2\sqrt{3} = 4\sqrt{3}$ square unit.

TRY yourself 2

Prove that : ΔABC is an isosceles triangle where : A (3 , 3) , B (5 , 9) and C (– 1 , 7)

! Remark 1

To prove that three given points are collinear (**i.e.** They lie on one straight line) we can find the distance between each two of these points , then prove that the greatest distance equals the sum of the two other distances.

Example 5 **Prove that :** The points A (– 2 , 7) , B (– 3 , 4) and C (1 , 16) are collinear.

Solution $\because AB = \sqrt{(-2+3)^2 + (7-4)^2} = \sqrt{1+9} = \sqrt{10}$ length unit.

, $BC = \sqrt{(-3-1)^2 + (4-16)^2} = \sqrt{16+144} = \sqrt{160} = 4\sqrt{10}$ length unit.

and $AC = \sqrt{(-2-1)^2 + (7-16)^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$ length unit.

$\therefore BC = AB + AC$ $\qquad \therefore$ A , B and C are collinear.

Remark 2

- To prove that the points A , B and C are the vertices of a triangle , we can find AB , BC and AC , then prove that the sum of the smaller two lengths is greater than the third length.
- To determine the type of the triangle ABC according to its angle measures (where $\overline{AC}$ is the longest side of the triangle ABC)
, we compare between $(AC)^2$ and $(AB)^2 + (BC)^2$ as the following :

1. If $(AC)^2 > (AB)^2 + (BC)^2$, then the triangle is obtuse-angled at B
2. If $(AC)^2 = (AB)^2 + (BC)^2$, then the triangle is right-angled at B
3. If $(AC)^2 < (AB)^2 + (BC)^2$, then the triangle is acute-angled.

Example 6 **Prove that :** The triangle whose vertices are A (3 , 2) , B (−4 , 1) and C (2 , −1) is right-angled , then find its area.

Solution

$\because AB = \sqrt{(3+4)^2 + (2-1)^2}$

$= \sqrt{49+1} = \sqrt{50}$ length unit.

, $BC = \sqrt{(-4-2)^2 + (1+1)^2}$

$= \sqrt{36+4} = \sqrt{40}$ length unit.

and $AC = \sqrt{(3-2)^2 + (2+1)^2}$

$= \sqrt{1+9} = \sqrt{10}$ length unit.

$\because (AC)^2 + (BC)^2 = 10 + 40 = 50$

, $(AB)^2 = 50$

$\therefore (AC)^2 + (BC)^2 = (AB)^2$

$\therefore \Delta ABC$ is right-angled at C

$\therefore$ The area of the triangle ABC $= \frac{1}{2} AC \times BC$

$= \frac{1}{2} \times \sqrt{10} \times \sqrt{40}$

$= \frac{1}{2} \times \sqrt{10} \times 2\sqrt{10} = 10$ square unit.

TRY by yourself 3

If A (−1 , −1) , B (2 , 3) and C (6 , 0)

, ***prove that :*** ΔABC is right-angled at B , then find its area.

Remark 3

If ABCD is a quadrilateral :

1. To prove that ABCD is a parallelogram , we prove that : $AB = CD$, $BC = AD$
2. To prove that ABCD is a rhombus , we prove that : $AB = BC = CD = DA$
3. To prove that ABCD is a rectangle , we prove that : $AB = CD$, $BC = AD$, $AC = BD$
4. To prove that ABCD is a square , we prove that : $AB = BC = CD = DA$, $AC = BD$

Example 7 If A $(3 , -2)$, B $(-5 , 0)$, C $(0 , -7)$ and D $(8 , -9)$,
prove that : ABCD is a parallelogram.

Solution

$\because AB = \sqrt{(3+5)^2 + (-2-0)^2} = \sqrt{64+4}$

$= \sqrt{68}$ length unit.

, $BC = \sqrt{(-5-0)^2 + (0+7)^2} = \sqrt{25+49}$

$= \sqrt{74}$ length unit.

, $CD = \sqrt{(0-8)^2 + (-7+9)^2} = \sqrt{64+4}$

$= \sqrt{68}$ length unit.

and $DA = \sqrt{(8-3)^2 + (-9+2)^2} = \sqrt{25+49} = \sqrt{74}$ length unit.

$\therefore AB = CD$, $BC = DA$ $\quad \therefore$ ABCD is a parallelogram.

Example 8 **Prove that :** The points A $(-1 , 4)$, B $(1 , 1)$, C $(-1 , -2)$ and D $(-3 , 1)$ are the vertices of a rhombus and graph it , then find its area.

Solution

$\because AB = \sqrt{(-1-1)^2 + (4-1)^2}$

$= \sqrt{4+9} = \sqrt{13}$ length unit.

, $BC = \sqrt{(1+1)^2 + (1+2)^2}$

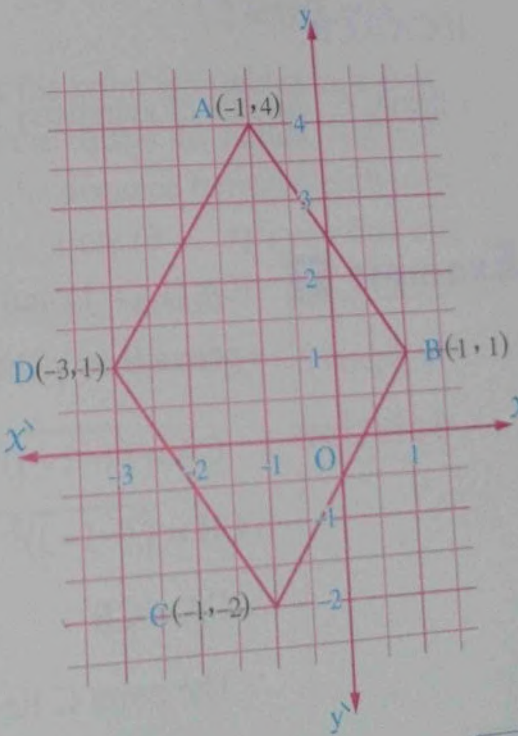
$= \sqrt{4+9} = \sqrt{13}$ length unit.

, $CD = \sqrt{(-1+3)^2 + (-2-1)^2}$

$= \sqrt{4+9} = \sqrt{13}$ length unit.

and $DA = \sqrt{(-3+1)^2 + (1-4)^2}$

$= \sqrt{4+9} = \sqrt{13}$ length unit.



$\therefore AB = BC = CD = DA$

$\therefore$ The quadrilateral ABCD is a rhombus.

$\because AC = \sqrt{(-1+1)^2 + (4+2)^2} = \sqrt{0+36} = \sqrt{36} = 6$ length unit.

$, BD = \sqrt{(1+3)^2 + (1-1)^2} = \sqrt{16+0} = \sqrt{16} = 4$ length unit.

$\therefore$ The area of the rhombus ABCD $= \frac{1}{2} \times 6 \times 4 = 12$ square unit.

TRY by yourself 4

Prove that : The points A $(-1, 3)$, B $(5, 1)$, C $(6, 4)$ and D $(0, 6)$ are the vertices of a rectangle , then find its area.

! Remark 4

- The axis of symmetry of a line segment is the straight line that is perpendicular to it at its midpoint.
- Any point on the axis of symmetry of a line segment is at equal distances from its terminals.

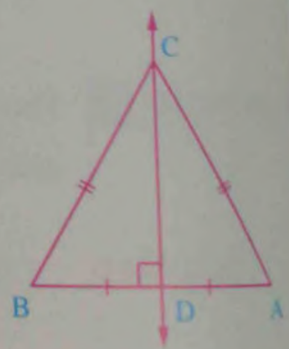
The converse is true , i.e. If a point is at equal distances from the two terminals of a line segment , then this point lies on the axis of this line segment.

For example:

In the opposite figure :

If CA = CB

, then C $\in$ the axis of symmetry of $\overline{AB}$



Example 9 If A $(1, -1)$ and B $(1, 3)$

, prove that : The point C $(-1, 1)$ lies on the axis of symmetry of $\overline{AB}$

Solution

$\because CA = \sqrt{(-1-1)^2 + (1+1)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ length unit.

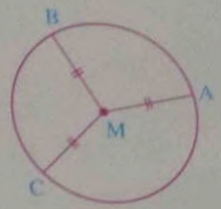
$, CB = \sqrt{(-1-1)^2 + (1-3)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ length unit.

$\therefore CA = CB$

$\therefore$ The point C lies on the axis of symmetry of $\overline{AB}$

Remark 5

- If $A \in$ the circle M , then the radius length of this circle $(r) = MA$
- To prove that : Three points as A , B and C lie on the same circle of centre M we prove that : $MA = MB = MC$
- Remember that :
  - The circumference of the circle $= 2\pi r$
  - The area of the circle $= \pi r^2$



Example 10 **Choose the correct answer from the given ones :**

1 The diameter length of the circle of centre A $(-2 , 3)$ and passing through B $(2 , -1)$ equals .... length unit.

(a) $8\sqrt{2}$ (b) $4\sqrt{2}$ (c) 5 (d) 4

2 A circle is of centre $(3 , -4)$ and its radius length is 5 length unit. Which of the following points belongs to this circle ?

(a) $(-3 , 4)$ (b) $(0 , 0)$ (c) $(5 , 0)$ (d) $(0 , 4)$

Solution

1 (a) **The reason :** $r = $ the length of $\overline{AB} = \sqrt{(2+2)^2 + (-1-3)^2}$

$$= \sqrt{(4)^2 + (-4)^2} = \sqrt{32}$$

$$= 4\sqrt{2} \text{ length unit.}$$

$\therefore$ The diameter length $= 2r = 2 \times 4\sqrt{2}$

$$= 8\sqrt{2} \text{ length unit.}$$

2 (b) **The reason :** The right answer is the point whose distance from the centre of the circle equals the radius length of the circle. Finding the distance between each point and the centre of the circle $(3 , -4)$, you find that $(0 , 0)$ is the right answer because

$$\sqrt{(3-0)^2 + (-4-0)^2} = \sqrt{9+16} = \sqrt{25} = 5 \text{ length unit} = r$$

Example 11 **Prove that :** The points A $(-6, 2)$, B $(0, 8)$ and C $(-8, 4)$ lie on the circle whose centre is M $(-4, 6)$ and find its area where $\pi \approx 3.14$

Solution

$\because$ $MA = \sqrt{(-6+4)^2 + (2-6)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ length units.

, $MB = \sqrt{(0+4)^2 + (8-6)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$ length units.

and $MC = \sqrt{(-8+4)^2 + (4-6)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$ length units.

$\therefore$ $MA = MB = MC$

$\therefore$ The points A , B and C lie on the circle M whose radius length $r = 2\sqrt{5}$ length units.

$\therefore$ The area of the circle $M = \pi r^2 \approx 3.14 \times \left(2\sqrt{5}\right)^2 \approx 62.8$ square units.

TRY yourself 5

Prove that : The points A $(-2, 0)$, B $(5, 1)$ and C $(6, -6)$ lie on the circle whose centre is M $(2, -3)$ and find the circumference of the circle in terms of π

Lesson 2

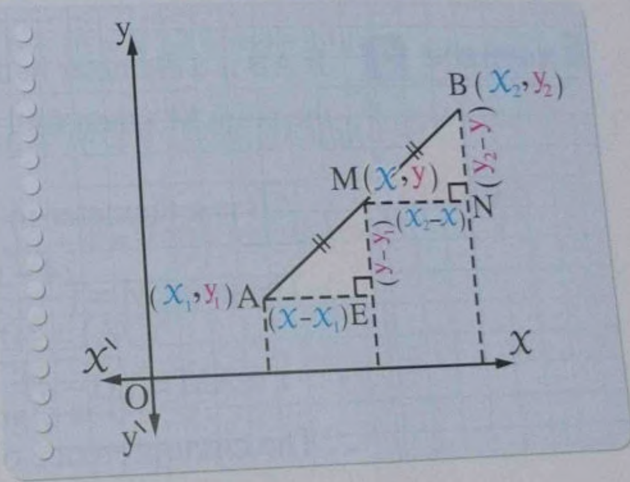
The two coordinates of the midpoint of a line segment

WATCH VIDEO

If A (x_1, y_1) and B (x_2, y_2) are two points in a coordinates plane

and M (x, y) is the midpoint of $\overline{AB}$

From the opposite figure :



Δ AEM and Δ MNB are congruent

$\therefore AE = MN$, $EM = NB$

$\therefore x - x_1 = x_2 - x$, $y - y_1 = y_2 - y$

$\therefore 2x = x_1 + x_2$, $2y = y_1 + y_2$

$\therefore x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

$$\therefore M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

For example:

If X $(3, -2)$, Y $(-1, -4)$ and M is the midpoint of $\overline{XY}$, then :

$M = \left(\frac{3 + (-1)}{2}, \frac{-2 + (-4)}{2}\right) = (1, -3)$

Example 1 If C (10 , – 4) is the midpoint of $\overline{AB}$ where A (4 , – 2) , find the point B

Solution Let B (x , y)

$\because$ C is the midpoint of $\overline{AB}$

$\therefore (10 , -4) = \left(\frac{x+4}{2} , \frac{y+(-2)}{2}\right)$

$\therefore \frac{x+4}{2} = 10$ $\quad \therefore x + 4 = 20$ $\quad \therefore x = 16$

, $\frac{y-2}{2} = -4$ $\quad \therefore y - 2 = -8$ $\quad \therefore y = -6$ $\quad \therefore B = (16 , -6)$

Notice that :
If (a , b) = (c , d) , then a = c , b = d

TRY by yourself 1

If C is the midpoint of $\overline{AB}$, then find the value of each of x and y in each of the following :

1. A (2 , 5) , B (– 2 , – 3) and C (x , y)
2. A (x , 4) , B (– 1 , – 6) and C (– 2 , y)

! Remark

If $\overline{AB}$ is a diameter in a circle of centre M , then M is the midpoint of $\overline{AB}$

Example 2 If $\overline{AB}$ is a diameter in the circle M where A (4 , – 1) and B (– 2 , 7) , find the point M , then find the circumference and the area of the circle.

Solution $\because$ $\overline{AB}$ is a diameter in the circle M $\quad \therefore$ M is the midpoint of $\overline{AB}$

$\therefore$ The point $M = \left(\frac{4+(-2)}{2} , \frac{-1+7}{2}\right) = (1 , 3)$

, $\because r = AM = \sqrt{(1-4)^2+(3+1)^2} = \sqrt{9+16} = \sqrt{25} = 5$ length units.

$\therefore$ The circumference of the circle $= 2\pi r = 2\pi \times 5 = 10\pi$ length units.

, the area of the circle $= \pi r^2 = \pi \times 5^2 = 25\pi$ square units.

Another method to calculate the radius length of the circle :

$\because AB = \sqrt{(-2-4)^2+(7+1)^2} = \sqrt{36+64} = \sqrt{100} = 10$ length units.

, $\because$ $\overline{AB}$ is a diameter $\quad \therefore r = \frac{1}{2} AB = 5$ length units.

, then complete the solution to find the circumference and the area of the circle.

TRY by yourself 2

If $\overline{AB}$ is a diameter in the circle M where A (4 , 1) and B (– 6 , 3) , then find the point M

Example 3 **Prove that :** The quadrilateral ABCD is a parallelogram where A (4 , 3) , B (0 , 2) , C (– 2 , – 3) and D (2 , – 2)

Solution $\because$ The two diagonals of the quadrilateral are $\overline{AC}$ and $\overline{BD}$

, the midpoint of $\overline{AC} = \left(\frac{4+(-2)}{2}, \frac{3+(-3)}{2}\right) = (1, 0)$

and the midpoint of $\overline{BD} = \left(\frac{0+2}{2}, \frac{2+(-2)}{2}\right) = (1, 0)$

$\therefore$ The midpoint of $\overline{AC}$ is the same midpoint of $\overline{BD}$

$\therefore$ The two diagonals bisect each other.

$\therefore$ ABCD is a parallelogram.

Notice that :
You can solve this example by using the distance between two points as the previous.

Example 4 **Prove that :** The points A (5 , 1) , B (1 , – 3) and C (– 5 , 3) are the vertices of a right-angled triangle at B , then find the point D that makes the figure ABCD a rectangle.

Solution $\because AB = \sqrt{(1-5)^2 + (-3-1)^2} = \sqrt{16+16} = \sqrt{32}$ length unit.

, $BC = \sqrt{(-5-1)^2 + (3+3)^2} = \sqrt{36+36} = \sqrt{72}$ length unit.

, $AC = \sqrt{(-5-5)^2 + (3-1)^2} = \sqrt{100+4} = \sqrt{104}$ length unit.

$\therefore (AB)^2 + (BC)^2 = 32 + 72 = 104 = (AC)^2$

$\therefore \Delta ABC$ is a right-angled triangle at B

Let D (x , y) such that the figure ABCD is a rectangle.

$\therefore \overline{AC}$ and $\overline{BD}$ bisect each other.

$\therefore$ The midpoint of $\overline{AC}$ = the midpoint of $\overline{BD}$

, $\because$ the midpoint of $\overline{AC} = \left(\frac{5-5}{2}, \frac{1+3}{2}\right) = (0, 2)$

, the midpoint of $\overline{BD} = \left(\frac{x+1}{2}, \frac{y-3}{2}\right)$

$\therefore \left(\frac{x+1}{2}, \frac{y-3}{2}\right) = (0, 2)$

$\therefore \frac{x+1}{2}=0$

$, \frac{y-3}{2}=2$ $\therefore y-3=4$ $\therefore y=7$

$\therefore D=(-1, 7)$

Example 5 **Prove that :** The triangle whose vertices are A $(-1, 4)$, B $(3, 1)$ and C $(-5, 1)$ is an isosceles triangle, then find its area.

Solution $\because AB=\sqrt{(3+1)^2+(1-4)^2}=\sqrt{16+9}$

$= 5$ length unit.

$, BC=\sqrt{(3+5)^2+(1-1)^2}=\sqrt{64}=8$ length unit.

$, AC=\sqrt{(-5+1)^2+(1-4)^2}=\sqrt{16+9}$

$= 5$ length unit.

$\therefore AB = AC$

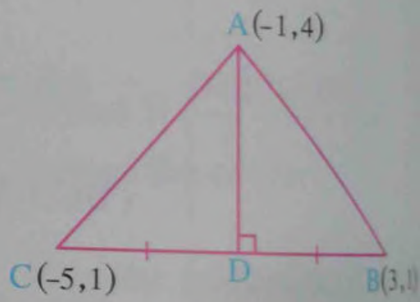
$\therefore \Delta ABC$ is an isosceles triangle.

Let D (x, y) be the midpoint of $\overline{BC}$

$\therefore D=\left(\frac{3-5}{2}, \frac{1+1}{2}\right)=(-1, 1)$

$, \because$ D is the midpoint of $\overline{BC}$

$\therefore \overline{AD} \perp \overline{BC}$



Illustrative drawing

$, \because AD=\sqrt{(-1+1)^2+(1-4)^2}=\sqrt{9}=3$ length unit.

$, BC = 8$ length unit

$\therefore$ The area of $\Delta ABC = \frac{1}{2} BC \times AD = \frac{1}{2} \times 8 \times 3 = 12$ square unit.

TRY by yourself 3

If C is the midpoint of $\overline{AB}$ where A $(2, 3)$, B $(4, -7)$ and C is the midpoint of $\overline{DE}$ where D $(-3, 5)$, find the point E

Lesson 3

The slope of the straight line

You studied before the slope of the straight line given two points on it.

If A and B are two points in the coordinates plane where $A(x_1, y_1)$ and $B(x_2, y_2)$

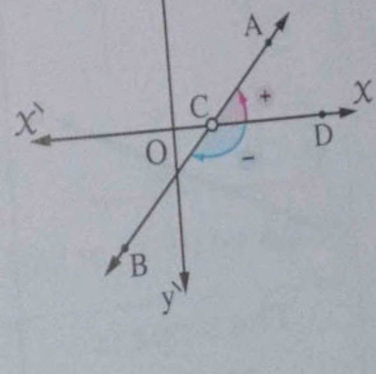
, then : The slope of the straight line $\overleftrightarrow{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}$

In this lesson , you will learn :

- How to find the slope of the straight line given the measure of the positive angle which this straight line makes with the positive direction of the x-axis.
- The relation between the slopes of two parallel straight lines.
- The relation between the slopes of two perpendicular straight lines.

And before studying these topics , you will study the positive and negative measures of an angle.

The positive measure and the negative measure of an angle

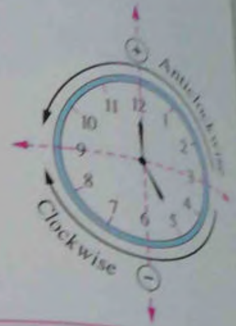


In the opposite figure :

If $\overleftrightarrow{AB}$ intersects the x-axis at the point C , then $\overleftrightarrow{AB}$ makes two angles with the positive direction of the x-axis.

• One of them is positive (i.e. It has a positive measure) taken from the positive direction of the x-axis to the straight line in the direction of anticlockwise and it is $\angle DCA$

- The another one is negative (**i.e.** It has a negative measure) taken from the positive direction of the X-axis to the straight line in the direction of clockwise and it is $\angle DCB$



The slope of the straight line

Definition

The slope of the straight line is the tangent of the positive angle which this straight line makes with the positive direction of the X-axis.

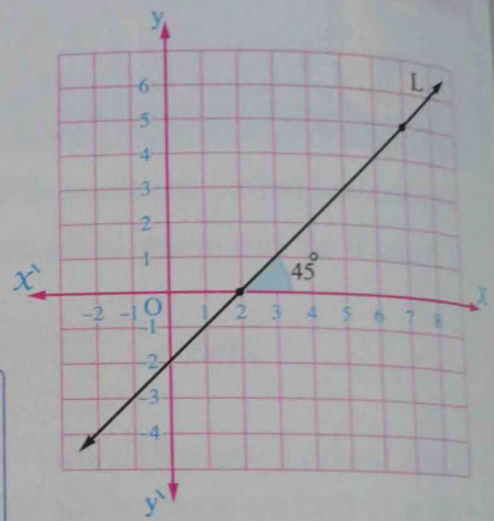
i.e. The slope of the straight line = $\tan \theta$ where θ is the measure of the positive angle which the straight line makes with the positive direction of the X-axis.

For example:

In the opposite figure :

The straight line L makes an angle of measure 45° with the positive direction of the X-axis, then :

the slope of the straight line L = $\tan 45° = 1$



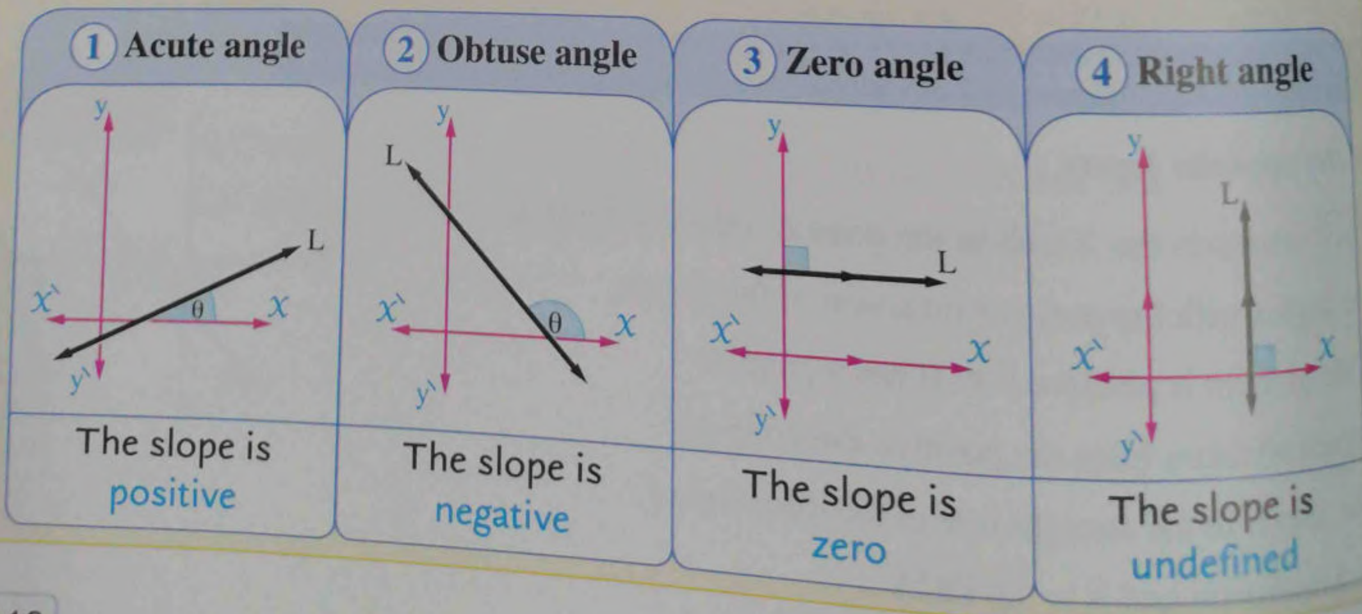
Notice that :

The straight line passes through the two points $(2, 0)$ and $(7, 5)$, then : the slope of the straight line

$$L = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-0}{7-2} = \frac{5}{5} = 1$$

Remark

The angle which the straight line L makes with the positive direction of the X-axis takes one of the following cases :



Example 1 **Find the slope of the straight line which makes a positive angle with the positive direction of X-axis where the measure of the angle is :**

1. 45°
2. $124^\circ\ 15'\ 12''$

Solution

1. The slope of the straight line $= \tan 45^\circ = 1$

Start → tan 4 5 =

2. The slope of the straight line $= \tan 124^\circ\ 15'\ 12'' \approx -1.4685$

Start → tan 1 2 4 °′″ 1 5 °′″ 1 2 °′″ =

Example 2 **Find the measure of the positive angle (θ) which the straight line makes with the positive direction of X-axis if the slope of the straight line is :**

1. 1.486
2. $-\frac{1}{\sqrt{3}}$

Solution

1. $\because m = \tan \theta$ $\qquad \therefore \tan \theta = 1.486$

$\because$ The slope is positive $\qquad \therefore \angle \theta$ is an acute angle.

Start → SHIFT tan 1 . 4 8 6 = °′″

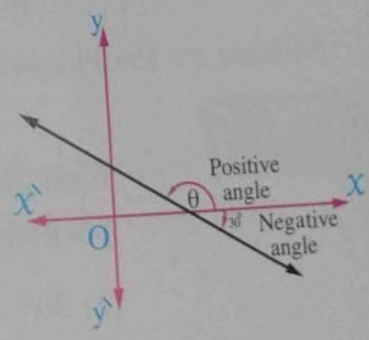
$\therefore m(\angle \theta) \approx 56^\circ\ 3'\ 41''$

2. $\because m = \tan \theta$ $\qquad \therefore \tan \theta = -\frac{1}{\sqrt{3}}$

, $\because$ the slope is negative $\qquad \therefore \angle \theta$ is an obtuse angle.

By using the calculator as follows :

Start → SHIFT tan (-) 1 ÷ √ 3 =



We will find the calculator gives the result -30°

Where the calculator is programmed to get the acute angle only either negative or positive.

But the required is the positive angle , so we find $m(\angle \theta)$ by finding the supplementary of the angle of measure 30°

Then : $m(\angle \theta) = 180^\circ - 30^\circ = 150^\circ$

Example 3 Find the measure of the positive angle (θ) which the straight line L makes with the positive direction of X-axis if the straight line (L) passes through the two points :

1 $\left(-2, \sqrt{3}\right), \left(1, 4\sqrt{3}\right)$ **2** $(-2, 3), (-3, 4)$

Solution

1 $\because$ The straight line L passes through the two points $\left(-2, \sqrt{3}\right), \left(1, 4\sqrt{3}\right)$

$\therefore$ The slope of the straight line L

$$= \frac{4\sqrt{3} - \sqrt{3}}{1-(-2)} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

Start → SHIFT tan √ 3 =

$\therefore m(\angle \theta) = 60^\circ$

Notice that : The slope is positive, then the angle is acute.

2 $\because$ The straight line passes through the two points $(-2, 3)$ and $(-3, 4)$

$\therefore$ The slope of the straight line L

$$= \frac{4-3}{-3-(-2)} = -1$$

By using the calculator as follows :

Start → SHIFT tan (-) 1 =

Notice that : The slope is negative, then the angle is obtuse.

We will find that, the calculator gives the result -45° (a negative acute angle)

We will find the positive obtuse angle as follows :

$m(\angle \theta) = 180^\circ - 45^\circ = 135^\circ$

TRY by yourself 1

1. Find the slope of the straight line which makes a positive angle with the positive direction of X-axis with measure :

   (1) 30° (2) $54^\circ\ 30'\ 6''$ (3) 120°

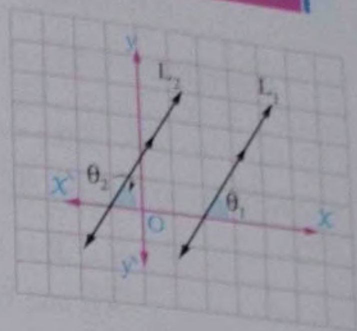
2. Find the measure of the positive angle which the straight line makes with the positive direction of X-axis if the slope of the straight line $= 6.2$

3. Find the measure of the positive angle (θ) which the straight line L makes with the positive direction of X-axis if the straight line L passes through the two points $(4, -1)$ and $(5, -3)$

The relation between the two slopes of two parallel straight lines

In the opposite figure :

If L_1 and L_2 are two parallel straight lines of slopes m_1 and m_2 respectively and make two positive angles with the positive direction of X-axis of measures θ_1 and θ_2 respectively , then



$\because L_1 \mathbin{/\!/} L_2$ $\therefore \theta_1 = \theta_2$ corresponding angles

$\therefore \tan\theta_1 = \tan\theta_2$ $\therefore m_1 = m_2$

thus we deduce the following :

If $\boxed{L_1 \mathbin{/\!/} L_2}$, then $\boxed{m_1 = m_2}$

i.e. If two straight lines are parallel , then their slopes are equal.

Also , we can deduce the opposite :

If $\boxed{m_1 = m_2}$, then $\boxed{L_1 \mathbin{/\!/} L_2}$

i.e. If the two straight lines have equal slopes , then the two straight lines are parallel.

Example 4 **Prove that :** The straight line which passes through the two points (2 , 3) and (–1 , 6) is parallel to the straight line which makes with the positive direction of X-axis a positive angle of measure 135°

Solution The slope of the first straight line $m_1 = \frac{6-3}{-1-2} = \frac{3}{-3} = -1$

, the slope of the second straight line $m_2 = \tan 135° = -1$

$\therefore m_1 = m_2$ $\therefore$ The two straight lines are parallel.

Example 5 If A (–1 , 2) , B (2 , 3) , C (– 4 , 1) and D (X , 2) are four points in the Cartesian coordinates plane and $\overleftrightarrow{AB} \mathbin{/\!/} \overleftrightarrow{CD}$, **find the value of :** X

Solution $\because \overleftrightarrow{AB} \mathbin{/\!/} \overleftrightarrow{CD}$

$\therefore$ The slope of the straight line passes through A (– 1 , 2) and B (2 , 3) is equal to the slope of the straight line passes through C (– 4 , 1) and D (X , 2)

$\therefore \frac{3-2}{2-(-1)} = \frac{2-1}{X-(-4)}$ $\therefore \frac{1}{3} = \frac{1}{X+4}$

$\therefore X + 4 = 3$ $\therefore X = -1$

Example 6 In the Cartesian coordinates plane , prove that the points A $(-1, 6)$, B $(3, -4)$ and C $(2, -1.5)$ are collinear.

Solution $\because$ The slope of $\overleftrightarrow{AB} = \frac{-4-6}{3-(-1)} = \frac{-10}{4} = -\frac{5}{2}$

, the slope of $\overleftrightarrow{BC} = \frac{-1.5-(-4)}{2-3} = \frac{2.5}{-1} = -2\frac{1}{2} = -\frac{5}{2}$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ $\quad \therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$

, $\because$ B is a common point between $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$

$\therefore$ A , B and C are collinear.

Notice that :

If the slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$, then A , B and C are collinear points.

TRY yourself 2

1. ***Prove that :*** The straight line L_1 passing through the two points $(1, 5)$ and $(-2, -1)$ is parallel to the straight line L_2 that passes through the two points $(0, -1)$ and $(5, 9)$

2. If the straight line $\overleftrightarrow{AB} \parallel$ the X-axis where A $(5, -4)$ and B $(-2, y)$, ***find the value of :*** y

The relation between the two slopes of two perpendicular (orthogonal) straight lines

If L_1 and L_2 are two straight lines of slopes m_1 and m_2

respectively and $\boxed{L_1 \perp L_2}$, then $\boxed{m_1 \times m_2 = -1}$, unless one of them is parallel to one of the coordinate axes.

i.e. The product of the slopes of the perpendicular straight lines $= -1$

and vice versa : If L_1 and L_2 are two straight lines of slopes m_1 and m_2

respectively and $\boxed{m_1 \times m_2 = -1}$, then $\boxed{L_1 \perp L_2}$

i.e. If the product of the two slopes of two straight lines equals -1 , then the two straight lines are perpendicular (orthogonal)

Example 7 **Prove that :** The straight line L_1 which passes through the two points $(-1, 4)$ and $(3, 7)$ is perpendicular to the straight line L_2 which passes through the two points $(1, 1)$ and $(4, -3)$

Solution $\because$ The slope of $L_1 = \frac{7-4}{3-(-1)} = \frac{3}{4}$, the slope of $L_2 = \frac{-3-1}{4-1} = -\frac{4}{3}$

, $\because$ the slope of $L_1 \times$ the slope of $L_2 = \frac{3}{4} \times -\frac{4}{3} = -1 \qquad \therefore L_1 \perp L_2$

Example 8 In the Cartesian coordinates plane, if the points A (1 , 7) , B (2 , 4) and C (5 , y) represent the vertices of a right-angled triangle at B ,
find the value of : y

Solution $\because$ The slope of $\overleftrightarrow{AB} = \frac{4-7}{2-1} = -3$, the slope of $\overleftrightarrow{BC} = \frac{y-4}{5-2} = \frac{y-4}{3}$,

$\because \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$ $\quad\therefore$ The slope of $\overleftrightarrow{AB} \times$ the slope of $\overleftrightarrow{BC} = -1$

$\therefore -3 \times \frac{y-4}{3} = -1$ $\quad\therefore y - 4 = 1$ $\quad\therefore y = 5$

! Remark

If $L_1 \perp L_2$, the slope of L_1 is m_1 and the slope of L_2 is m_2 where $m_1 \in \mathbb{R}^*$, $m_2 \in \mathbb{R}^*$
, then $m_2 = \frac{-1}{m_1}$, $m_1 = \frac{-1}{m_2}$

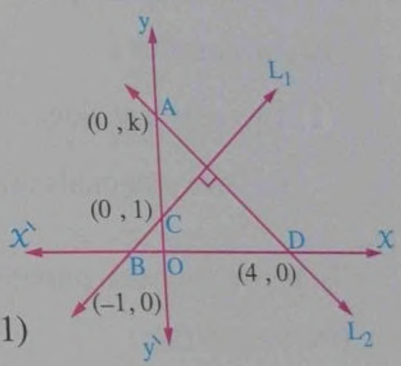
For example:

- If the slope of the straight line L is 2 , then the slope of the perpendicular to it $= -\frac{1}{2}$
- If the slope of the straight line L is $-\frac{2}{3}$, then the slope of the perpendicular to it $= \frac{3}{2}$

Example 9 **In the opposite figure :**

If $L_1 \perp L_2$

Find : The value of k



Solution $\because$ The straight line L_1 passes through the two points B (– 1 , 0) and C (0 , 1)

$\therefore$ The slope of $L_1 = \frac{1-0}{0-(-1)} = 1$

, $\because$ the straight line L_2 passes through the two points A (0 , k) and D (4 , 0)

$\therefore$ The slope of $L_2 = \frac{0-k}{4-0} = -\frac{k}{4}$ (1)

, $\because L_1 \perp L_2$, the slope of $L_1 = 1$ $\quad\therefore$ The slope of $L_2 = -1$ (2)

From (1) and (2) : $\therefore -\frac{k}{4} = -1$ $\quad\therefore k = 4$

TRY yourself 3

1. If A (– 2 , 5) , B (1 , 2) and C (3 , 4) are three points in a Cartesian coordinates plane , ***prove that :*** $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$
2. ***Prove that :*** The straight line which passes through the two points (7 , –1) and (5 , –3) is perpendicular to the straight line which makes with the positive direction of x-axis an angle of measure 135°

Remarks to solve the problems on quadrilateral

To prove that a quadrilateral is a trapezium , we prove that :

Two opposite sides are parallel and the other two sides are not parallel.

To prove that a quadrilateral is a parallelogram , we prove only one of the following properties :

1. Each two opposite sides are parallel.
2. Each two opposite sides are equal in length.
3. Two opposite sides are parallel and equal in length.
4. The two diagonals bisect each other.

To prove that a quadrilateral is a rectangle, rhombus or square, we prove at first that the quadrilateral is a parallelogram, then :

- **To prove that the parallelogram is a rectangle, we prove only one of the following two properties :**

  (1) Two adjacent sides are perpendicular.

  (2) The two diagonals are equal in length.

- **To prove that the parallelogram is a rhombus, we prove only one of the following two properties :**

  (1) Two adjacent sides are equal in length.

  (2) The two diagonals are perpendicular.

- **To prove that the parallelogram is a square, we prove only one of the following properties :**

  (1) Two adjacent sides are perpendicular and equal in length.

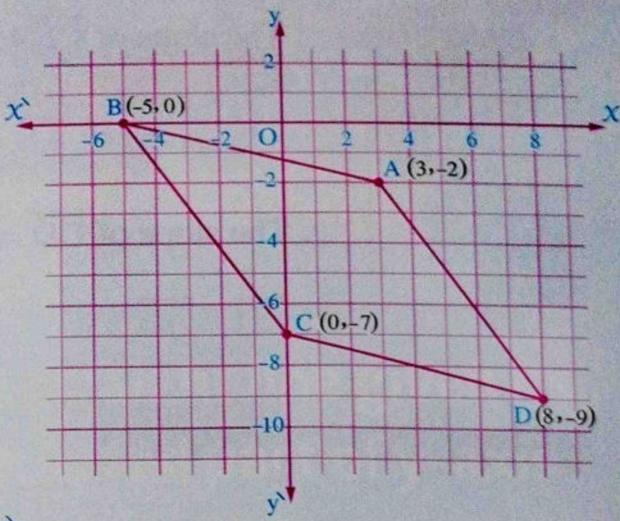
  (2) Two adjacent sides are perpendicular and its diagonals are perpendicular.

  (3) Two diagonals are equal in length and perpendicular.

  (4) Two adjacent sides are equal in length and its two diagonals are equal in length.

Example 10 On a Cartesian coordinates plane , represent the points A $(3 , -2)$, B $(-5 , 0)$, C $(0 , -7)$ and D $(8 , -9)$, then prove that the quadrilateral ABCD is a parallelogram.

Solution



$\because$ The slope of $\overleftrightarrow{AB} = \frac{0-(-2)}{-5-3}$

$= \frac{2}{-8} = -\frac{1}{4}$

, the slope of $\overleftrightarrow{CD} = \frac{-9-(-7)}{8-0}$

$= \frac{-2}{8} = -\frac{1}{4}$

$\therefore$ The slope of $\overleftrightarrow{AB}$

= the slope of $\overleftrightarrow{CD}$

$\therefore \overleftrightarrow{AB} \,//\, \overleftrightarrow{CD}$ (1)

$\because$ The slope of $\overleftrightarrow{AD} = \frac{-9-(-2)}{8-3} = \frac{-7}{5}$, the slope of $\overleftrightarrow{BC} = \frac{-7-0}{0-(-5)} = \frac{-7}{5}$

$\therefore$ The slope of $\overleftrightarrow{AD}$ = the slope of $\overleftrightarrow{BC}$

$\therefore \overleftrightarrow{AD} \,//\, \overleftrightarrow{BC}$ (2)

From (1) and (2) : $\therefore$ The quadrilateral ABCD is a parallelogram.

Example 11 **Prove that :** The points A $(2 , -2)$, B $(8 , 4)$, C $(5 , 7)$ and D $(-1 , 1)$ are vertices of the rectangle ABCD

Solution $\because$ The slope of $\overleftrightarrow{AB} = \frac{-2-4}{2-8} = \frac{-6}{-6} = 1$

, the slope of $\overleftrightarrow{CD} = \frac{7-1}{5-(-1)} = \frac{6}{6} = 1$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{CD}$ $\therefore \overleftrightarrow{AB} \,//\, \overleftrightarrow{CD}$ (1)

$\because$ The slope of $\overleftrightarrow{AD} = \frac{-2-1}{2-(-1)} = \frac{-3}{3} = -1$

, the slope of $\overleftrightarrow{BC} = \frac{4-7}{8-5} = \frac{-3}{3} = -1$

$\therefore$ The slope of $\overleftrightarrow{AD}$ = the slope of $\overleftrightarrow{BC}$ $\therefore \overleftrightarrow{AD} \,//\, \overleftrightarrow{BC}$ (2)

From (1) and (2) we deduce that the quadrilateral ABCD is a parallelogram.

$\because$ The slope of $\overleftrightarrow{AB} \times$ the slope of $\overleftrightarrow{BC} = 1 \times -1 = -1$

$\therefore \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$ $\therefore$ The quadrilateral ABCD is a rectangle.

Example 12 On a Cartesian coordinates plane , represent the points A $(-3, -3)$, B $(3, 1)$, C $(1, 5)$ and D $(-2, 3)$, then prove that the quadrilateral ABCD is a trapezium.

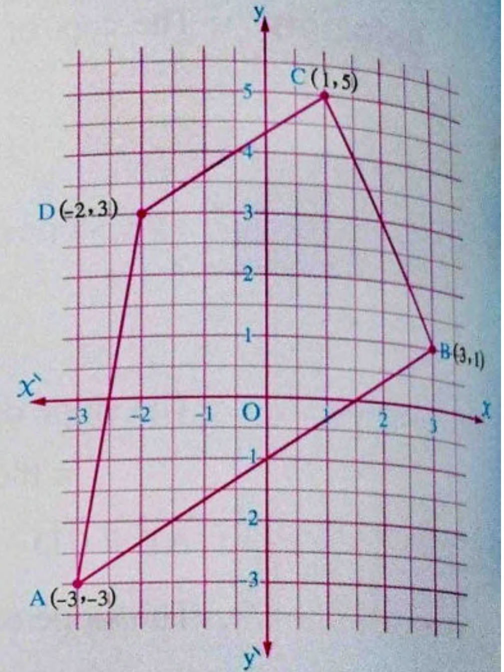
Solution

$\because$ The slope of $\overleftrightarrow{CD} = \frac{5-3}{1-(-2)} = \frac{2}{3}$

, the slope of $\overleftrightarrow{AB} = \frac{1-(-3)}{3-(-3)} = \frac{4}{6} = \frac{2}{3}$

$\therefore$ The slope of $\overleftrightarrow{CD}$ = the slope of $\overleftrightarrow{AB}$

$\therefore \overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$ (1)



The slope of $\overleftrightarrow{BC} = \frac{5-1}{1-3} = -2$

, the slope of $\overleftrightarrow{AD} = \frac{3-(-3)}{-2-(-3)} = 6$

$\therefore$ The slope of $\overleftrightarrow{BC} \neq$ the slope of $\overleftrightarrow{AD}$

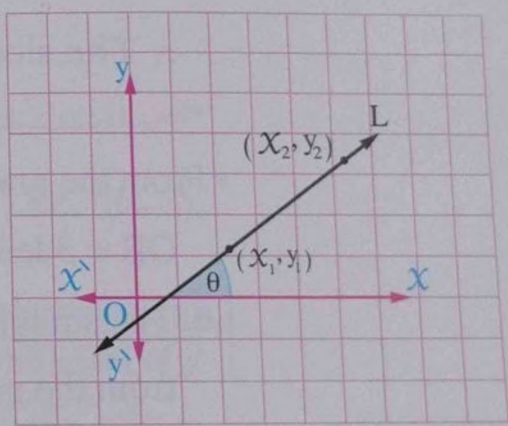
$\therefore \overleftrightarrow{BC}$ is not parallel to $\overleftrightarrow{AD}$ (2)

From (1) and (2) :

$\therefore$ The quadrilateral ABCD is a trapezium.

Lesson 4 The equation of the straight line given its slope and the intercepted part of y-axis

We studied before that the relation : $a\,X + b\,y + c = 0$ where $a \neq 0$, $b \neq 0$ together is a linear relation represented graphically by a straight line and we can find its slope (m) by one of the following methods :



1 $m = \dfrac{\text{vertical change}}{\text{horizontal change}} = \dfrac{y_2 - y_1}{X_2 - X_1}$

Where (X_1 , y_1) and (X_2 , y_2) are two points on the straight line

2 $m = \tan \theta$

Where θ is the measure of the positive angle which the straight line makes with the positive direction of the X-axis.

• **We will continue our study about this subject by studying how :**

- To find the slope of the straight line and the length of the intercepted part from y-axis if we know the equation of the straight line.
- To find the equation of the straight line if we know its slope and the length of the intercepted part from the y-axis.

First Finding the slope of the straight line and the length of the intercepted part of y-axis

Prelude example Represent graphically the relation : $2x - y + 3 = 0$ and from the graph , find the slope of the straight line which represents the relation and the intercepted part of the y-axis by the straight line.

Solution To graph the straight line which represents the relation , find two points of the points of the straight line at least , to facilitate that , put one of the variables x or y in a side of the equation

$\because 2x - y + 3 = 0 \qquad \therefore -y = -2x - 3 \qquad \therefore y = 2x + 3$

At $x = 0 \qquad \therefore y = 0 + 3 = 3$

$\therefore (0 , 3)$ is one of the points of the straight line.

At $x = -1 \qquad \therefore y = -2 + 3 = 1$

$\therefore (-1 , 1)$ is one of the points of the straight line.

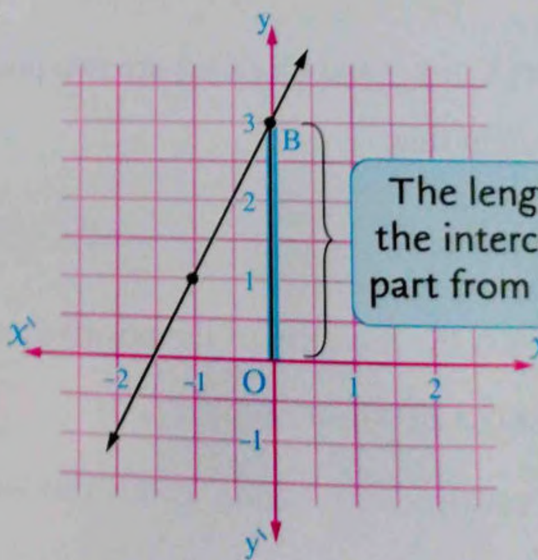
i.e. The straight line passes through the two points $(0 , 3)$ and $(-1 , 1)$

$\therefore$ The slope of the straight line $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-3}{-1-0} = \dfrac{-2}{-1} = 2$

- From the graph , we find that :

  OB = 3 length units.

i.e. The straight line intercepts from the positive part from y-axis $\boxed{\text{3 length units}}$



Observing the graph of the straight line : $y = 2x + 3$

We find that :

- The slope of the straight line = the coefficient of $x = 2$
- The length of the intercepted part from y-axis = | absolute term | = | 3 | = 3 length units.

The slope of the straight line

$y = 2x + 3$

The length of the intercepted part from y-axis

i.e.

If the equation of a straight line is in the form : $y = m x + c$, then :

- The slope of the straight line $= m$
- The length of the intercepted part from y-axis $= |c|$

and it passes through the point $(0 , c)$

WATCH VIDEO

Example 1 Find the slope of the straight line : $2 x + 5 y - 15 = 0$, then find the intercepted part of y-axis.

Solution Write the equation of the straight line in the form : $y = m x + c$

$\therefore 5 y = -2 x + 15$ $\qquad \therefore y = \frac{-2}{5} x + 3$

$\therefore$ The slope of the straight line $= \frac{-2}{5}$ and the intercepted part of the positive part of y-axis is of length = 3 length units.

Remark

In the previous example , observing the equation in the form : $2 x + 5 y - 15 = 0$, we find that :

- The slope of the straight line $= \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-2}{5}$
- The straight line cuts y-axis at the point $\left(0 , \frac{-\text{absolute term}}{\text{coefficient of } y}\right)$ **i.e.** $(0 , 3)$

i.e. The straight line intercepts a part of y-axis of length $= \left|\frac{-\text{absolute term}}{\text{coefficient of } y}\right|$

$= |3| = 3$ length units.

i.e.

If the equation of a straight line is in the form : $a x + b y + c = 0$, then

- The slope of the straight line $= \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b}$
- The straight line cuts y-axis at the point $\left(0 , \frac{-c}{b}\right)$

i.e. The length of the intercepted part from y-axis $= \left|\frac{-c}{b}\right|$

For example:

- The straight line whose equation is : $x - 2y + 3 = 0$

  Its slope $= \frac{-1}{-2} = \frac{1}{2}$ and cuts y-axis at the point $\left(0 , \frac{3}{2}\right)$

  i.e. The straight line intercepts a part of length $\frac{3}{2}$ length unit from the positive part of y-axis.

- The straight line whose equation is : $3x + y + 4 = 0$

  Its slope $= -3$ and cuts y-axis at the point $(0 , -4)$

  i.e. The straight line intercepts a part of length 4 length units from the negative part of y-axis.

Example 2 If the straight line that passes through the two points $(-1 , 7)$ and $(9 , 3)$ is perpendicular to the straight line whose equation is : $x + ky - 13 = 0$, **find the value of :** k

Solution Let the slope of the straight line that passes through the two points $(-1 , 7)$ and $(9 , 3)$ be m_1

$$\therefore m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-7}{9-(-1)} = \frac{-4}{10} = \frac{-2}{5}$$

Let the slope of the straight line whose equation is : $x + ky - 13 = 0$ be m_2

$$\therefore m_2 = \frac{-a}{b} = \frac{-1}{k}$$

$\because$ The two straight lines are perpendicular

$\therefore m_1 \times m_2 = -1$ $\qquad \therefore \frac{-2}{5} \times \frac{-1}{k} = -1$

$\therefore \frac{2}{5k} = -1$ $\qquad \therefore -5k = 2$ $\qquad \therefore k = \frac{-2}{5}$

TRY by yourself 1

1. If the two straight lines : $3y + x - 7 = 0$ and $y = kx + 5$ are perpendicular , then find the value of : k
2. Find the measure of the positive angle which is made by the straight line whose equation is : $3x - 3y + 5 = 0$ with the positive direction of x-axis.
3. Find the length of the intercepted part from y-axis by the straight line whose equation is : $2y = 3x + 12$

Second Finding the equation of the straight line given its slope and the length of intercepted part of y-axis

The straight line whose slope = m and cuts y-axis at the point $(0, c)$ its equation is in the form :

$$\boxed{y = m x + c}$$

Example 3 **Find the equation of the straight line :**

1 Whose slope $= -\frac{3}{4}$ and intercepts from the positive part of y-axis 3 length units.

2 Whose slope = 2 and intercepts from the negative part of y-axis 7 length units.

Solution $y = m x + c$

1 $\because m = -\frac{3}{4}$, $c = 3$ $\therefore$ The equation is : $y = -\frac{3}{4} x + 3$

2 $\because m = 2$, $c = -7$ $\therefore$ The equation is : $y = 2 x - 7$

Example 4 Find the equation of the straight line which makes with the positive direction of x-axis a positive angle of measure 135° and intercepts from the positive part of y-axis a part of length 7 length units.

Solution $\because$ The slope $= \tan 135° = -1$

$\therefore$ The equation of the straight line is : $y = -x + 7$

! Remarks

1. The equation of the straight line which passes through the origin point O $(0, 0)$ is $\boxed{y = m x}$, where m is the slope of the straight line.
2. The equation of x-axis is $\boxed{y = 0}$
3. The equation of y-axis is $\boxed{x = 0}$
4. The equation of the straight line which is parallel to x-axis and passes through the point $(0, \ell)$ is $\boxed{y = \ell}$
5. The equation of the straight line which is parallel to y-axis and passes through the point $(k, 0)$ is $\boxed{x = k}$

Example 5 Find the equation of the straight line which passes through the two points $(1 , -1)$ and $(2 , 2)$

Solution Let the equation of the straight line be in the form $y = m X + c$

$\because$ The slope $(m) = \frac{y_2 - y_1}{X_2 - X_1} = \frac{2-(-1)}{2-1} = 3$

$\therefore$ The equation of the straight line is in the form : $y = 3 X + c$

$\because (1 , -1)$ belongs to the straight line.

$\therefore -1 = 3 \times 1 + c$ $\therefore c = -1 - 3 = -4$

$\therefore$ The equation of the straight line is : $y = 3 X - 4$

Example 6 Find the equation of the straight line which passes through the point $(1 , 2)$ and parallels the straight line $2 X + 3 y - 6 = 0$

Solution $\because$ The slope of the given straight line $= \frac{-\text{coefficient of } X}{\text{coefficient of } y} = \frac{-2}{3}$

$\therefore$ The slope of the required straight line $= \frac{-2}{3}$

$\therefore$ The equation of the required straight line is : $y = -\frac{2}{3} X + c$

$\because$ The straight line passes through the point $(1 , 2)$

$\therefore 2 = -\frac{2}{3} \times 1 + c$ $\therefore c = \frac{8}{3}$

$\therefore$ The equation of the required straight line is : $y = -\frac{2}{3} X + \frac{8}{3}$

Example 7 Find the equation of the straight line which passes through the point $(2 , 3)$ and perpendicular to the straight line passing through the two points A $(3 , -4)$ and B $(5 , -3)$

Solution $\because$ The slope of the straight line which passes through the two points $(3 , -4)$ and $(5 , -3)$ equals $\frac{-3-(-4)}{5-3} = \frac{1}{2}$

$\therefore$ The slope of the required straight line $= -2$

$\therefore$ The equation of the required straight line is $y = -2 X + c$

$\because$ The straight line passes through the point $(2 , 3)$

$\therefore$ It satisfies the equation.

$\therefore 3 = -2 \times 2 + c$ $\therefore c = 7$

$\therefore$ The equation of the required straight line is : $y = -2 X + 7$

TRY yourself 2

1. Find the equation of the straight line which intercepts from the positive part of y-axis 5 length units and it is parallel to the straight line passing through the two points $(-2, 3)$ and $(-1, -6)$

2. Find the equation of the straight line which passes through the point $(3, 4)$ and perpendicular to $\overleftrightarrow{AB}$, where $A(2, -3)$ and $B(5, 4)$

Example 8 ABC is a triangle whose vertices are $A(1, 2)$, $B(-2, 3)$, $C(-4, -3)$, $\overline{AD}$ is a median of it, find the equation of $\overleftrightarrow{AD}$

Solution

$\because$ $\overline{AD}$ is a median of ΔABC

$\therefore$ D is the midpoint of $\overline{BC}$

$\therefore D = \left(\frac{-2+(-4)}{2}, \frac{3+(-3)}{2}\right) = (-3, 0)$

$\therefore$ The slope of $\overleftrightarrow{AD} = \frac{2-0}{1-(-3)} = \frac{1}{2}$

$\therefore$ The equation of $\overleftrightarrow{AD}$ is : $y = \frac{1}{2}x + c$

$\because$ $\overleftrightarrow{AD}$ passes through the point $A = (1, 2)$

$\therefore$ It satisfies its equation

$\therefore 2 = \frac{1}{2} \times 1 + c$ $\qquad$ $\therefore c = \frac{3}{2}$

$\therefore$ The equation of $\overleftrightarrow{AD}$ is : $y = \frac{1}{2}x + \frac{3}{2}$

TRY by yourself 3

ABC is a triangle whose vertices are $A(-1, 5)$, $B(4, -2)$ and $C(-3, 0)$

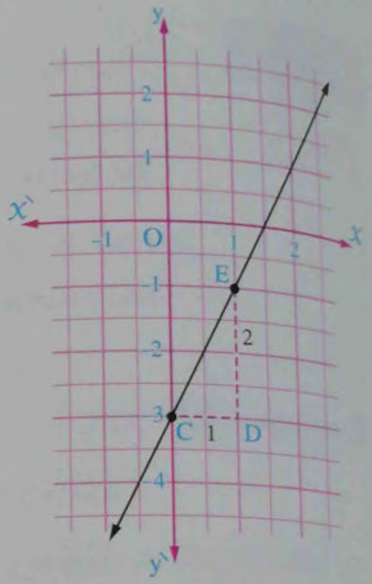
Find the equation of the straight line passing through A and perpendicular to $\overline{BC}$

Example 9 Using the slope and the intercepted part of y-axis, represent graphically the straight line whose equation is $y = 2x - 3$

Solution The slope of the straight line $= 2 = \frac{2}{1} = \frac{\text{vertical change}}{\text{horizontal change}}$

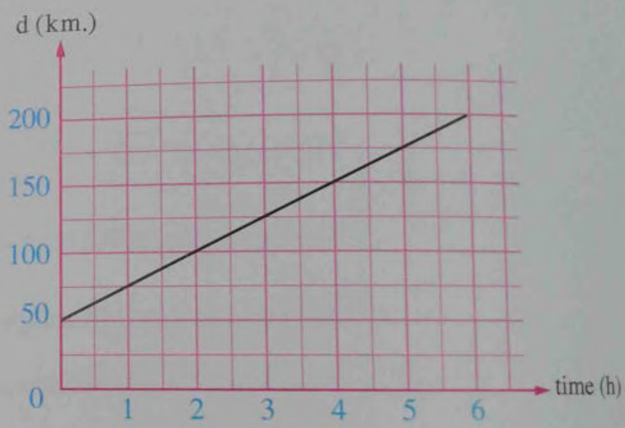
and the straight line passes through the point $C(0, -3)$

From the point C , we move horizontally towards the right one unit (the horizontal change (+1)) to reach the point D , then we move vertically towards up two units (the vertical change (+2)) to reach the point E, then $\overleftrightarrow{CE}$ is the graph of the equation of the straight line $y = 2X - 3$



Example 10

The opposite graph represents the motion of a car moving with a uniform velocity where the distance (d) is measured in km. and the time (t) in hours , **find :**



1. The distance (d) at the beginning of the motion.
2. The velocity of the car.
3. The equation of the straight line representing the motion of the car.

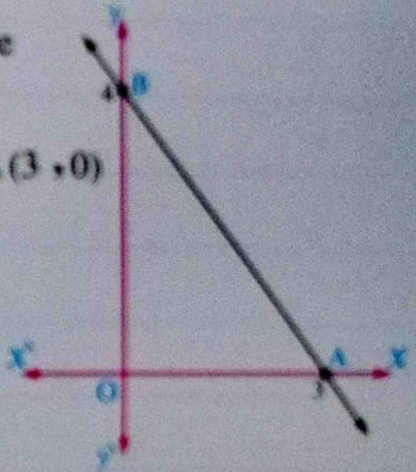
Solution

1. The distance (d) at the beginning of the motion = 50 kilometres.
2. The velocity of the car = the slope of the straight line passing through the two points (0 , 50) and (6 , 200) $= \frac{200 - 50}{6 - 0} = \frac{150}{6}$ = 25 km./hr.
3. The equation of the straight line is : d = m t + c

   i.e. $\boxed{d = 25\,t + 50}$

Example 11 Find the equation of the straight line which intercepts from the coordinate axes (X-axis and y-axis) two positive parts with lengths 3 and 4 length units respectively, then find the area of the triangle included between the straight line and the two axes.

Solution $\because$ The straight line intercepts from the positive part of X-axis 3 length units.



$\therefore$ The straight line passes through the point A (3 , 0)

$\because$ The straight line intercepts from the positive part of y-axis 4 length units.

$\therefore$ The straight line passes through the point B (0 , 4)

$\therefore$ The straight line passes through the two points A (3 , 0) and B (0 , 4)

Let the equation of the required straight line be $y = m X + c$

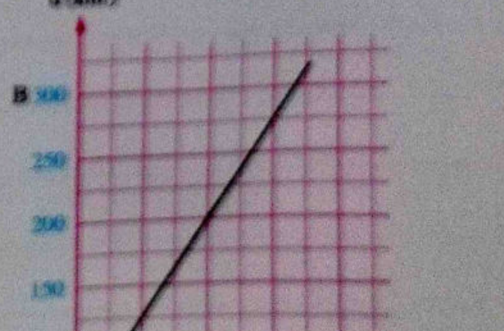
, where the slope (m) $= \frac{4-0}{0-3} = -\frac{4}{3}$ $\therefore y = -\frac{4}{3} X + c$

, $\because c = 4$ $\therefore$ The equation is : $y = -\frac{4}{3} X + 4$

, the area of Δ ABO $= \frac{1}{2} \times AO \times BO = \frac{1}{2} \times 3 \times 4 = 6$ square units.

TRY yourself 4

A person moved between the cities A and B using his car with a uniform velocity and the opposite graph represents the relation between the distance (d) in kilometres and the time (t) in hours.



Answer the following :

1. What is the uniform velocity of the car ?
2. Find the equation of the straight line representing the motion of the car.
3. Find the distance between the car and O (0 , 0) after 3 hours from the beginning of the motion.

EL-MOASSER

By a group of supervisors

EXERCISES

3rd PREP. 2025

FIRST TERM

Maths

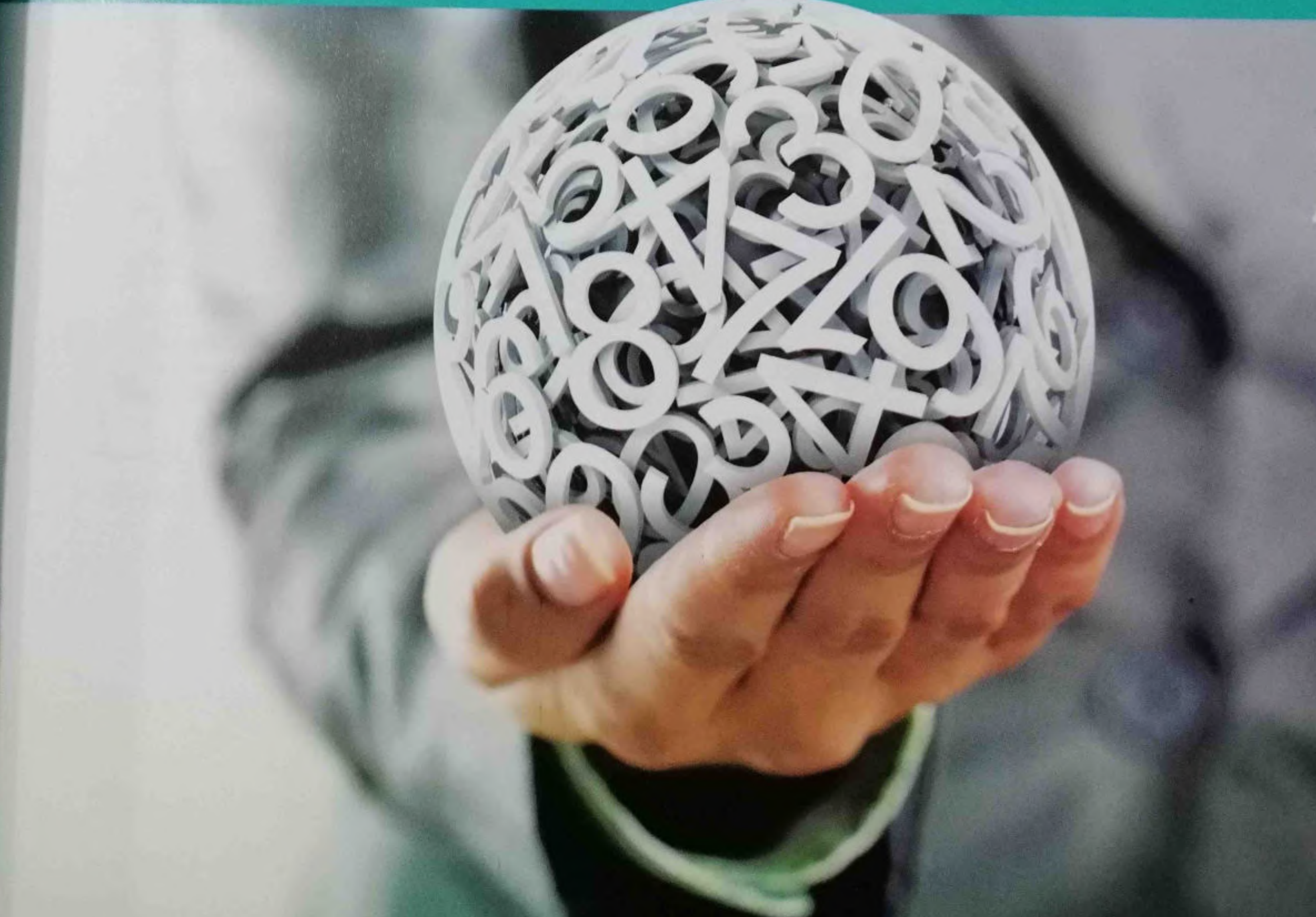
Contents

First — Algebra and Statistics

Second — Trigonometry and Geometry

First

Algebra and Statistics



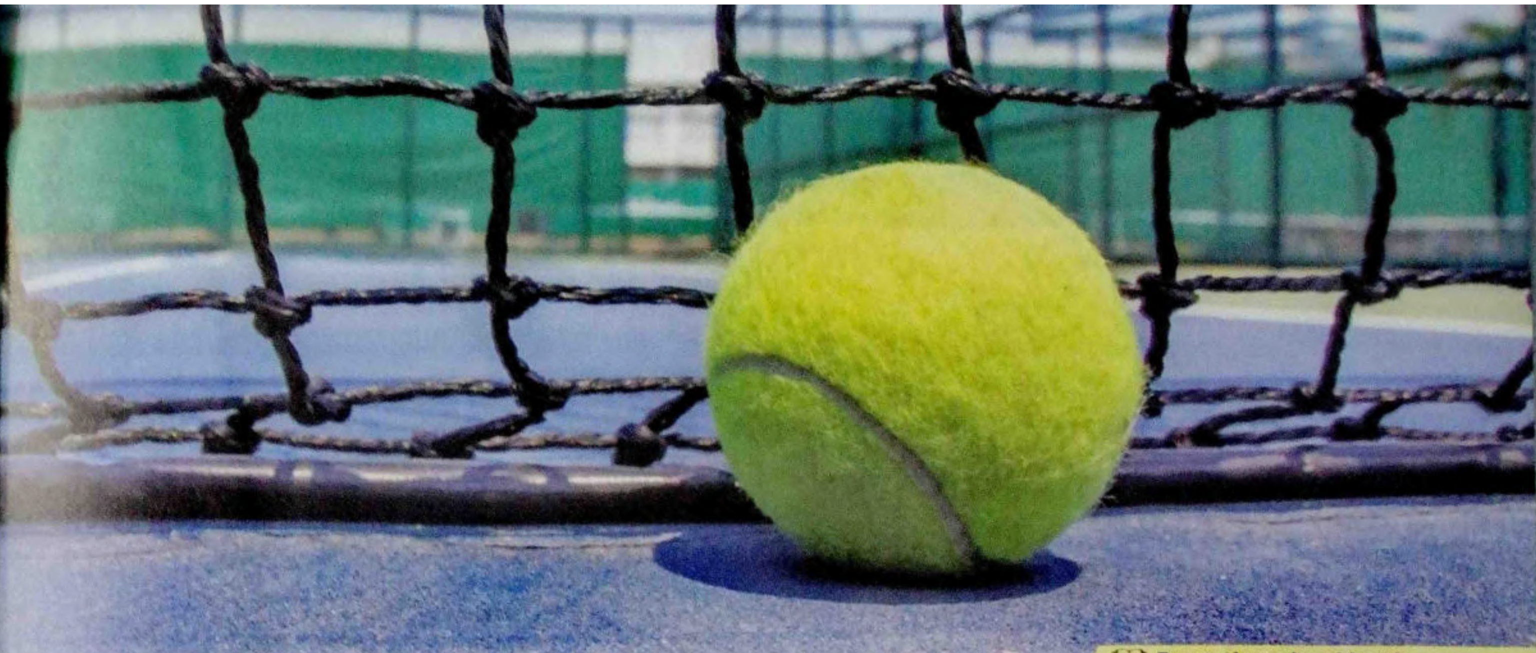
UNIT ONE

Relations and functions

Exercises of the unit :

1. Cartesian product.
2. Relation - Function (mapping).
3. The symbolic representation of the function - Polynomial functions.
4. The study of some polynomial functions.

Scan the QR code to solve an interactive test on each lesson



From the school book

Exercise **1**

Cartesian product

● Remember ● Understand ● Apply ● Problem Solving

Interactive test

First Problems on the equality of two ordered pairs

1 **Find the values of a and b in each of the following if :**

1 $(a , b) = (-5 , 9)$

2 $(a , b) = \left(\sqrt{25} , \sqrt[3]{27}\right)$

3 $(a - 2 , b + 1) = (2 , -3)$

4 $(6 , b - 3) = (2 - a , -1)$

5 $(a - 7 , 26) = (-2 , b^3 - 1)$

6 $(a , b) = (2 - a , 2 b - 3)$

7 $(a^5 , b^2 - 1) = \left(32 , \sqrt[3]{27}\right)$

8 $(a , 7) = (b^2 , b)$

9 $(2 a , 7) = (2 b + 1 , a)$

10 $(3 , b) = (5 a - 1 , 4 a)$

2 **Choose the correct answer from those given :**

1 If $(X - 1 , 11) = (8 , y + 3)$, then $\sqrt{X + 2y} =$ .... *(Port Said 19)*

(a) 5 (b) ± 5 (c) $\sqrt{17}$ (d) 25

2 If $(X + 2 , y) = (2 , 3)$, then $X^5 y + 1 =$ .... *(El-Sharkia 20)*

(a) 3 (b) 2 (c) zero (d) 1

3 If $\left(3^X , \sqrt{y}\right) = (1 , 4)$, then $X + y =$ .... *(El-Gharbia 18)*

(a) 2 (b) 3 (c) 16 (d) 17

4 If $(X^3 , y^2) = (1 , 4)$, $X > y$, then $X y =$ .... *(New Valley 22 – Ismailia 23)*

(a) 4 (b) 2 (c) -2 (d) -4

5 If $(X - 3 , 2^y) = (2 , 16)$, then $(y , X) =$ ....

(a) $(1 , 4)$ (b) $(5 , 4)$ (c) $(4 , 1)$ (d) $(4 , 5)$

Second Problems on the Cartesian product of two finite sets

3 If $X = \{1, 2\}$, $Y = \{3, 4, 5\}$, **find $X \times Y$ and represent it by :**

1 The arrow diagram.
2 The Cartesian diagram.

4 If $X = \{3, 4, 8\}$, **find X^2 and represent it :**

1 By an arrow diagram.
2 By a Cartesian diagram.

5 If $X = \{1, 2, 3\}$, $Y = \{4\}$, **find :**

1 $X \times Y$
2 $Y \times X$
3 Y^2
4 $n(X^2)$

6 If $X = \{2, -1\}$, $Y = \{4, 0\}$, $Z = \{4, 5, -2\}$, **find :**

1 $X \times Y$
2 $Y \times Z$
3 X^2
4 $n(X \times Z)$
5 $n(Y^2)$
6 $n(Z^2)$

Choose the correct answer from those given :

1 If A and B are two sets , then the set $\{(x, y) : x \in A , y \in B\}$ expresses .... *(El-Dakahlia 16)*

(a) $n(A \times B)$ (b) $A \times B$ (c) $n(B \times A)$ (d) $B \times A$

2 If $X = \{1, 2\}$, then $X \times \varnothing =$ ....

(a) X (b) $\varnothing$ (c) $\{0\}$ (d) $\{(1, 0), (2, 0)\}$

3 If $X = \{2\}$, $Y = \{3\}$, then $X \times Y =$ .... *(Giza 17)*

(a) 6 (b) $\{6\}$ (c) $(2, 3)$ (d) $\{(2, 3)\}$

4 If $X = \{3\}$, then $X^2 =$ .... *(Cairo 13 – El-Sharkia 17)*

(a) 9 (b) $(3, 3)$ (c) $\{9\}$ (d) $\{(3, 3)\}$

5 If $X = \{3\}$, then $n(X^2) =$ .... *(Qena 20)*

(a) 1 (b) 9 (c) $\{3, 3\}$ (d) 3

6 If $X = \{1, 2\}$ and $Y = \{3, 4\}$, then $(3, 4) \in$ .... *(Qena 11 – Suez 19 – Qena 23)*

(a) $X \times Y$ (b) $Y \times X$ (c) X^2 (d) Y^2

7 If $n(X) = 2$, $Y = \{1, 2\}$, then $n(X \times Y) =$ .... *(Giza 15)*

(a) 4 (b) 3 (c) 5 (d) 6

8 If $n(X) = 3$, $n(X \times Y) = 12$, then $n(Y) =$ .... *(Cairo 18 – El-Menia 19 – Port Said 20 – Ismailia 22 – EL-Beheira 24)*

(a) 4 (b) 9 (c) 15 (d) 36

9 If $n(X^2) = 9$, then $n(X) = \dots\dots$

(a) 2 (b) 3 (c) 9 (d) 81

10 If $n(X^2) = 4$, $n(X \times Y) = 6$, then $n(Y^2) = \dots\dots$ *(Giza 20)*

(a) 4 (b) 9 (c) 16 (d) 12

11 If X is a non-empty set , $n(X) = n(X \times Y)$, then $n(Y) = \dots\dots$ *(Damietta 18)*

(a) 1 (b) 2 (c) 3 (d) 4

12 If X and Y are two sets where $n(X \times Y) = 11$

, then $n(X) + n(Y) = \dots\dots$ *(El-Dakahlia 23)*

(a) 8 (b) 9 (c) 11 (d) 12

13 If $a \in X^2$, where $X = \{x : 5 < x < 7 , x \in \mathbb{N}\}$, then $a = \dots\dots$ *(El-Sharkia 20)*

(a) 36 (b) $\{36\}$ (c) $(6 , 6)$ (d) $[5 , 7]$

14 If $(3 , 5) \in \{3 , 6\} \times \{x , 8\}$, then $x = \dots\dots$

(Kafr El-Sheikh 18 – Port Said 19 – Alex. 20 – Beni suef 22)

(a) 8 (b) 6 (c) 5 (d) 3

15 If $\{2\} \times \{x , y\} = \{(2 , 4) , (2 , 3)\}$, then $x - y = \dots\dots$

(El-Sharkia 15 – Kafr El-Sheikh 20 – Port Said 24)

(a) 1 (b) -1 (c) ± 1 (d) 0

---

8 If $X \times Y = \{(2 , 6) , (2 , 9) , (3 , 6) , (3 , 9) , (5 , 6) , (5 , 9)\}$, **find :** X and Y

9 If $X \times Y = \{(1 , 1) , (1 , 3) , (1 , 5)\}$, **find :**

1 X and Y 2 $Y \times X$ 3 Y^2

(Giza 16 – Souhag 19 – El-Kalyoubia 20 – Luxor 22)

10 If $X^2 = \{(1 , 1) , (1 , 2) , (2 , 1) , (2 , 2)\}$, **find :** X

11 If $Y \times X = \{(1 , 3) , (2 , 3) , (3 , 3)\}$, **find :** X^2

12 If $X = \{1 , 2 , 3 , 4\}$, $Y = \{3 , 4 , 5\}$, **represent X and Y by Venn diagram , then find :**

1 $(X \cap Y) \times Y$ 2 $(X - Y) \times Y$ 3 $(Y - X) \times X$

13 If $X = \{3 , 4\}$, $Y = \{4 , 5\}$ and $Z = \{6 , 5\}$, **then find :**

1 $X \times (Y \cap Z)$ 2 $(X - Y) \times Z$ 3 $(X - Y) \times (Y - Z)$

(El-Dakahlia 13 – El-Monofia 18 – El-Menia 19)

14 If $X = \{1\}$, $Y = \{2 , 3\}$, $Z = \{2 , 5 , 6\}$
, represent each of X , Y and Z by Venn diagram , then find :

First : 1 $X \times Y$ 2 $Y \times Z$
3 $X \times Z$ 4 Y^2

Second : $(X \times Y) \cup (Y \times Z)$ Third : $X \times (Y \cap Z)$

Fourth : $(X \times Y) \cap (X \times Z)$ Fifth : $(Z - Y) \times (X \cup Y)$

15 If $(X - Y) \times Y = \{(1 , 2) , (1 , 3)\}$, $n(X \times Y) = 6$
Find : 1 X , Y 2 $(X \cap Y) \times Y$ *(El-Sharkia 24)*

Third Problems on the Cartesian product of two infinite sets

16 Identify the following points on a perpendicular graphical net of the Cartesian product $\mathbb{R} \times \mathbb{R}$:
A (4 , 5) , B (6 , – 3) , C (– 2 , 7) , D (– 1 , 6) , E (– 4 , – 5)
, M (0 , 6) , K (9 , 0)
Then mention the quadrant that each point is located on the perpendicular graphical net or the axis it belongs to.

17 Choose the correct answer from those given :

1 Which of the following points lies on the second quadrant ?
(a) (3 , 2) (b) (– 4 , 5) (c) (– 3 , – 2) (d) (2 , – 3)

2 If the point (a – b , 5) lies on the y-axis , then .... *(Giza 18)*
(a) a = b (b) a + b = 0 (c) a ≠ b (d) a – b = 5

3 If the point (5 , b – 7) is located on the X-axis ,
then b = ... *(Alex. 11 – North Sinai 16 – Qena 17 – Cairo 18 – El-Kalyoubia 20)*
(a) 2 (b) 5 (c) 7 (d) 12

4 If the point (X , 7) lies on the y-axis , then $5X + 1$ = ... *(El-Beheira 17)*
(a) zero (b) 1 (c) 5 (d) 6

5 If $\left(X + 1 , \sqrt[3]{27}\right) = (-1 , y)$, then the point (X , y) lies in the .... quadrant. *(El-Fayoum 20)*
(a) first (b) second (c) third (d) fourth

6 If b < 3 , then the point (5 , b – 3) lies in the .... quadrant. *(Cairo 16)*
(a) first (b) second (c) third (d) fourth

7 If $x \in \mathbb{R}_-$, then the point $(-x , \sqrt[3]{x})$ lies in the .... quadrant. *(El-Monofia 20)*

(a) first (b) second (c) third (d) fourth

8 If the point (a , b) lies in the fourth quadrant , then a × b .... zero.

(a) = (b) > (c) < (d) ≥

9 If the point (x , y) lies in the third quadrant , then the point (x^3 , y^2) lies in the .... quadrant. *(El-Monofia 22)*

(a) first (b) second (c) third (d) fourth

10 If the point $(2a , 3b) \in \overleftrightarrow{xx}$, then $\frac{b}{a}$ = ... (where $a \neq 0$)

(a) zero (b) $\frac{2}{3}$ (c) 2 (d) 3

11 If $(|x| , 4) = (3 , y^2)$ and the point (x , y) lies in the second quadrant , then $x + y$ = ... *(El-Sharkia 14)*

(a) 7 (b) 1 (c) – 1 (d) – 7

12 If a < zero , b > zero , then the point which lies in the second quadrant is .... *(El-Fayoum 18)*

(a) (a , b) (b) (– a , b) (c) (a , – b) (d) (– a , – b)

13 If the point $(x - 4 , 2 - x)$ where $x \in \mathbb{Z}$ is located in the third quadrant , then x = ... *(El-Monofia 17 – Port Said 19 – El-Beheira 20 – South Sinai 22 – Assiut 23)*

(a) 2 (b) 3 (c) 4 (d) 6

14 If the point $(k^2 - 4 , k)$ lies on the negative part of y-axis , then k = ... *(El-Sharkia 18)*

(a) ± 2 (b) 4 (c) – 2 (d) 2

---

18 If A (– 2 , 0) , B (– 2 , 3) , C (2 , 3) , identify on the perpendicular square net $\mathbb{R}^2$ the points A , B , C and find the area of Δ ABC « 6 square units »

Fourth Problems on the Cartesian product of two intervals

19 If X = [– 2 , 3] , find the location which represents X × X

Show which of the following points belongs to the Cartesian product of X × X

A (1 , 2) , B (3 , – 1) , C (– 1 , 4) and D (– 2 , 0)

---

20 If X = [– 2 , 3] , Y = [0 , 4] , find the region which represents each of :

1 X × Y 2 Y × X 3 Y^2

For excellent pupils

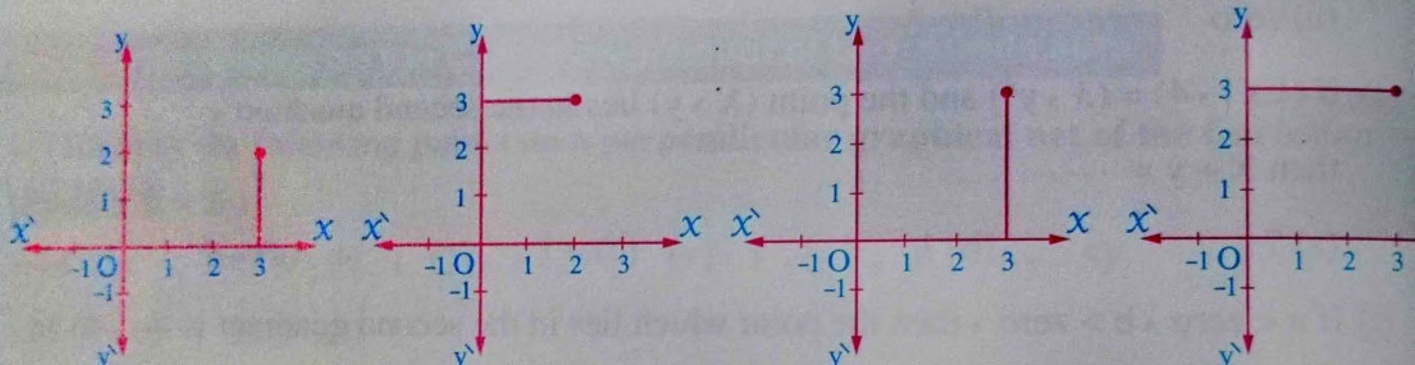
21 **Choose the correct answer from those given :**

1. If $X - Y = \{7\}$, $Y - X = \{2, 4\}$, $X \cap Y = \{6\}$, then $(X \times Y) \cap (Y \times X) = \ldots\ldots\ldots$

(a) $\{(6, 6)\}$ (b) $\{(7, 2), (7, 4)\}$

(c) $\{(2, 7), (4, 7)\}$ (d) $\{(7, 6)\}$

2. $\{3\} \times [0, 2]$ is represented graphically in the figure ………



(a) (b) (c) (d)

22 If $X \subset Y$, $X \times Y = \{(a, 1), (a, 2), (a, 3), (2, 1), (2, 2), (2, 3)\}$

, **find the values of : a**

23 If $X \subset Y$, $n(X \times Y) = 6$, $4 \in X$ and $(1, 7) \in X \times Y$,

then find X , Y and $X \times Y$

(Damietta

From the school book

Exercise 2 Relation - Function (mapping)

Remember · Understand · Apply · Problem Solving

Interactive test

First Problems on relation and function from a set to another set

1 Choose the correct answer from those given :

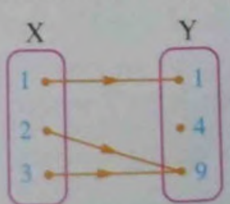
1. If f is a function from the set X to the set Y , then X is called ....
   (a) the range of the function f
   (b) the domain of the function f
   (c) the codomain of the function f
   (d) the rule of the function f

2. If f is a function from the set X to the set Y , then Y is called ....
   (a) the domain of the function.
   (b) the codomain of the function.
   (c) the range of the function.
   (d) the rule of the function.

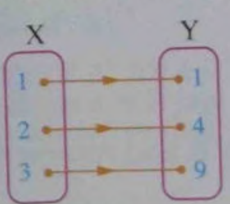
3. If the relation $R = \{(4, 3), (1, 3), (2, 5)\}$, then R represents a function where its range is .... *(El-Kalyoubia 17)*
   (a) $\{1, 2, 4\}$
   (b) $\{4, 1, 2, 3, 5\}$
   (c) $\{3, 5\}$
   (d) $\mathbb{N}$

4. If R is a function from X to Y where $X = \{2, 4, 5\}$, $Y = \{6, 7\}$ and $R = \{(2, 6), (a, 6), (5, 6)\}$, then a = ...
   (a) 4
   (b) 5
   (c) 12
   (d) 6

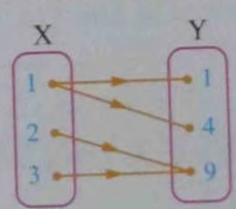
2 Which of the following relations represents a function from X to Y ? If the relation represents a function , then find the function range :



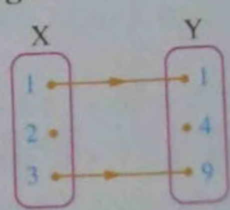
R_1



R_2

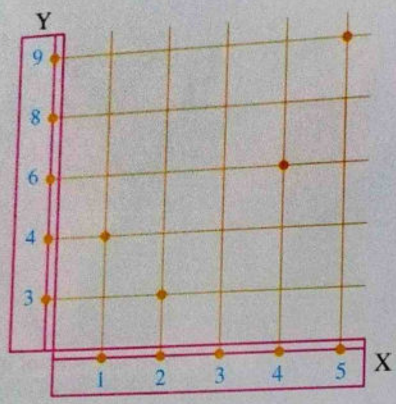


R_3

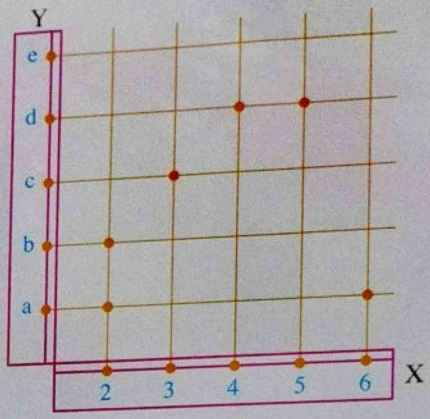


R_4

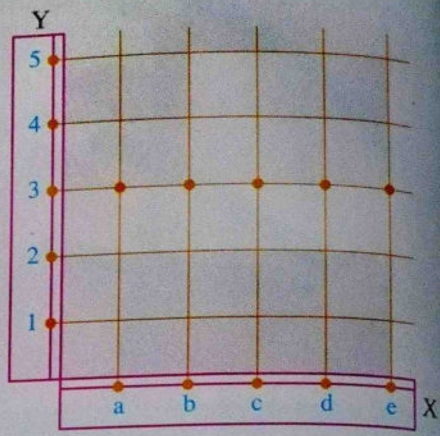
3 **Show which of the following Cartesian diagrams represents a function , then mention the set of each function and its range :**



t_1



t_2



t_3

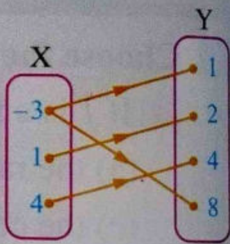
4 If $X = \{a , b , c\}$, $Y = \{2 , 4 , 6 , 8 , 10\}$, which of the following relations is a function from X to Y and which is not with giving reasons , if the relation is a function , state its range :

1 $R_1 = \{(a , 2) , (b , 4)\}$

2 $R_2 = \{(a , 2) , (b , 4) , (b , 6) , (c , 8)\}$

3 $R_3 = \{(a , 2) , (b , 8) , (c , 10)\}$

5 The opposite arrow diagram represents a relation R from the set X to the set Y , where : $X = \{-3 , 1 , 4\}$, $Y = \{1 , 2 , 4 , 8\}$



1 Write R

2 Is R a function ? Why ?

3 Find the value of x if $(x , 2) \in R$

(Souhag 16 – Beni Suef 17)

6 If $X = \{1 , 2 , 3\}$, $Y = \{1 , 3 , 6 , 9 , 12\}$ and R is a relation from X to Y , where "a R b" means "$a = \frac{1}{3} b$" for each $a \in X$, $b \in Y$

Write R and show that it is a function and write its range. *(El-Monofia 15 – Souhag 17 – Matrouh 19)*

7 If $X = \{4 , 6 , 8 , 10\}$, $Y = \{2 , 3 , 4 , 5\}$ and R is a relation from X to Y , where "a R b" means "$a = 2 b$" for each $a \in X$, $b \in Y$

Write R and represent it by an arrow diagram.

(Aswan 11)

8 If $X = \{1 , 3 , 4 , 5\}$, $Y = \{1 , 2 , 3 , 4 , 5 , 6\}$ and R is a relation from X to Y , where "a R b" means "$a + b = 7$" for each $a \in X$, $b \in Y$

Write R and represent it by an arrow diagram and also by a Cartesian diagram.

(El-Menia 11 – Beni Suef 15 – Port Said 17)

9 If $X = \{0 , 1 , 4 , 7\}$, $Y = \{1 , 3 , 5 , 6\}$ and R is a relation from X to Y where "a R b" means "$a + b < 8$" for each $a \in X$, $b \in Y$ Write R and represent it by an arrow diagram.

Is R a function ? And why ?

(El-Kalyoubia 11 – Alex. 18)

10 If $X = \{2, 4, 5, 7\}$, $Y = \{4, 5, 6, 7, 9\}$ and R is a relation from X to Y where "a R b" means "$a \le b$" for each $a \in X$ and $b \in Y$

Write R and represent it by an arrow diagram and also by a Cartesian diagram.

---

11 If $X = \{1, 2, 3, 4\}$, $Y = \{y : y \in \mathbb{N} , y \text{ is an even number} \le 10\}$ where $\mathbb{N}$ is the set of natural number and R is a relation from X to Y where "a R b" means "$a = \frac{1}{2} b$" for each $a \in X$, $b \in Y$

1. Write R and represent it by an arrow diagram.
2. Show that R is a function from X to Y and find its range. *(El-Monofia 17)*

---

12 If $X = \{1, 2, 3\}$, $Y = \{2, 3, 7\}$ and R is a relation from X to Y , where "a R b" means "a + b = a prime number" for each $a \in X$, $b \in Y$

1. Write R and represent it by an arrow diagram. Is R a function ?
2. If 2 a R 3 , then find the value of a

---

13 If $X = \{-1, 0, 1, 2, 3\}$, $Y = \{0, 1, 4, 6, 9\}$ and R is a relation from X to Y , where "a R b" means "$a^2 = b$" for each $a \in X$, $b \in Y$

1. Write R and represent it by a Cartesian diagram.
2. Is R a function ? And why ? *(Red Sea 16 – Qena 18 – Giza 23)*

---

14 If $X = \{-2, -1, 1, 2\}$, $Y = \left\{\frac{1}{8}, \frac{1}{3}, 1, 3, 8\right\}$ and R is a relation from X to Y , where "a R b" means "$a^3 = b$" for each $a \in X$, $b \in Y$

Write R and represent it by an arrow diagram and also by a Cartesian diagram.

---

15 If $X = \{-1, 1, 2\}$ and $Y = \{-1, 1, 4, 8\}$ and R is a relation from X to Y where "a R b" means "$a = \sqrt[3]{b}$" for all $a \in X$, $b \in Y$

Find R , then prove that R is function and find the range. *(Luxor 24)*

---

16 If $X = \{-2, -1, 0, 1, 2\}$ and $Y = \left\{4, 2, \frac{3}{2}, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right\}$ and R is a relation from X to Y where "a R b" means "$b = 2^a$" for each $a \in X$, $b \in Y$ Write R and represent it by an arrow diagram. Prove that R represents a function and mention its range.

---

17 If $X = \{2, 5, 8\}$ and $Y = \{10, 16, 24, 30\}$ and R is a relation from X to Y where "a R b" means "a is a factor of b" for each $a \in X$, $b \in Y$

Write R and represent it by an arrow diagram and by a Cartesian diagram. Is R a function ? And why ?

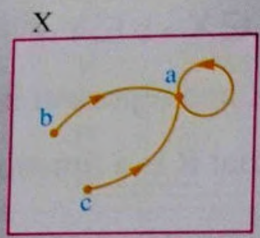
18 If X = {2 , 3 , 4} , Y = {6 , 8 , 10 , 11 , 15} and R is a relation from X to Y , where "a R b" means "a divides b" for each a ∈ X , b ∈ Y

Write the relation R

Second Problems on relation and function from a set to itself

19 **Choose the correct answer from those given :**

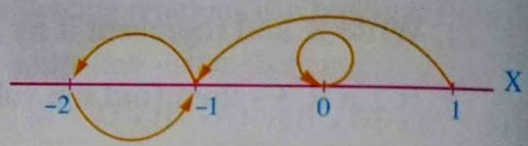
1 The opposite diagram represents a function on X , its range is .... *(Port Said 22)*



(a) {a} (b) {a , b , c}

(c) {a , b} (d) {b , c}

2 The opposite figure represents a function on X , its range is ....



(a) {1 , 0 , −1 , −2} (b) {1 , 0 , −1}

(c) {0 , −1 , −2} (d) {1 , −1 , −2}

20 If X = {1 , 2 , 3 , 4} , which of the following arrow diagrams represents a function on the set X ?

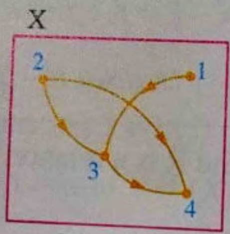


Fig. (1)

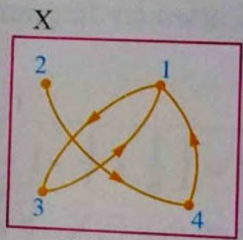


Fig. (2)

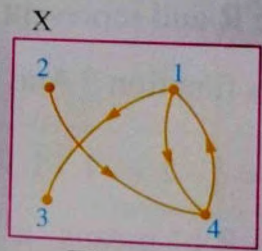


Fig. (3)

21 If X = {6 , 4 , 2 , 0 , −2 , −4 , −6} and R is a relation on X where "a R b" means "a is the additive inverse of b" for each a ∈ X , b ∈ X

Write R and represent it by an arrow diagram and show with reason if R is a function or not , and if R is a function , mention its range.

22 If $X = \{0 , 1 , 2 , \frac{1}{2}\}$ and R is a relation on X where "a R b" means "a is the multiplicative inverse of b" for each a ∈ X , b ∈ X Write R and represent it by an arrow diagram and show if R is a function or not.

23 If X = {1 , 2 , 3 , 6 , 11} and R is a relation on X where "a R b" means "a + 2 b = an odd number" for each a ∈ X , b ∈ X

Write R and represent it by an arrow diagram. Is R a function ? And why ?

24 If $X = \{x : x \in \mathbb{N} , 1 \leq x \leq 3\}$ and R is a relation on X where "a R b" means "a + b is divisible by 3" for each $a \in X , b \in X$

Write R and represent it by an arrow diagram , then mention if R is a function or not.

And if R is a function , mention its range. *(Luxor 16)*

---

25 If $X = \{1 , 2 , 4 , 6 , 10\}$ and R is a relation on X where "a R b" means "a is a multiple of b" for each $a \in X , b \in X$

Write R and represent it by an arrow diagram and also by a Cartesian diagram.

Is R a function ? And why ?

---

26 If $X = \{-2 , -1 , 0 , 1 , 2\}$ and R is a relation on X where "a R b" means "$b = |a|$" for each $a \in X$ and $b \in X$

Write R and represent it by an arrow diagram and show whether R is a function or not.

---

27 If $X = \{-2 , 2 , 5\}$, $Y = \{3 , 7 , \ell\}$ and R is a function from X to Y where "a R b" means "$b = a^2 - 1$" for each $a \in X$ and $b \in Y$

1. Find the value of ℓ
2. Represent R by an arrow diagram.

---

28 If $X = \{0 , 4 , 16\}$, $Y = \{0 , 2 , 4\}$, show which of the following relations represents a function from X to Y :

1. R_1 where "a R_1 b" means "$a = b^2$" for each $a \in X , b \in Y$
2. R_2 where "a R_2 b" means "$a = \sqrt{b}$" for each $a \in X , b \in Y$
3. R_3 where "a R_3 b" means "$\frac{1}{2} a = b$" for each $a \in X , b \in Y$

---

29 If R is a relation on the set of natural numbers ($\mathbb{N}$) where "a R b" means "$a \times b = 12$" for each $a \in \mathbb{N} , b \in \mathbb{N}$:

1. If x R 4 , then find the value of x
2. If y R 3 y , then find the value of y

---

30 If $X = \{1 , 0 , -1\}$, R_1 is the relation of the additive inverse on X and R_2 is the relation of the multiplicative inverse on X

Find $R = R_1 \cap R_2$ is R a function on X ?

31 If $X = \{1, 2, 3\}$, $Y = \{13, 31, 65, 23\}$ and R is a relation from X to Y where "a R b" means "a is a digit of the number b" for each $a \in X$, $b \in Y$

1. Write R and represent it by an arrow diagram.
2. Show which of the following is true , giving reasons : 2 R 65 , 1 R 31 , 3 R 13
3. Write by listing method : $M = \{(y, 23) : (y, 23) \in R\}$

---

32 If $A = \{-1, 1, 2\}$, $B = \{d : d \in \mathbb{N}\}$ and R is a relation from A to B where "x R y" means "$y = 2x + 3$" for each $x \in A$, $y \in B$

Write R and represent it by an arrow diagram.

---

33 If $X = \{1, 2, 3\}$, $Y = \{3, 4, 5\}$, show with reasons which of the following represents a relation from X to Y :

1. $L = \{(1, 3), (3, 3), (5, 3)\}$
2. $M = \{(2, 4), (1, 3), (3, 3), (3, 4)\}$

---

34 If $X = \{1, 3, 5\}$ and R is a function on X where $R = \{(a, 3), (b, 1), (1, 5)\}$

Find : 1. The range of the function.

2. The numerical value of the expression : $a + b$

(El-Kalyoubia 20 – Damietta 22 – El-Beheira 23)

For excellent pupils

35 If $X = \{-2, -1, 0, 1, 2\}$, $Y = [0, 4[$ and R is a relation from X to Y where "a R b" means "$a^2 = b$" for each $a \in X$, $b \in Y$

Write R and mention whether R is a function from X to Y or not. Give reasons.

---

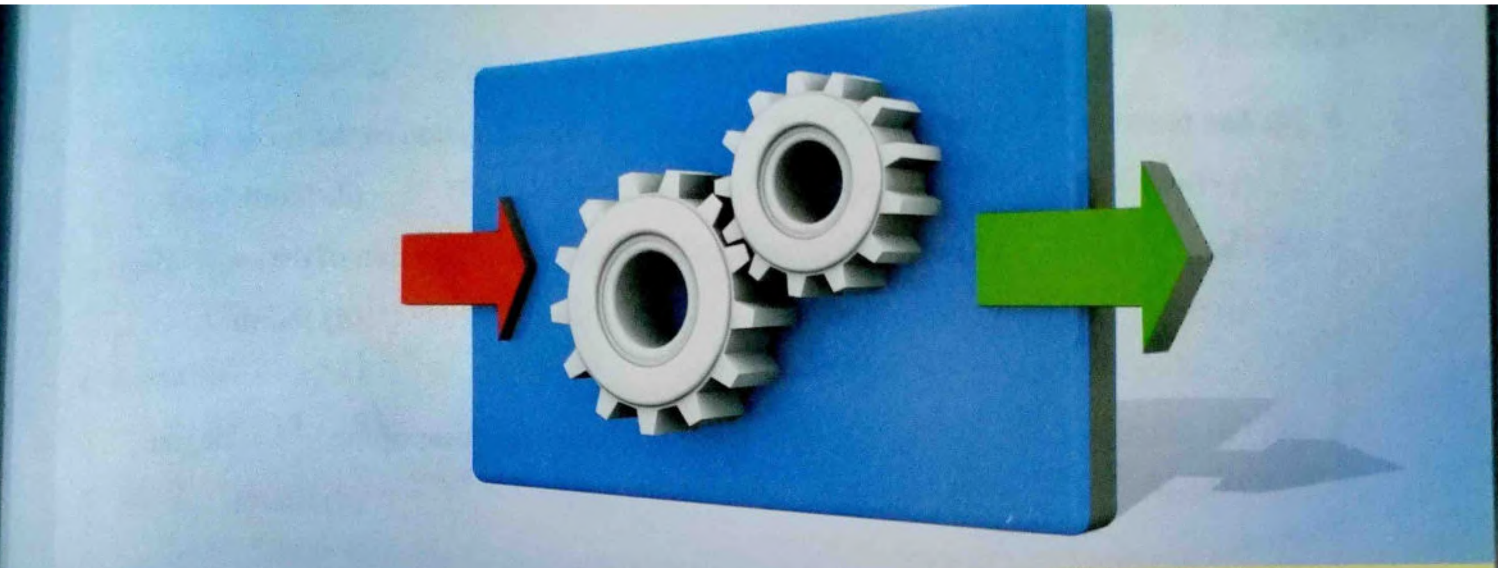
36 If f is a function from X to Y where "a R b" means "a divides b" for each $a \in X$, $b \in Y$, $X \cup Y = \{2, 3, 5, 11, 14, 9, 35\}$, $n(X) = 3$ and $n(X \times Y) = 12$

Find each of X and Y and write R of the function f and find its range.

---

37 If f is a function from X to Y where "a R b" means "a is a multiple of b" for each $a \in X$, $b \in Y$, $n(X) = 4$, $n(Y) = 2$ and $X \cup Y = \{4, 8, 9, 27\}$

Find each of X and Y and write R of the function f and find its range.



From the school book

Exercise

3 The symbolic representation of the function - Polynomial functions

● Remember ● Understand ● Apply ● Problem Solving

Interactive test

1 Choose the correct answer from those given :

1 The set of images of the elements of the domain of the function is called .... *(Damietta 15 – Matrouh 16)*

(a) the rule. (b) the domain. (c) the range. (d) the codomain.

2 If the function $f : X \longrightarrow Y$, then the range of the function $f \subset$ .... *(Cairo 17)*

(a) $X \times Y$ (b) X (c) $Y \times X$ (d) Y

3 Which of the following functions is polynomial ?

(a) $f : f(X) = X(X^2 + X^{-2} - 4)$ (b) $f : f(X) = X^3 + X^2 + 3$

(c) $f : f(X) = X^2 + \sqrt{X} + 8$ (d) $f : f(X) = \sqrt[3]{X} + 8$

4 All the following functions are polynomials except ....

(a) $f : f(X) = 2X - 5$ (b) $f : f(X) = 3$

(c) $f : f(X) = X\left(X + \frac{1}{X} - 2\right)$ (d) $f : f(X) = \frac{X}{2} - 7$

5 The function f where $f(X) = X^4 - 2X^3 + 7$ is a polynomial function of the .... degree. *(Suez 15 – South Sinai 19)*

(a) first (b) second (c) third (d) fourth

6 The function $f : f(X) = X(X - 2X^2)$ is a polynomial function of the .... degree.

(a) first (b) second (c) third (d) fourth

7 The function $f : f(X) = X^2 - (X^2 - 3X)$ is a polynomial function of the .... degree.

(a) first (b) second (c) third (d) fourth

(Port Said 16)

8 The function $f : f(X) = X^2 (X - 3)^2$ is a polynomial function of the .... degree.

(a) first (b) second (c) third (d) fourth

9 The function $f : f(X) = (X - 5)^3$ is a polynomial function of the .... degree. *(Qena 11)*

(a) zero (b) second (c) third (d) fourth

10 If $f(X) = X^n - 2$, $f(3) = 7$, then $f(X)$ is of the .... degree. *(Suez 24)*

(a) first (b) second (c) third (d) fourth

11 If $f(X) = X^2 - X + 3$, then $f(-2) =$ ....

(a) -2 (b) -1 (c) 5 (d) 9

12 If $f(X) = X^2 - \sqrt{2}X$, then $f(\sqrt{2}) =$ .... *(El-Dakahlia 11)*

(a) 4 (b) 2 (c) 6 (d) zero

13 If the function $f : \mathbb{Z} \longrightarrow \mathbb{Z}$ where $f(X) = X^2$, then $f(2) + f(-2) =$ ....

(a) 0 (b) 4 (c) 8 (d) -8

14 If $f(X) = kX + 8$, $f(2) = 0$, then $k =$ .... *(El-Sharkia 15 – El-Dakahlia 20)*

(a) 8 (b) 6 (c) 4 (d) -4

15 If $f(X) = X - 5$ and $\frac{1}{2} f(a) = 3$, then $a =$ ....

(a) 2 (b) 8 (c) 11 (d) 16

16 If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = X^{k-2} + 3$, $f(2) = 11$, then $k =$ .... *(El-Sharkia 20)*

(a) 5 (b) 3 (c) 2 (d) -3

17 If $(-1, 0) \in$ the set of the function f where $f(X) = mX + 2$, then $m =$ ....

(a) 0 (b) -1 (c) 2 (d) -2

18 If $(3, y) \in$ the set of the function f where $f(X) = X + 2$, then $y =$ ....

(a) 5 (b) 3 (c) 2 (d) 1

19 If $(a, a) \in$ the set of the function f where $f(X) = 2X + 3$, then $a =$ ....

(a) 2 (b) 3 (c) -3 (d) -2

20 If $f(X+3) = X-3$, then $f(7) = \dots\dots$ (El-Dakahlia 19)

(a) 4 (b) 1 (c) 7 (d) 10

21 If $X = \{2, 4, 6\}$, $n(Y) = 4$ and the function $f : X \longrightarrow Y$, $f(X) = X^2 - 1$, then Y may be ……

(a) $\{3, 7, 13\}$ (b) $\{3, 15, 25, 45\}$

(c) $\{3, 15, 35\}$ (d) $\{3, 15, 25, 35\}$

22 If $f(X) = n X^2 + 2 X^n - 3$, then the possible values of n such that f is a function of the second degree is …… (El-Dakahlia 16)

(a) $\{2, 3\}$ (b) $\{1, -1\}$ (c) $\{2, 1, 0\}$ (d) $\{2, 1\}$

---

2 If $f : \mathbb{R} \longrightarrow \mathbb{R}$, mention the degree of f , then find $f(-2)$, $f(0)$, $f\left(\frac{1}{2}\right)$ when :

1 $f(X) = 3 - 2X$ 2 $f(X) = X^2 - 4$

---

3 If $f(X) = 2X^2 - 5X + 2$, then prove that : $f(2) = f\left(\frac{1}{2}\right)$ (Luxor 14)

---

4 If $f(X) = 2X - 1$, then prove that : $f(2) - 3f(1) = \text{zero}$ (El-Gharbia 11)

---

5 If $f(X) = X^2 - 3X$, $g(X) = X - 3$ (El-Menia 17 – Alex.18 – Qena 19 – Port Said 20 – Alex. 24)

1 **Find :** $f\left(\sqrt{2}\right) + 3g\left(\sqrt{2}\right)$ 2 **Prove that :** $f(3) = g(3) = 0$

---

6 If $f(X) = X^2 - 2X - 5$, then prove that : $f\left(1+\sqrt{6}\right) = f\left(1-\sqrt{6}\right) = 0$

---

7 The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX^2 + bX + 5$, a = zero and b is a real number not equal to zero.

1 Find the degree of the function f

2 If $f(3) = 11$, then find the value of b (El-Menia 18) « 2 »

---

8 If $f(X) = 5X - b$ and $h(X) = X - 2b$ and $f(1) + h(3) = -7$

, **then find :** $f(3) + h(1)$ « 1 »

---

9 If the function $f : \mathbb{Z} \longrightarrow \mathbb{N}$ where $f(X) = (X-3)^2$ and the function $t : \mathbb{Z} \longrightarrow \mathbb{N}$ where $t(X) = X - 3$, then find the value of X which makes : $f(X) = t(X)$ « 3 or 4 »

---

10 If f is a function on X where $X = \{3, 4, 5, 6\}$

and $f(3) = 3$, $f(4) = 5$, $f(5) = 5$, $f(6) = 5$

1 Represent f by an arrow diagram.

2 Write the set of f and mention its range. (Ismailia 15)

● Remember ● Understand ○ Apply Problem Solving

11 If $X = \{0 , 1 , 3\}$, $Y = \{1 , 2 , 3 , 4 , 5 , 7\}$ and the function $f : X \longrightarrow Y$ where $f(X) = 5 - X$

1. Find the range of f
2. Draw a Cartesian diagram for the function f

(New Valley 17)

12 If the function $t : \mathbb{N} \longrightarrow \mathbb{N}$ where $\mathbb{N}$ is the set of natural numbers , $t : X \longrightarrow 2X + 3$

1. Find : $t(0)$, $t(1)$, $t(2)$, $t(3)$, $t(4)$, $t(5)$
2. Represent five elements of the elements of t on a part of the square net of the Cartesian product $\mathbb{N} \times \mathbb{N}$
3. What is the range of t ?

13 If the function $f : \mathbb{Z} \longrightarrow \mathbb{Z}$ where $\mathbb{Z}$ is the set of integers , $f(X) = X^2 - 2X - 3$

1. Find : $f(4)$, $f(3)$, $f(2)$, $f(1)$, $f(0)$, $f(-1)$, $f(-2)$
2. Draw a part of the perpendicular square net of the Cartesian product $\mathbb{Z} \times \mathbb{Z}$ and represent on it seven elements of the elements of f
3. If $f(X) = 5$, find the value of X

« 4 or – 2

14 If $f(X) = aX + b$, $f(a) = b$, **find the value of :** $a b^2 + 5$

(El-Sharkia 19) « 5

15 If the set of the function $f = \{(1 , 3) , (2 , 5) , (3 , 7) , (4 , 9) , (5 , 11)\}$

Write :

1. The domain of the function f
2. The range of the function f
3. The rule of the function f

(Damietta 16 – North Sinai 17 – Luxor 1

For excellent pupils

16 If $f(X) = 2X^2 + bX + c$ and $f(X) = 0$, when $X \in \{0 , 3\}$, find the value of each of b and c

« – 6 , 0

From the school book

Exercise 4

The study of some polynomial functions

● Remember ● Understand ● Apply ● Problem Solving

Interactive test

First Problems on the linear function and the constant function

Choose the correct answer from those given :

1 If $f(\mathcal{X}) = 7$, then $f(-3) =$ .... *(Giza 17)*

(a) 7 (b) – 7 (c) 21 (d) – 21

2 If $f(\mathcal{X}) = 2$, then $3 f(\sqrt{2}) =$ ....

(a) $f(3\sqrt{2})$ (b) 6 (c) 3 (d) 2

3 If $f(\mathcal{X}) = 2$, then $f(3) - f(1) =$ .... *(El-Dakahlia 13)*

(a) $f(2)$ (b) 2 (c) zero (d) 10

4 If $f(\mathcal{X}) = 5$, then $\frac{f(5)}{f(10)} =$ ....

(a) 5 (b) $\frac{1}{2}$ (c) 1 (d) 10

5 If f is a function such that $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(\mathcal{X}) = 3$, then $\frac{f(6)}{f(\text{zero})} =$ .... *(El-Dakahlia 17)*

(a) 6 (b) 1 (c) 3 (d) undefined.

6 If $f(\mathcal{X}) = 3$, then $\frac{2 f(3)}{3 f(2)} =$ .... *(Alex. 05)*

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 1 (d) $\frac{32}{23}$

7 If $f(X) = -7$, then $f(X+7) =$ ....

(a) -7 (b) 0 (c) 7 (d) 14

(El-Dakahlia 09)

8 If $f(2X) = 4$, then $f(-X) =$ ....

(a) -2 (b) -4 (c) 4 (d) 2

9 The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 5$ is represented by a straight line intersecting the y-axis at the point ....

(a) $(5, 0)$ (b) $(0, 5)$ (c) $(-5, 0)$ (d) $(0, -5)$

10 The linear function defined by the rule $y = 2X - 1$ is represented by a straight line intersecting the y-axis at the point .... *(Matrouh 20)*

(a) $(0, 1)$ (b) $(0, -1)$ (c) $(1, 0)$ (d) $(-1, 0)$

11 The linear function defined by the rule $f(X) = 3X + 6$ is represented by a straight line intersecting the X-axis at the point ....

(a) $(0, -2)$ (b) $(-2, 0)$ (c) $(0, -6)$ (d) $(-6, 0)$

12 The function f where $f(X) = 3X$ is represented graphically by a straight line which passes through the point ....

(Beni Suef 17)

(a) $(3, 3)$ (b) $(3, 0)$ (c) $(0, 0)$ (d) $(0, 3)$

13 If the straight line which represents the function $f : f(X) = 2X - a$ passes through the origin point , then $a =$ ....

(El-Fayoum 17)

(a) -2 (b) 2 (c) zero (d) 3

14 If the point $(a, 3)$ lies on the straight line representing the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 4X - 5$, then $a =$ .... *(New Valley 20 – Damietta 22 – Matrouh 24)*

(a) 2 (b) 3 (c) 4 (d) 5

15 If the point $(a, 4)$ is one of the points of the function $g : \mathbb{R} \longrightarrow \mathbb{R}$ where $g(X) = 2X + b$, then $6a + 3b =$ ....

(El-Dakahlia 17)

(a) 12 (b) 9 (c) 6 (d) 3

2 Represent the following functions graphically , where $X \in \mathbb{R}$:

1 $f : f(X) = 5$

2 $f : f(X) = -4$

3 $f : f(X) = 0$

4 $f : f(X) = 2\frac{1}{\ }$

3 **Represent each of the following linear functions graphically and find the points of intersection of the straight line which represents each of them with the coordinate axes , where $X \in \mathbb{R}$:**

1 $f : f(X) = X$
2 $f : f(X) = -X$
3 $f : f(X) = 3X$
4 $f : f(X) = -2X$
5 $f : f(X) = X + 2$
6 $f : f(X) = 2 - X$
7 $f : f(X) = 3X - 1$
8 $f : f(X) = -2X + 3$
9 $f : f(X) = \frac{1}{2}X$
10 $f : f(X) = 5 - \frac{1}{2}X$

4 If the function $f : f(X) = aX^2 + 5X + 4$ is a linear function **find :**

1 The value of a
2 $f(-2)$ *(Qena 23)* « zero , – 6 »

5 If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 6X - a$ intersects the y-axis at the point (b , 2) , find the value of each of a , b *(Aswan 20)* « – 2 , 0 »

6 If the function $f : f(X) = 3X - 6$ is represented by a straight line passing through the point (a , 2 a) , find the value of a , then find the intersection point of the straight line with the y-axis. *(El-Gharbia 20)* « 6 , (0 , – 6) »

7 If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 2X + a$ and $f(3) = 9$, **find :**

1 The value of a
2 The coordinates of the intersection point of the straight line representing the function with the X-axis. *(Giza 20)* « 3 , $(-\frac{3}{2}, 0)$ »

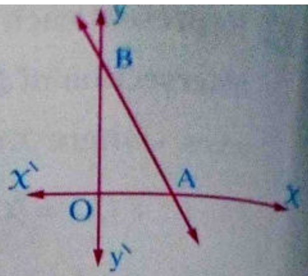
8 If the straight line representing the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$ cuts a positive part of the y-axis of length 3 units and passes through the point (1 , 5) **, find the value of each of :** a , b *(Kafr El-Sheikh 20)* « 2 , 3 »

9 If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = aX + b$ intersects the X-axis at the point (3 , 0) and intersects the y-axis at the point (0 , – 3) , then find the values of the two constants a and b and find the value of $f(1)$ *(El-Sharkia 17)* « 1 , – 3 , – 2 »

10 If X = {2 , 3 , 6} , Y = {3 , 4 , 5 , 6 , 7 , 8} and r : X $\longrightarrow$ Y where $r(X) = 9 - X$

1 Find the set of images of the elements of the set X by the function r
2 Is r a linear function ? "state the reason" *(El-Dakahlia 14)*

11 The opposite figure represents the function f where $f(X) = 4 - 2X$



Find :

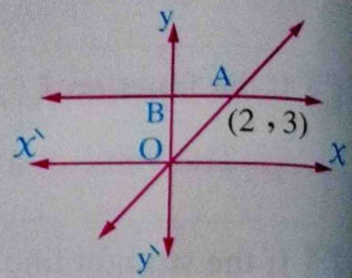
1 The coordinates of A , B

2 The area of Δ AOB

(Ismailia 16 – Luxor 19 – El-Kalyoubia 23) « (2 , 0) , (0 , 4) , 4 square units »

---

12 **In the opposite figure :**



The constant function f is represented graphically by the straight line $\overleftrightarrow{BA}$ and the linear function g is represented graphically by the straight line $\overleftrightarrow{OA}$ where A = (2 , 3)

1 Write the rule of the function f and the rule of the function g

2 Find the value of : $f(-10) + g(6)$

(El-Sharkia 14) « 12 »

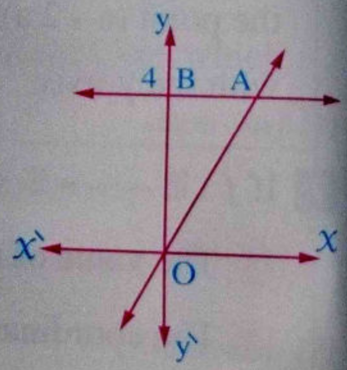
---

13 The opposite figure shows the straight line $\overleftrightarrow{AB}$ which represents the function $f : f(X) = 4$

, if $\overleftrightarrow{AO}$ represents the linear function $g : g(X) = nX + k$ and the area of the triangle ABO equals 4 square units

, then find the values of n and k

, where O is the origin point.



(El-Dakahlia 17) « 2 , 0 »

---

14 **In the opposite figure :**

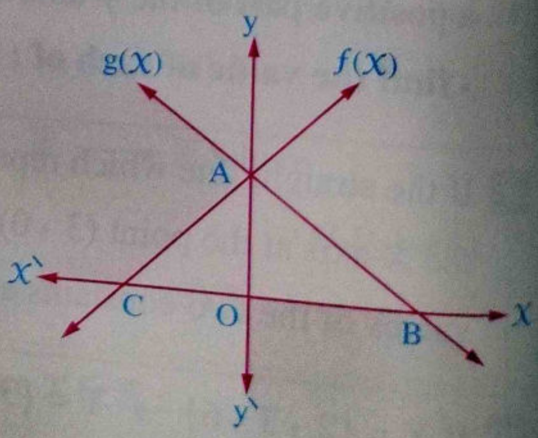
$\overleftrightarrow{AC}$ represents the linear function $f(X) = X + 3$

, $\overleftrightarrow{AB}$ represents the linear function $g(X) = mX + k$

If length of $\overline{BC}$ = 7 length units , **find :**

1 The value of k , m

2 g (8)



(El-Dakahlia 23) « 3 , $\frac{-3}{4}$, –3 »

15 While Karim was reading a book , he found that after 3 hours , 50 pages remained and after 6 hours , 20 pages remained. If the relation between the time (t) and the number of remained pages (b) is a linear relation :

1. Represent graphically the relation between t and b , then find the algebraic relation between the two variables.
2. What is the time that should be taken to finish the book ?
3. What is the number of pages remaining when Karim began to read ? *(Ismailia 20)*

Second Problems on the quadratic function

16 **Choose the correct answer from those given :**

1. If the point (3 , 2) is the vertex of the curve of the quadratic function f , then the equation of the line of symmetry is ....

   (a) $X = 3$ (b) $X = 2$ (c) $y = 3$ (d) $y = -3$

2. The vertex of the curve of the function $f : f(X) = 2X^2 - 4X + 5$ is ....

   (a) $(-1 , 11)$ (b) $(1 , 3)$ (c) $(2 , 5)$ (d) $(3 , 11)$

3. The equation of the axis of symmetry of the curve of the function $f : f(X) = X^2$ is ....

   (a) $X = 1$ (b) $X = 0$ (c) $y = 1$ (d) $y = 0$

4. The equation of the axis of symmetry of the curve of the function $f : f(X) = (X - 2)^2$ is ....

   (a) $X = 0$ (b) $X = 2$ (c) $X = -2$ (d) $X = -4$

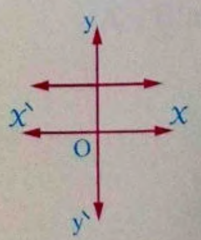
5. If the curve of the function f such that $f(X) = X^2 + c$ passes through the point (0 , 2) , then c = ...

   (a) -4 (b) -2 (c) 2 (d) 4

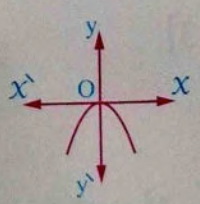
6. If $(-2 , y)$ belongs to the curve of the function $f : f(X) = X^2 + 1$, then y = ...

   (a) -3 (b) -1 (c) 3 (d) 5

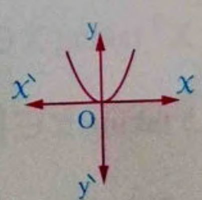
7. The graph of the function $f : f(X) = X^2 - 2X + 1$ is the graph number .... *(Giza 08)*



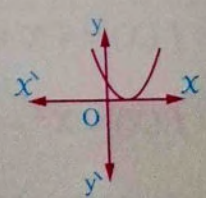
(a)



(b)

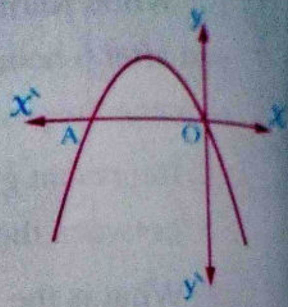


(c)



(d)

8 The opposite figure represents the curve of a quadratic function , A $(-4, 0)$, then the equation of the axis of symmetry is $X =$ .... *(El-Dakahlia 19)*



(a) 1 (b) -1

(c) -2 (d) 0

9 The maximum value of the function $f : f(X) = -2X^2 + 4X + 3$ is ....

(a) -1 (b) 1 (c) 3 (d) 5

10 If $f(X) = X^2$, $X \in [-2, 2]$, then $f(X) \in$ .... *(El-Dakahlia 08)*

(a) $]0, 4]$ (b) $]0, 4[$ (c) $[0, 4]$ (d) $[-4, 4[$

17 Represent each of the following functions graphically and from the graph , deduce the coordinates of the vertex of the curve , the equation of the line of symmetry and the maximum or minimum value of the function , where $X \in \mathbb{R}$:

1 $f : f(X) = 2X^2$ taking $X \in [-2, 2]$

2 $f : f(X) = X^2 + 1$ taking $X \in [-3, 3]$ *(Beni Suef 14 – El-Fayoum 16 – Ismailia 24)*

3 $f : f(X) = X^2 - 2$ taking $X \in [-3, 3]$ *(Alex. 22 – El Gharbia 23 – El-Menia 24)*

4 $f : f(X) = 2 - X^2$ taking $X \in [-3, 3]$ *(Damietta 22 – N. Sinai 23 – Suez 24)*

5 $f : f(X) = X^2 - 2X$ taking $X \in [-2, 4]$ *(Qena 11 – Cairo 18 – Kafr El-Sheikh 20)*

6 $f : f(X) = X^2 + 2X + 1$ taking $X \in [-4, 2]$ *(El-Gharbia 22 – Aswan 24)*

7 $f : f(X) = (X - 2)^2$ taking $X \in [-1, 5]$ *(El-Gharbia 20 – Qena 23 – El-Beheira 24)*

8 $f : f(X) = X(X - 2) - 3$ taking $X \in [-2, 4]$ *(El-Dakahlia 17)*

9 $f : f(X) = 3 - 2X - X^2$ taking $X \in [-4, 2]$

10 $f : f(X) = 4X + 3 - 2X^2$ taking $X \in [-2, 3]$

11 $f : f(X) = X^2 - 4X + 5$ taking $X \in [0, 5]$

12 $f : f(X) = 1 - 3X + X^2$ taking $X \in [-1, 4]$

18 If the curve of the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = m - X^2$ intersects the X-axis at the point $(-2, b)$, **find the value of :** $m^b + 2m$ *(El-Sharkia 15 - New Valley 24)* « 9 »

19 If $f(X) = a + X^2$, $\ell(X) = c$ are two polynomial functions where $3f(2) + 3\ell(X) = 6$, find the numerical value of : $2f(0) + 2\ell(7)$ where a and c are constants.

(El-Dakahlia 19) « -4 »

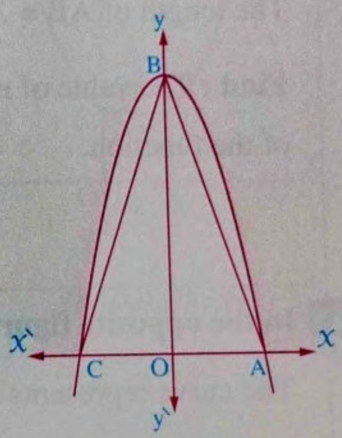
20 If $f : f(X) = kX^2 + (3k + 2)X + 6$, and X-coordinate of the vertex of the curve is -2, **find :**

1. The value of k
2. The minimum or maximum value of function f *(El-Dakahlia 23)* « 2, -2 »

21 The opposite figure represents the curve of the function f where $f(X) = 9 - X^2$

Find :

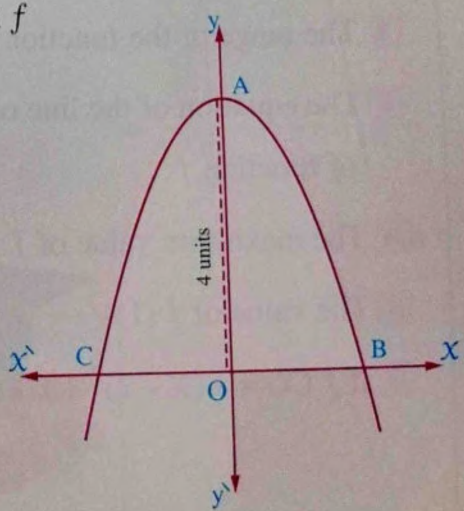
1. The coordinates of A and C
2. The area of the triangle ABC



(Kafr El-Sheikh 18) « $(3, 0)$, $(-3, 0)$, 27 square units »

22 The opposite figure represents the curve of the function f where $f(X) = m - X^2$, if OA = 4 units

Find :
1. The value of m
2. The coordinates of B and C
3. The area of the triangle with vertices A, B and C



(North Sinai 16 – Luxor 18 – Giza 20) « 4, $(2, 0)$, $(-2, 0)$, 8 square units »

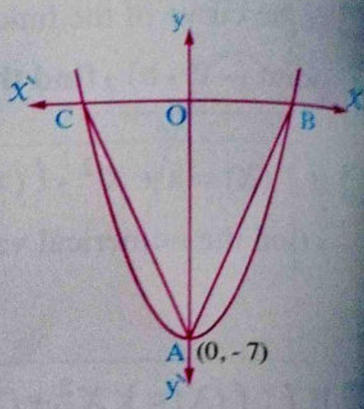
23 The opposite figure represents the curve of the function $f : f(X) = \ell X^2 - 7$

, the area of the triangle ABC = 21 square units

, A (0 , − 7)

Find the coordinates of the point B

, then find the value of ℓ



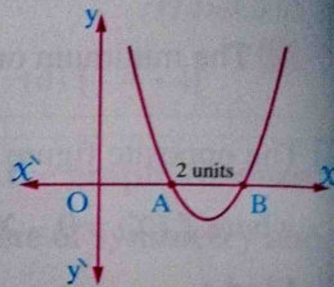
(El-Dakahlia 18) « (3 , 0) , $\frac{7}{9}$ »

24 The opposite figure represents the curve of the function f :

$f(X) = X^2 - 6X + m$

The length of $\overline{AB}$ = 2 length units

Find : The value of m , then find the minimum value of the function.



(El-Sharkia 24) « 8 , − 1 »

25 **In the opposite figure :**

The curve represents a function of the second degree f :

1 Write the domain of f

Use the graph to find :

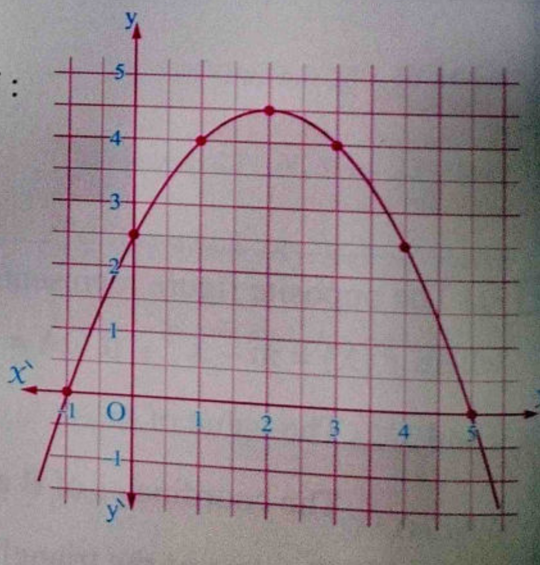
2 The range of the function f

3 The equation of the line of symmetry of the curve of function f

4 The maximum value of f

5 The value of $f(1)$

6 If $f(X) = a(X - 2)^2 + k$, then find the numerical value of : a + k



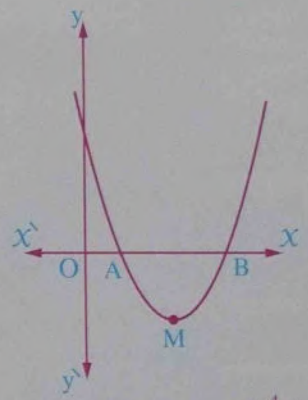
(El-Dakahlia 16)

For excellent pupils

26 **In the opposite figure :**

If the curve of the function f intersects the X-axis at the two points : A (1 , 0) , B (4 , 0) and M is the point of the vertex of the curve and $f(-2) + f(7) = 8$

, **find :** $f(-2)$



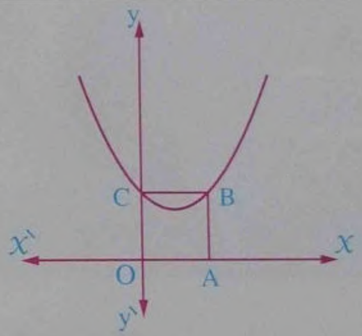
« 4 »

27 **In the opposite figure :**

The drawn curve represents the quadratic function $f : f(X) = X^2 - (k - 2) X - k + 4$

If ABCO is a square

, **find the value of :** k



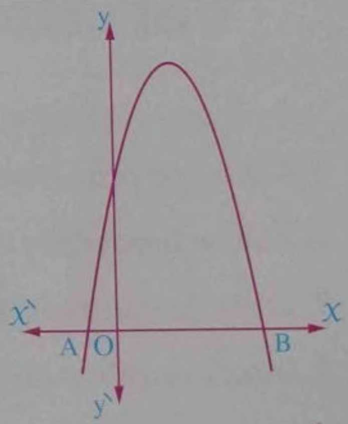
(El-Dakahlia 19) « 3 »

28 **In the opposite figure :**

The curve represents the function $f : f(X) = -X^2 + 4X + k - 1$ and intersects the X-axis at the two points A and B

If OB = 5 OA

, **find the value of :** k



« 6 »

UNIT TWO

Ratio, proportion, direct variation and inverse variation

Exercises of the unit :

5. Ratio and proportion.
6. Follow properties of proportion.
7. Continued proportion.
8. Direct variation and inverse variation.

Scan the QR code to solve an interactive test on each lesson

From the school book

Exercise 5

Ratio and proportion

● Remember ● Understand ● Apply ● Problem Solving

Interactive test

1 Choose the correct answer from those given :

1 If a , b , 2 and 3 are proportional , then $\frac{a}{b}$ = ... *(Matrouh 19)*

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

2 The fourth proportional for the numbers 4 , 8 and 8 is .... *(North Sinai 19)*

(a) 4 (b) 8 (c) 12 (d) 16

3 The third proportional for the numbers 4 , 12 , ... , 48 is .... *(Kafr El-Sheikh 19)*

(a) 7 (b) 32 (c) 16 (d) 36

4 If X , 3 , 4 and 6 are proportional , then X = ... *(Damietta 22)*

(a) 0 (b) 1 (c) 2 (d) 3

5 The second proportional for the numbers 2 , ... , 8 , 12 is .... *(El-Menia 18)*

(a) 4 (b) 6 (c) 3 (d) 2

6 If 2 , 3 , 6 and $X - 1$ are proportional , then X = ... *(El-Monofia 18)*

(a) 18 (b) 9 (c) 20 (d) 10

7 If 3 , a − 1 , a + 1 and 5 are proportional , then a = ...

(a) 3 (b) 4 (c) ± 3 (d) ± 4

8 If $7X = 3y$, then $\frac{X}{y}$ = ...

(a) $\frac{7}{3}$ (b) $\frac{3}{10}$ (c) $\frac{10}{3}$ (d) $\frac{3}{7}$

9 If 5 a − 4 b = 0 , then a : b = ...

(a) 4 : 5 (b) 4 : 9 (c) 5 : 4 (d) 5 : 9

10 If $\frac{a}{3} = \frac{b}{5}$, then $5a - 3b + 4 =$ .... *(El-Monofia 19)*

(a) 3 (b) 4 (c) 5 (d) 6

11 If $\frac{a}{3} = \frac{b}{4}$, then $8a - 6b + 4 =$ .... *(El-Kalyoubia 20)*

(a) 3 (b) 4 (c) 5 (d) 6

12 If $\frac{3a}{5b} = \frac{1}{2}$, then $\frac{a}{b} =$ .... *(Red Sea 11 – Alex. 20)*

(a) $\frac{6}{5}$ (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

13 If $2a = 3b$, then $\frac{3a}{2b} =$ .... *(El-Dakahlia 18 – El-Fayoum 23)*

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{9}{4}$ (d) $\frac{4}{9}$

14 If $4x = 5y$, then $\frac{5y}{4x} =$ .... *(Qena 11)*

(a) 1 (b) 2 (c) 3 (d) 4

15 If $3a = 5b$, then $\frac{3a}{b} =$ .... *(El-Fayoum 17)*

(a) 3 (b) 5 (c) $\frac{3}{5}$ (d) $\frac{5}{8}$

16 If $2x = 7y$, then $\left(\frac{x}{y}\right)^{-1} =$ .... *(El-Fayoum 09)*

(a) $\frac{2}{7}$ (b) $\frac{7}{2}$ (c) $\frac{49}{4}$ (d) $\frac{4}{49}$

17 If a , b , 2 and 3 are proportional , then $\frac{b}{a} =$ .... *(El-Kalyoubia 17)*

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) 3 (d) 2

18 If a , x , b and $2x$ are proportional quantities , then $\frac{a}{b} =$ ....

(Aswan 17 – Qena 23 – El-Monofia 24)

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

19 If 5 a , 2 , 3 b and 7 are four proportional quantities , then $\frac{a}{b} =$ .... *(Souhag 13)*

(a) $\frac{3}{7}$ (b) $\frac{6}{35}$ (c) $\frac{3}{5}$ (d) $\frac{3}{2}$

20 If $4x^2 = 9y^2$, then $\frac{x}{y} =$ .... *(Beni Suef 16)*

(a) $\frac{9}{4}$ (b) $\frac{3}{2}$ (c) $\pm\frac{2}{3}$ (d) $\pm\frac{3}{2}$

21 If $\frac{5a - 7b}{2a + 3b} = 0$, then $\frac{b}{a} =$ ....

(a) $\frac{5}{7}$ (b) $\frac{7}{5}$ (c) $\frac{3}{10}$ (d) $\frac{10}{3}$

22 If $\frac{a + 2b}{a - b} = \frac{2}{3}$, then $\frac{b}{a} =$ .... *(Alexandria 11 – El-Monofia 20)*

(a) $\frac{1}{8}$ (b) 8 (c) $-\frac{1}{8}$ (d) -8

23 If a , b , c and d are proportional quantities , then ....

(a) $\frac{b}{d} = \frac{a}{c}$ (b) $\frac{a}{c} = \frac{d}{b}$ (c) $\frac{b}{c} = \frac{a}{d}$ (d) $ab = cd$

24 If $4x^2 + 9y^2 = 12xy$, then $\frac{x}{y} = \dots\dots\dots$ *(El-Kalyoubia 09)*

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $-\frac{2}{3}$ (d) $-\frac{3}{2}$

25 If $a : b = 2 : 3$, $b : c = 5 : 6$, then $a : c = \dots\dots\dots$ *(El-Sharkia 24)*

(a) 1 : 3 (b) 3 : 5 (c) 2 : 3 (d) 5 : 9

26 The ratio between the area of a square shaped region of side length ℓ cm. to the area of another square shaped region of side length 2ℓ cm. is .... *(El-Monofia 13)*

(a) 1 : 2 (b) ℓ : 4 (c) 1 : 4 (d) 4 : 1

---

2 Find each of the following :

1 The first proportional for the numbers : ... , $\sqrt{8}$, 7 and $14\sqrt{2}$

2 The third proportional for the quantities : a , (a + b) , ... and $(a^2 - b^2)$

3 The fourth proportional for the quantities : (a + b) , (a – b) , (a – b) and ...

---

3 Find the value of x in each of the following , if :

1 $(2x - 3) : (x - 5) = 1 : 4$ « 1 »

2 $(x - 5) : (5x + 3) = 2 : 3$ « –3 »

3 $(x^2 - 8) : (2x^2 + 1) = 1 : 3$ « ±5 »

4 $(x^2 + 10x) : (2x^2 - 3) = 24 : 5$ where x is an integer. « 2 »

---

4 If $\frac{x - 2y}{x + 3y} = \frac{1}{3}$ **, find :** $\frac{y}{x}$ *(Aswan 15)* « $\frac{2}{9}$ »

---

5 If $\frac{2x + 3}{2x - 3} = \frac{2y + 5}{2y - 5}$ **, prove that :** $\frac{x}{y} = \frac{3}{5}$

---

6 If $x^2 - 4y^2 = 3xy$ **, find :** $x : y$ « –1 : 1 or 4 : 1 »

---

7 If $3x^2 - 10xy + 7y^2 = 0$, $x \neq y$ **, find the ratio :** $x : y$ « 7 : 3 »

---

8 If $x^2 - 4xy + 4y^2 = 0$ **, find the value of :** $\frac{x + 3y}{3x - y}$ *(Luxor 24)* « 1 »

---

9 If $\frac{x}{y} = \frac{2}{3}$ **, find the value of the ratio :** $\frac{3x + 2y}{6y - x}$ *(Alex. 22 – Qena 24)* « $\frac{3}{4}$ »

10 If $\frac{a}{b} = \frac{3}{5}$, **find the value of :** $7a + 9b : 4a + 2b$ *(Qena 15 – Cairo 20 – Aswan 24)* « 3 »

11 If $4a = 3b$, **then find the value of :**

1 $\frac{4a+b}{2a-b}$ « 8 »

2 $\frac{b^2-a^2}{a^2-b^2}$ « −1 »

12 If $\frac{a}{b} = \frac{1}{3}$, $\frac{c}{d} = \frac{7}{2}$, **find the ratio :** $\frac{2ac+bd}{bc-3ad}$ « $\frac{4}{3}$ »

13 If $7X - 3y : X + y = 3 : 1$, **find the ratio :** $12X + 9y : 11X - 3y$ « 2 : 1 »

14 If $\frac{21X+a}{7X+b} = \frac{a}{b}$, where $X \neq 0$, **then find the value of :** $\frac{a+2b}{2a}$ *(Ismailia 13)* « $\frac{5}{6}$ »

15 Find the number that if it is added to each of the numbers 3 , 5 , 8 and 12, they become proportional. *(South Sinai 17 – Assiut 18 – El-Gharbia 22)* « 2 »

16 Find the number which is subtracted from each of the following numbers to be proportional 16 , 21 , 14 and 18 « 6 »

17 **Prove that : a , b , c and d are proportional quantities if :**

1 $\frac{a+b}{b} = \frac{c+d}{d}$ *(El-Fayoum 09 – Qena 22)*

2 $\frac{a}{b-a} = \frac{c}{d-c}$ *(El-Sharkia 15 – Aswan 20 – Alex. 22)*

3 $\frac{a-b}{a+b} = \frac{c-d}{c+d}$

4 $\frac{a^2-2c^2}{b^2-2d^2} = \frac{a^2}{b^2}$ where a , b , c and d are positive quantities.

18 If $a : b : c = 5 : 7 : 3$ and $a + b = 27.6$, **find the value of each of :** a , b and c « 11.5 , 16.1 , 6.9 »

19 If $a : b : c = 3 : 4 : 5$, **find the numerical value of the expression :** $\frac{a^2+b^2+c^2}{a(b+c)}$ « $\frac{50}{27}$ »

20 If $2a = 3b = 4c$, **find :** $a : b : c$ « 6 : 4 : 3 »

21 If $4a = 3b = 6c$ and $a + b + c = 36$, find the value of each a , b and c *(El-Fayoum 22)* « 12 , 16 , 8 »

22 **Answer the following :**

1. Find the number which if it is added to the two terms of the ratio 7 : 11 , it will be 2 : 3 *(El-Fayoum 18 – Giza 19 – Aswan 22 – Suez 23 – Red Sea 24)* « 1 »

2. Find the number that if we subtract thrice of it from each of the two terms of the ratio $\frac{49}{69}$, the ratio becomes $\frac{2}{3}$ *(Giza 12 – El-Beheira 20)* « 3 »

3. Find the number which if its square is added to each of the two terms of the ratio 7 : 11 , it becomes 4 : 5 *(Suez 17 – El-Monofia 20 – South Sinai 24)* « 3 or – 3 »

4. Find the positive number which if we add its square to each of the two terms of the ratio 5 : 11 , it becomes 3 : 5 *(Giza 19 – Beni Suef 20 – El-Monofia 22 – Alex. 24)* « 2 »

5. What is the number which is subtracted from the antecedent of the ratio 15 : 13 and added to its consequent to become 3 : 4 ? *(Luxor 20)* « 3 »

6. Two integers , the ratio between them is 3 : 7 and if we subtracted 5 from each term , the ratio between them becomes 1 : 3 , find the two numbers. *(Ismailia 20 – Monofia 23)* « 15 , 35 »

7. Two integers , the ratio between them is 2 : 3 , if you add to the first 7 and subtract from the second 12 , the ratio between them becomes 5 : 3
Find the two numbers. *(El-Sharkia 22 – El-Gharbia 23)* « 18 , 27 »

8. Two positive real numbers , the ratio between them is 4 : 7 and the square of the small number exceeds 5 times the great number by 39 , find the two numbers. « 12 , 21 »

9. **In the opposite figure :**
Alaa shaded $\frac{5}{6}$ the area of the circle , $\frac{2}{3}$ the area of the triangle.
Find the ratio between the area of the circle and the area of the triangle.

(Giza 08) « 2 : 1 »

23 Through the interest of the Egyptian authorities in the villages , a budget of 1.85×10^6 pounds was set for one of the villages to build a school , a medical unit and a youth centre. If the cost of the school is $\frac{3}{2}$ of the cost of the medical unit and the cost of the medical unit is $\frac{5}{6}$ of the cost of the youth centre , what is the cost of each of them ?

« 7.5×10^5 , 5×10^5 , 6×10^5 »

24 If the rate of success in one of the governorates of the third preparatory is 83% and the rate of success for boys is 79% and the rate of success of girls is 89% , find the ratio between the number of boys and the number of girls in this governorate.

« 3 : 2 »

25 The length of a piece of wire is 152 cm. , it is divided into two parts of ratio 11 : 8 , a circular shape is made from the long part and a square shape is made from the short part. Find the ratio between the area of the square and the area of the circle. $\left(\pi = \frac{22}{7}\right)$ « 32 : 77 »

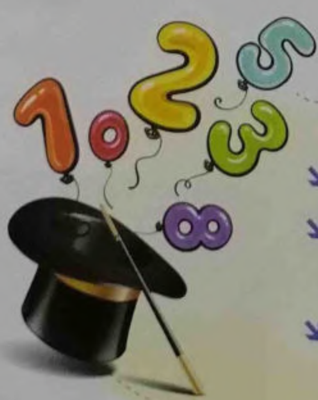
For excellent pupils

26 Four proportional numbers , the fourth proportional equals the square of the second proportional, the first proportional decreases the second proportional by 2 , the third proportional = 8 , find the four numbers.

« 2 , 4 , 8 , 16 or – 4 , – 2 , 8 , 4 »

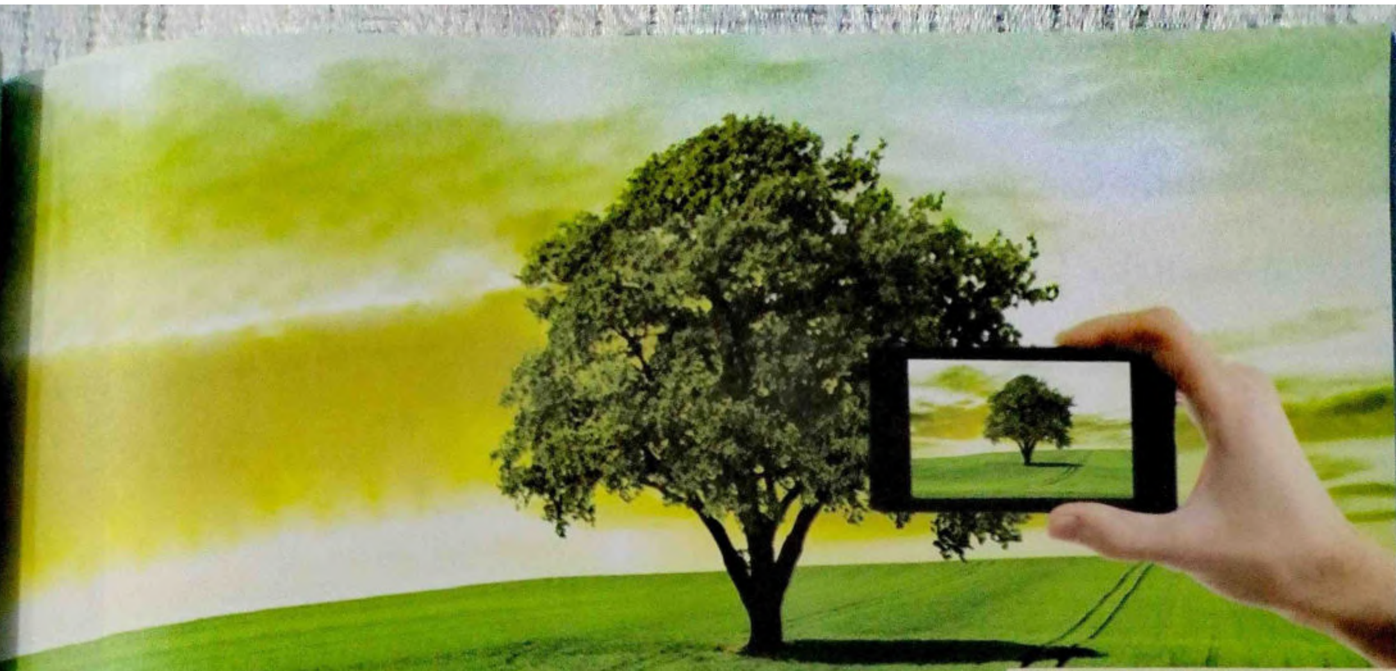
27 Find the positive number which if its multiplicative inverse is added to the consequent of the ratio $\frac{2}{3}$, it will become $\frac{3}{5}$

« 3 »



Wonders of numbers

- Choose an integer between 100 , 1000
- Multiply it by 7 , then multiply the product by 11 and multiply the product by 13
- Do it using different numbers and notice the product each time !



From the school book

Exercise 6 Follow properties of proportion

● Remember ● Understand ○ Apply Problem Solving

Interactive test

1 Choose the correct answer from those given :

1 If $\frac{a}{b} = \frac{c}{d} = \frac{h}{m}$, then $\frac{a+c+h}{b+d+m} = \text{..........}$ *(El-Sharkia 20)*

(a) $\frac{a}{b} + \frac{c}{d} + \frac{h}{m}$ (b) $\frac{c}{h}$ (c) $\frac{c}{a}$ (d) $\frac{c}{d}$

2 If $\frac{a}{b} = \frac{c}{d} = \frac{5}{8}$, then $\frac{b+d}{a+c} = \text{..........}$ *(El-Fayoum 22)*

(a) $\frac{5}{8}$ (b) $\frac{8}{5}$ (c) $\frac{13}{8}$ (d) $\frac{5}{13}$

3 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{3}{5}$, then $\frac{a-2c+e}{b-2d+f} = \text{..........}$

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

4 If $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$, then each ratio equals .... *(El-Fayoum 19)*

(a) $\frac{x+y+z}{3}$ (b) $\frac{x+2y-z}{3}$ (c) $\frac{x-y+z}{10}$ (d) $\frac{x-y}{5}$

5 If $\frac{4}{x} = \frac{7}{y} = \frac{a}{y-x}$, then a = ...

(a) – 3 (b) 3 (c) 11 (d) 28

6 If $\frac{\ell}{3} = \frac{m}{8} = \frac{\ell + \frac{1}{2} m}{b}$, then b = ...

(a) 24 (b) 11 (c) 8 (d) 7

7 If $\frac{x}{5} = \frac{y}{4} = \frac{x + 2y}{k}$, then k = ... *(El-Gharbia 17 – Port Said 23)*

(a) 9 (b) 13 (c) 14 (d) 8

8 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then $\frac{a + 2c + 3e}{b + 2d + 3f} = \frac{.......}{5f}$

(a) 5 a (b) 5 c (c) 5 e (d) 5 a + 5 c + 5 e

9 If $\frac{a}{2} = \frac{b}{3}$, then $\frac{b - a}{b + a}$ = ...

(a) $\frac{1}{5}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

10 If $\frac{a}{b} = \frac{c}{d} = 5$, then $\frac{2a - 3c}{2b - 3d}$ = ...

(a) 10 (b) 15 (c) 5 (d) 1

11 If $\frac{6x}{4y} = \frac{3z}{9\ell} = 10$, then $\frac{3x + z}{2y + 3\ell}$ = ...

(a) 50 (b) 30 (c) 20 (d) 10

12 If $\frac{a}{b} = \frac{c}{d} = m$, where $m \neq 0$, then $\frac{a \times c}{b \times d}$ = ... *(Cairo 17)*

(a) $2m^2$ (b) m^2 (c) m (d) 2 m

13 If $\frac{x}{5} = \frac{y}{7} = m$, then $\frac{2x + y}{17}$ = ...

(a) 3 m (b) 2 m (c) 17 m (d) m

14 If $\frac{a}{4} = \frac{b}{5} = k$, then $\frac{4a + 4b}{9}$ = ...

(a) k (b) 2 k (c) 3 k (d) 4 k

15 If $\frac{a}{4} = \frac{b}{5}$, 2 a + 3 b = 46 , then a = ...

(a) 2 (b) 4 (c) 5 (d) 8

16 If $\frac{a}{b} = \frac{2}{3}$, $\frac{a}{c} = \frac{4}{5}$, then b : c = ...

(a) 3 : 4 (b) 5 : 6 (c) 6 : 5 (d) 4 : 3 *(El-Gharbia 17)*

17 If $\frac{x}{y + 1} = \frac{y}{z - 2} = \frac{z}{x + 3} = \frac{2}{3}$, then $x + y + z$ = ...

(a) 3 (b) 4 (c) 6 (d) 8

2 If a , b , c and d are proportional quantities , prove that :

1 $\frac{3a + c}{5a - 2c} = \frac{3b + d}{5b - 2d}$ *(Assiut 17 – S. Sinai 23)*

2 $\frac{3a - 2c}{5a + 3c} = \frac{3b - 2d}{5b + 3d}$ *(Suez 16 – Kafr El-Sheikh 18 – South Sinai 22 – Souhag 24)*

3 $\frac{a^2+c^2}{ab+cd}=\frac{a}{b}$ *(El-Monofia 11)*

4 $\frac{a^2+c^2}{b^2+d^2}=\frac{ac}{bd}$ *(El-Monofia 16 – El-Kalyoubia 17 – El-Gharbia 18)*

5 $\frac{ac}{bd}=\left(\frac{a-c}{b-d}\right)^2$ *(Suez 18 – Aswan 22 – El-Monofia 23 – Giza 23)*

6 $\left(\frac{a+b}{c+d}\right)^2=\frac{2a^2-3b^2}{2c^2-3d^2}$

7 $\sqrt{\frac{3a^2-5c^2}{3b^2-5d^2}}=\frac{a}{b}$ where a , b , c and d are positive quantities.

8 $\sqrt[3]{\frac{5a^3-3c^3}{5b^3-3d^3}}=\frac{a+c}{b+d}$ *(El-Kalyoubia 19)*

9 $\frac{a^2-2ac+c^2}{ac}=\frac{b^2-2bd+d^2}{bd}$ *(Ismailia 18)*

3 If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ **, prove that :**

1 $\frac{a+5c}{b+5d}=\frac{c-3e}{d-3f}$

2 $\frac{2a+7c-4e}{2b+7d-4f}=\frac{a-8e}{b-8f}$

3 $\frac{2a^4b^2+3a^2e^2-5e^4f}{2b^6+3b^2f^2-5f^5}=\frac{a^4}{b^4}$

4 $\sqrt{\frac{5a^2-7ce}{5b^2-7df}}=\frac{2a+c}{2b+d}$

4 If $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$ **, prove that :**

1 $\frac{2y-z}{3x-2y+z}=\frac{1}{2}$ *(Port Said 19 – Beni Suef 20 – Port Said 22 – Port Said 23 – Alex 24)*

2 $\sqrt{3x^2+3y^2+z^2}=2x+y$ *(El-Menia 12 – Souhag 16 – Damietta 19)*

5 If $x=\frac{y}{2}=\frac{z}{3}$ **, then prove that :** $\frac{x+y-2z}{x-3z}=\frac{3}{8}$ *(Assiut 17)*

6 If $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}$ **, prove that :** $2a-5b+3c$ = one of the given ratios.

7 If $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}=\frac{2a-b+5c}{3x}$ **, then find the value of :** x

(Qena 17 – Luxor 18 – Aswan 19 – El-Kalyoubia 20 – El-Beheira 22 – Assiut 23 – Matrouh 24) « 7 »

8 If $\frac{a}{2}=\frac{b}{7}=\frac{c}{3}$ **, find the value of :** $\frac{a+2b}{b-c}$ *(North Sinai 09)* « 4 »

9 If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{2}{3}$, and $5a-3c+e=18$

Find the value of : $5b-3d+f$ *(El-Dakahlia 23)* « 27 »

10 If $\frac{a}{4X+y}=\frac{b}{X-4y}$, **prove that :** $\frac{a+b}{5X-3y}=\frac{a-b}{3X+5y}$ *(Damietta 12 – El-Dakahlia 19)*

11 If $\frac{X+y}{19}=\frac{y+z}{7}$, **prove that :** $\frac{X+2y+z}{13}=\frac{X-z}{6}$

12 If $\frac{y}{X-z}=\frac{X}{y}=\frac{X+y}{z}$, prove that each ratio is equal to 2 (unless $X+y=0$), then find $X:y:z$ *(El-Beheira 18)* « 4 : 2 : 3 »

13 If $\frac{X}{a-b+c}=\frac{y}{b-c+a}=\frac{z}{c-a+b}$, **prove that :** $\frac{X+y}{a}=\frac{y+z}{b}$ *(Port Said 09)*

14 If $\frac{X}{2a+b}=\frac{y}{2b-c}=\frac{z}{2c-a}$, **then prove that :** $\frac{2X+y}{4a+4b-c}=\frac{2X+2y+z}{3a+6b}$

(El-Beheira 17 – El-Kalyoubia 18 – Matrouh 19)

15 If $\frac{a}{2X-y}=\frac{b}{2y-X}$, **prove that :** $\frac{2a+b}{a+2b}=\frac{X}{y}$

16 If $\frac{a}{2X+y}=\frac{b}{3y-X}=\frac{c}{4X+5y}$, **prove that :** $\frac{a+2b}{4b+c}=\frac{7}{17}$

17 If $\frac{X+y}{7}=\frac{y+z}{5}=\frac{z+X}{8}$, **prove that :** $\frac{X+y+z}{X-z}=5$ *(El-Monofia 16 – El-Gharbia 22 – Assiut 24)*

18 If $\frac{a+b}{4}=\frac{b+c}{5}=\frac{c+a}{7}$, **prove that :** $\frac{a+b+c}{8}=\frac{a}{3}$ *(Kafr El-Sheikh 15)*

19 If $\frac{X+y}{3}=\frac{y+z}{8}=\frac{z+X}{6}$, **prove that :** $\frac{X+y+z}{2X+3y+3z}=\frac{17}{50}$ *(Kafr El-Sheikh 20)*

20 If $\frac{X+y}{5}=\frac{y+z}{8}=\frac{z+X}{7}$, **prove that :** $\frac{X}{2}=\frac{y}{3}=\frac{z}{5}$ *(New Valley 17)*

21 If $\frac{X+y}{25}=\frac{X-y}{11}=\frac{X+y-z}{8}$, **prove that :** $X:y:z=18:7:17$

22 If $\frac{a+3b}{X+6y}=\frac{3b+5c}{6y+10z}=\frac{5c+a}{10z+X}$, **prove that :** $\frac{a}{b}=\frac{X}{2y}$ and find $a:b:c$ « $X:2y:2z$ »

23 If $\frac{a}{3X+4y}=\frac{b}{5X-2y}=\frac{c}{y+2X}$, **prove that :** $13X(3c-2a)+5y(a+2b)=0$

24 If $\frac{X}{7}=\frac{y}{3}$, **prove that :** $(2X-3y)$, $(X+2y)$, 10 and 26 are proportional

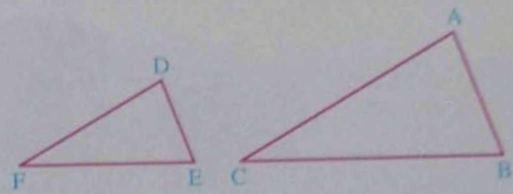
25 If $\frac{a}{b} = \frac{3}{5}$ and $\frac{a}{c} = \frac{3}{7}$ **, find the value of the expression :** $a + b + c$ in terms of a «5 a»

26 If $\frac{a}{b} = \frac{2}{3}$, $\frac{a}{c} = \frac{3}{5}$ and $a + b + c = 75$ **, find the value of each of :** a , b and c

(Red Sea 16) « 18 , 27 , 30 »

Geometric Application

27 **In the opposite figure :**



If $\Delta ABC \sim \Delta DEF$

where DF : AC = 2 : 3 and the perimeter of $\Delta DEF = 22$ cm.

, find the perimeter of : ΔABC « 33 cm. »

For excellent pupils

28 If $\frac{a}{x - y + z} = \frac{b}{x + y - z} = \frac{c}{y + z - x}$, prove that each ratio $= \frac{a x + b y + c z}{x^2 + y^2 + z^2}$

29 If $\frac{2x + y}{x} = \frac{4y + z}{y} = \frac{4z + 3x}{z}$, find the ratio $x : y : z$

, then prove that : $\frac{2x + y + z}{3x - y + 2z} = \frac{4}{3}$ « 1 : 3 : 3 »

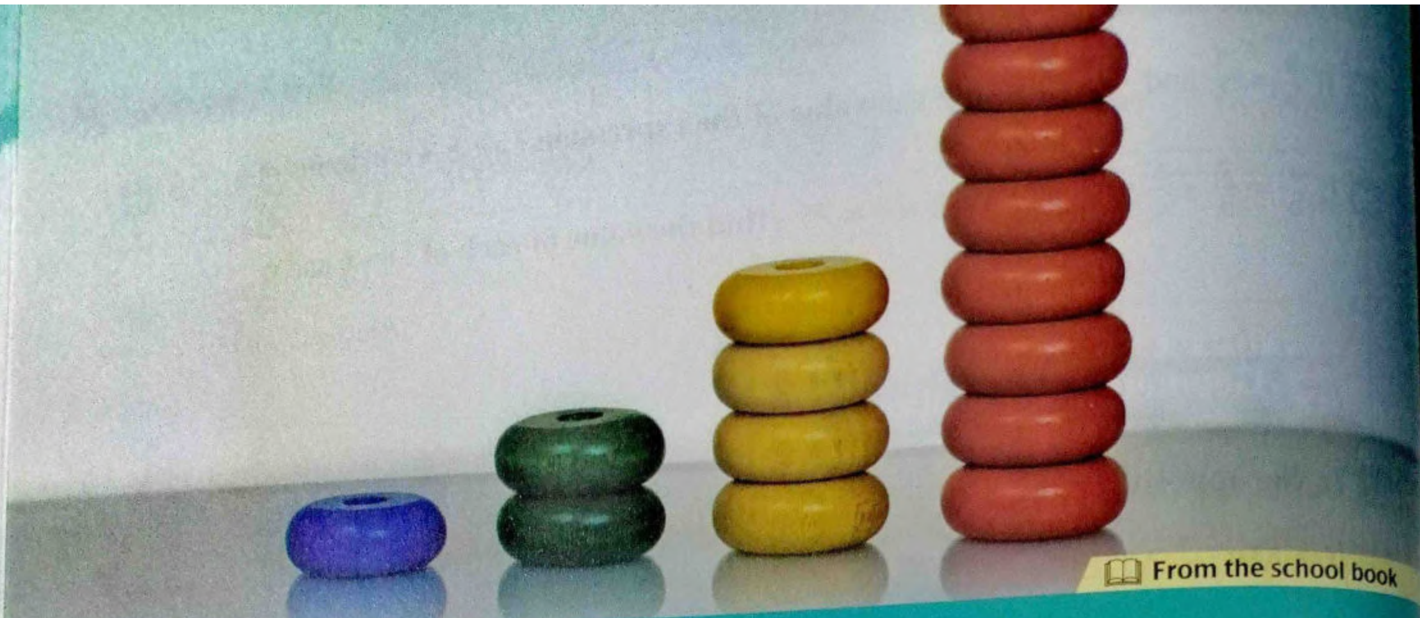
30 If $\frac{a + 2b}{5} = \frac{3b - c}{3} = \frac{c - a}{2}$ **, prove that :**

1. $a + b - c = \text{zero}$
2. $\frac{3b - a}{2b + c} = \frac{5}{7}$

Wonders of numbers

2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder.

Try it yourself !



From the school book

Exercise 7

Continued proportion

Remember Understand Apply Problem Solving

Interactive test

1 Find the middle proportional between :

1) $3, 27$ | 2) $9, 25$ | 3) $-2, -8$ *(Giza 09)*

4) $\frac{1}{5}, 125$ | 5) $2a, 8ab^2$ | 6) $(\ell + m)^2, (\ell - m)^2$

2 Find the third proportional of each of the following :

1) $6, 12$ | 2) $x^2, -5x$ | 3) $x^2, -3x^2$

3 If b is the middle proportional between a and c , prove that :

1) $\frac{a}{c} = \frac{b^2}{c^2}$ *(Red sea 23)*

2) $\frac{2a + 3b}{2b + 3c} = \frac{a}{b}$ *(Port said 22 – Suez 23)*

3) $\frac{a - b}{b - c} = \frac{a + 3b}{3c + b}$ *(Souhag 22)*

4) $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$ *(Cairo 20 – El-Dakahlia 24)*

5) $\left(\frac{b - c}{a - b}\right)^2 = \frac{c}{a}$ *(El-Menia 24)*

6) $\frac{a^3 + b^3}{b^3 + c^3} = \frac{a^2}{cb}$ *(El-Monofia 11 - Qena 24)*

7) $\frac{a^3 - 4b^3}{b^3 - 4c^3} = \frac{b^3}{c^3}$

8) $\frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{c}{a} = \frac{c^2}{b^2}$

9) $\frac{a^2 + ab + b^2}{b^2 + bc + c^2} = \frac{a^2 - b^2}{b^2 - c^2}$

10) $\frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{2a}{c}$ *(Port Said 17 – El-Dakahlia 19 – Suez 22 – El-Gharbia 24)*

11) $\frac{a + b + c}{a^{-1} + b^{-1} + c^{-1}} = b^2$ *(New Valley 22)*

12) $\frac{ac}{b(b + c)} = \frac{a}{a + b}$ *(El-Gharbia 17)*

13) $\frac{a - b}{a - c} = \frac{b}{b + c}$ *(Giza 22)*

4 If a , b , c and d are in continued proportion , prove that :

1. $\frac{a-2b}{b-2c}=\frac{3b+4c}{3c+4d}$ *(El-Monofia 24)*
2. $\frac{3a+5c}{3b+5d}=\frac{a-4c}{b-4d}$
3. $\frac{3a-5c}{a-b+c}=\frac{3b-5d}{b-c+d}$
4. $\frac{a-d}{a+b+c}=\frac{a-2b+c}{a-b}$
5. $\frac{c^2-d^2}{a-c}=\frac{b\,d}{a}$ *(Matrouh 17 – El-Beheira 18 – South Sinai 20 – El-Gharbia 22 – El-Dakahlia 23)*
6. $\frac{a^2-3c^2}{b^2-3d^2}=\frac{b}{d}$ *(El-Beheira 15 – Alex. 17 – Beni Suef 18 – El-Beheira 23 – El-Sharkia 24)*
7. $\frac{a\,b-c\,d}{b^2-c^2}=\frac{a+c}{b}$ *(Qena 16 – El-Monofia 17 – El-Monofia 22)*
8. $\frac{a}{b+d}=\frac{c^3}{c^2\,d+d^3}$ *(Alex. 19 – El-Fayoum 20)*
9. $\frac{a^2+b^2+c^2}{b^2+c^2+d^2}=\frac{a\,c}{b\,d}$ *(El-Dakahlia 11)*
10. $\frac{2a+3d}{3a-4d}=\frac{2a^3+3b^3}{3a^3-4b^3}$
11. $\frac{a+5b}{b+5c}=\sqrt{\frac{b}{d}}$
12. $\sqrt[3]{\frac{5a^3-3c^3}{5b^3-3d^3}}=\frac{a+c}{b+d}$ *(Alexandria 11)*
13. $\left(\frac{a+b}{b+c}\right)^3=\frac{a}{d}$ *(El-Sharkia 15)*
14. $\frac{a^2+d^2}{c\,(a+c)}=\frac{b}{d}+\frac{d}{b}-1$

5 Choose the correct answer from those given :

1. The third proportional of the two numbers 9 and – 12 is .... *(El-Beheira 11)*
   (a) – 16 (b) 8 (c) 16 (d) 108
2. The middle proportional between a and c is .... *(Beni Suef 20)*
   (a) $\sqrt{a+c}$ (b) $\frac{a+c}{2}$ (c) $\pm\sqrt{a\,c}$ (d) a c
3. If the number 6 is the positive proportional mean of the two numbers 2 and m , then m = ... *(Aswan 13)*
   (a) 8 (b) 12 (c) 18 (d) 36
4. If x , y , z are in continued proportion , then x = ... *(Luxor 20)*
   (a) $\pm\sqrt{y\,z}$ (b) y z (c) $\frac{y^2}{z}$ (d) $\frac{y}{z}$
5. If ℓ , m and n are in continued proportion , then $m^2-\ell\,n$ = ...
   (a) – 1 (b) 0 (c) 1 (d) 2
6. If 7 , x and $\frac{1}{y}$ are in continued proportion , then $x^2\,y$ = ... *(El-Beheira 19 – Ismailia 22)*
   (a) 7 (b) $\frac{1}{7}$ (c) 14 (d) 49
7. If $\frac{a}{b}=\frac{b}{c}=\frac{c}{5}=2$, then a = ... *(El-Monofia 12)*
   (a) 5×2^2 (b) 40 (c) 10 (d) 2×5^3

(El-Sharkia 13)

8 If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = 2$, then $\frac{a}{d} = \dots\dots$

(a) 2 (b) 4 (c) 8 (d) 16

9 If $6a^2b^2$, $3ab$ and c are proportional quantities , then $c = \dots\dots$

(a) -3 (b) $3ab$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

(Cairo 09)

10 The proportional mean between $(x-2)$ and $(x+2)$ is $\dots\dots$

(a) $\sqrt{x+2}$ (b) x^2-4 (c) $\pm\sqrt{x^2-4}$ (d) $\sqrt{x^2-4}$

11 The number which is added to each of the numbers 1 , 3 and 6 to become in continued proportion is $\dots\dots$ (Damietta 13)

(a) 1 (b) 2 (c) 3 (d) 6

12 If a , b , c and d in continued proportion , and $a+b+c=5$, $b+c+d=7$, then $\frac{a}{b} = \dots\dots$ (Alex 23)

(a) $\frac{5}{7}$ (b) $\frac{7}{5}$ (c) $-\frac{5}{7}$ (d) $-\frac{7}{5}$

6 If a , 3 , 9 and b are in continued proportion , find the value of each of a and b

(Luxor 16) « 1 , 27 »

7 If 3 , ℓ , 12 and m are in continued proportion , find the value of each of ℓ and m « ± 6 , ± 24 »

8 If 2 , a , b , 54 are in continued proportion , find the value of : a + b (El-Kalyoubia 24)

9 Find the number that if we subtract it from each of the numbers 3 , 7 , 19 , then they become in continued proportion. (Luxor 17) « 1 »

10 If b is the middle proportional between a and c and $a = 4c = 4$, **then find the value of :** $a^2 + b^2 + c^2$

(El-Fayoum 17) « 21 »

11 If b is the middle proportional between a and c , c is the middle proportional between b and d , **prove that :** $\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{d^2} = \frac{a}{c} + \frac{b}{d} + \frac{ac}{bd}$

12 If $y^2 = xz$, **prove that :** $\frac{x(x-y)}{y(y-z)} = \frac{y^2}{z^2}$

13 If $b^2 = ac$ and $c^2 = bd$, **prove that :** $\frac{2a+3d}{3a-4d} = \frac{2a^3+3b^3}{3a^3-4b^3}$

14 If $\frac{a^2+b^2}{b^2}=\frac{b^2+c^2}{c^2}$, prove that b is the middle proportional between a and c where a c is a positive quantity. *(Alexandria 15 – Beni Suef 15)*

---

15 If a , b , c and d are in continued proportion , **prove that :** (b + c) is the middle proportional between (a + b) and (c + d)

---

16 If 5 a , 6 b , 7 c and 8 d are positive quantities in continued proportion ,

prove that : $\sqrt[3]{\frac{5\,a}{8\,d}}=\sqrt{\frac{5\,a+6\,b}{7\,c+8\,d}}$

---

17 If b is the middle proportional between a and c , **prove that :** $\frac{a^4+b^4+c^4}{a^{-4}+b^{-4}+c^{-4}}=b^8$

Geometric Application

18 X , y and z are three proportional side lengths in a triangle , X + y = 15 cm. and y + z = 22.5 cm. **Find :** X : y « 2 : 3 »

---

19 ABC is a triangle in which m (∠ C) = 60° , if the measures of its angles ∠ A , ∠ B and ∠ C respectively are in continued proportion.

, **find :** m (∠ A) and m (∠ B) « 60° , 60° »

For excellent pupils

20 If $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=2$, **find the solution set of the equation :** $a\,X^2-2\,b\,X+c=0$ « $\{\frac{1}{2}\}$ »

---

21 If 5 is the middle proportional between X and y , find the middle proportional between $\left(X+\frac{1}{y}\right)$ and $\left(y+\frac{1}{X}\right)$ « ± 5.2 »

Exercise 8

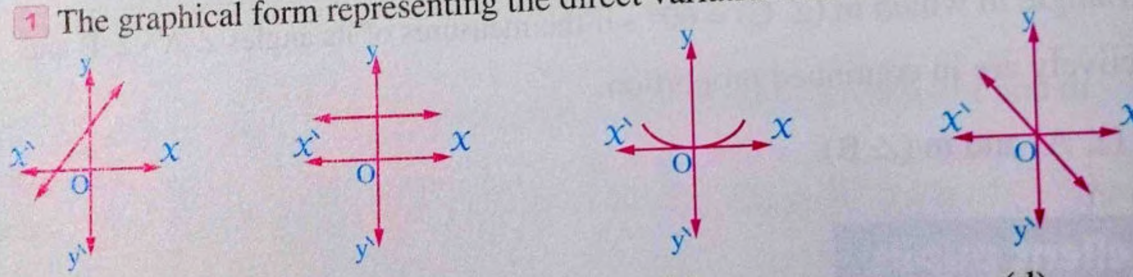
Direct variation and inverse variation

Remember · Understand · Apply · Problem Solving

Interactive test

1 Choose the correct answer from those given :

1 The graphical form representing the direct variation between x and y is ....



(a) (b) (c) (d) *(El-Sharkia 16)*

2 If $2xy = 5$, then $y \propto$ .... *(Aswan 24)*

(a) $\frac{1}{x}$ (b) $x - 5$ (c) x (d) $x + 5$

3 If $y = 9x$, then $y \propto$ .... *(North Sinai 24)*

(a) x (b) $\frac{1}{x}$ (c) $2x + 7$ (d) $\frac{1}{x^2}$

4 If $xy = 5$, then $y \propto$ .... *(New Valley 24)*

(a) x^{-1} (b) x (c) $5x$ (d) x^2

5 If $\frac{y}{x} = 5$, $x \neq 0$, then $y \propto$ .... *(New Valley 23 – El-Beheira 24)*

(a) $\frac{1}{x}$ (b) x (c) $x + 5$ (d) $x - 5$

6 If $\frac{x}{3} = \frac{5}{y}$, then $x \propto$ .... *(Ismailia 24)*

(a) y (b) 5 y (c) $\frac{1}{y}$ (d) y^2

7 Which of the following relations represents an inverse variation between the two variables X and y ? (El-Beheira 15)

(a) $y = X + 5$ (b) $y = 4X$ (c) $\frac{X}{y} = \frac{5}{7}$ (d) $Xy = 11$

8 The relation which represents a direct variation between the two variables X and y is .... (Souhag 20)

(a) $Xy = 5$ (b) $y = X + 3$ (c) $\frac{X}{3} = \frac{4}{y}$ (d) $\frac{X}{5} = \frac{y}{2}$

9 If $y = mX$ where m is a constant $\neq 0$, which of the following is wrong ?

(a) $y \propto X$ (b) $X \propto y$ (c) $X = \frac{1}{m} y$ (d) $X \propto \frac{1}{y}$

10 If the area of the rectangle equals 30 cm^2 and one of the both dimensions is X and the other dimension is y , then $y \propto$ .... (New Valley 22)

(a) X (b) $\frac{1}{X}$ (c) $30 + X$ (d) $30 - X$

11 If y varies inversely as X^2 , k is a constant , then ....

(a) $y = kX^2$ (b) $y = k - X^2$ (c) $y = \frac{k}{X^2}$ (d) $y = \frac{k}{X}$

12 If $y \propto X$, $y = 2$ when $X = 8$, then what is the value of y when $X = 12$?

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) 3 (d) 48

13 If $y \propto \frac{1}{X}$, $y = 3$ when $X = 20$, then what is the value of y when $X = 12$?

(a) $\frac{5}{9}$ (b) 1.8 (c) 5 (d) 8

14 If $y \propto \frac{1}{X}$, $X = 1$ when $y = 4$, then the relation between X and y is .... (Port Said 24)

(a) $Xy = 1$ (b) $\frac{X}{y} = 4$ (c) $\frac{y}{X} = 4$ (d) $Xy = 4$

15 If $y \propto X$ and $y = 5$ when $X = 3$, then the constant proportional equals ....

(a) 15 (b) 5 (c) 3 (d) $\frac{5}{3}$

16 If y varies inversely with X and $X = \sqrt{3}$ when $y = \frac{2}{\sqrt{3}}$, then the constant proportional equals .... (Beni Suef 15 – El-Beheira 16 – New Valley 20)

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 2 (d) 6

17 If Xy^5 = constant , then X varies inversely as .... (Ismailia 08)

(a) $\frac{1}{5}$ (b) y^5 (c) y (d) y^2

18 If $y \propto \frac{1}{\sqrt{X}}$, then X varies .... (Matrouh 09)

(a) directly as y^2 (b) inversely as y^2 (c) inversely as y (d) inversely as $\sqrt{y}$

19 If $y^2 + 4X^2 = 4Xy$, then .... (Alexandria 15 – South Sinai 19)

(a) $y \propto X$ (b) $y \propto X^2$ (c) $y \propto \frac{1}{X}$ (d) $y \propto \frac{1}{X^2}$

(El-Monofia 16)

20 If $x^2 y^2 + \frac{1}{4} = x y$, then ....

(a) $x \propto y$ (b) $y \propto x$ (c) $2x \propto 5y$ (d) $y \propto \frac{1}{x}$

(Ismailia 14)

21 If $\frac{y+3}{y} = \frac{x+2}{x}$ where $x \neq y \neq$ zero , then $y \propto$ ....

(a) x (b) $\frac{1}{x}$ (c) $x + 2$ (d) $x + 5$

22 If the total cost of a trip is (y) , some of it is constant (a) and the other is directly proportional with the number of participants (x) , then .... (Ismailia 11 – El-Menia 24)

(a) $y = a x$ (b) $y = \frac{a}{x}$

(c) $y = a + \frac{m}{x}$ (m is a constant $\neq 0$) (d) $y = a + m x$ (m is a constant $\neq 0$)

---

2 If y varies directly as x and $y = 20$ as $x = 7$

Find : x when $y = 40$ « 14 »

---

3 If a varies inversely as b and $a = 12$ as $b = 8$, **find :**

1 The value of a as $b = 1.5$

2 The value of b as $a = 2$ « 64 , 48 »

---

4 If $y \propto x$ and $y = 14$ when $x = 42$, **find :**

(Port Said 18 – South Sinai 19 – Port Said 20 – Ismailia 22 – Giza 23 – El-Menia 24)

1 The relation between x and y

2 The value of y when $x = 60$ « $y = \frac{1}{3} x$, 20 »

---

5 If $y \propto \frac{1}{x}$ and $y = 3$ when $x = 2$, **find :**

(North Sinai 19 – Cairo 20 – El-Kalyoubia 22 – Alex 23 – Alex. 24)

1 The relation between x and y

2 The value of y when $x = 1.5$ « $x y = 6$, 4 »

---

6 If $y \propto \frac{1}{x}$ and $x = 3$ as $y = 10$, find y when :

$x \in \{1 , 2 , 3 , 4 , 5\}$

« 30 , 15 , 10 , 7.5 , 6 »

---

7 If $y \propto$ the multiplicative inverse of the expression $\frac{1}{x^2}$, then find the relation between x and y , if $y = 4$ as $x = 3$, then find the value of y as $x = 9$

(El-Sharkia 08) « $y = \frac{4}{9} x^2$, 36 »

---

8 If $y \propto x^3$ and $y = 64$ as $x = 2$, find the relation between x and y and find the value of y as $x = \frac{1}{2}$

(Luxor 20) « $y = 8 x^3$, 1 »

9 If y varies inversely as $\sqrt{X}$ and $y = 2$ as $X = 16$, find the value of y as $X = 32$ «$\sqrt{2}$»

10 If $y^2 \propto X^3$, find the relation between X and y where $y = 3$ as $X = 2$ *(Qena 09)* «$y^2 = \frac{9}{8}X^3$»

11 If $y^2 \propto \frac{1}{\sqrt[3]{X}}$ and $X = 8$ as $y = 3$, find X as $y = 1.5$ «512»

12 If $y \propto (X + 1)$ and $X = 3$ when $y = 2$, then find the relation between X and y *(Matrouh 09)* «$y = \frac{1}{2}(X + 1)$»

13 If $\frac{5X - 3y}{3X + 5y} = 1$ for all the values of $X \in \mathbb{R}_+$, $y \in \mathbb{R}_+$, **prove that :** $y \propto X$

14 If $\frac{a + 2b}{6} = \frac{b + 3c}{3}$, **then prove that :** $a \propto c$ *(El-Fayoum 17 – South Sinai 23)*

15 If $\frac{21X - y}{7X - z} = \frac{y}{z}$, **prove that :** $y \propto z$ *(El-Kalyoubia 18 – Damietta 23 – Assiut 24)*

16 If $X^2 y^2 - 6Xy + 9 = 0$, **then prove that :** y varies inversely as X *(Damietta 13 – South Sinai 14)*

17 If $4a^2 + 9b^2 = 12ab$, **prove that :** a varies directly as b *(Matrouh 17)*

18 If $X^4 y^2 - 14X^2 y + 49 = 0$, **prove that :** $y \propto \frac{1}{X^2}$ *(El-Dakahlia 24)*

19 If $(4X + 7y) \propto (X + 2y)$ where $X \in \mathbb{R}$ and $y \in \mathbb{R}$, **then prove that :** $y \propto X$

20 If $\left(\frac{a}{y} - \frac{a}{X}\right) \propto (X - y)$ where a is a constant, $X \neq y \neq 0$,
then prove that : X varies inversely as y

21 Which of the following tables represents the direct variation and which of them represents the inverse variation and which does not represent the direct variation nor the inverse variation with mentioning the reason in each case :

X	y
3	20
5	12
4	15
6	10

X	y
2	9
4	18
12	54
16	72

X	y
5	9
10	18
15	27
25	45

X	y
3	6
–2	–9
–18	1
9	–2

22 From the data in the following table , answer the following questions :

X	2	4	6
y	6	3	2

1. Show the type of variation between X and y
2. Find the constant of variation.
3. Find the value of y at $X = 3$
4. Find the value of X at $y = 2\frac{2}{5}$

(Ismailia 18 – Luxor 22) « 12 , 4 , 5 »

---

23 From the opposite table :

X	1	2	b	4	6
y	12	a	36	48	72

1. Show the type of variation between X and y
2. Find the value of each of a and b

« a = 24 , b = 3 »

---

24 If $y = z + 5$, z changes inversely with X and $y = 6$ when $X = 2$, then find the relation between y and X , then find the value of y when $X = 1$ *(El-Monofia 17)* « $y = \frac{2}{X} + 5$, 7 »

---

25 If $y = a + b$ where a is a constant , b varies directly with X , $y = 3$ when $X = 0$ and $y = 5$ when $X = 3$, find the relation between X and y then find the value of y when $X = 7$

« $y = 3 + \frac{2}{3} X$, $7\frac{2}{3}$ »

---

26 If $y = a - 9$ and $y \propto \frac{1}{X^2}$ and $a = 18$ when $X = \frac{2}{3}$, find the relation between y and X , then deduce the value of y when $X = 1$

(Suez 18 – Luxor 19 – El-Gharbia 22 – Luxor 23) « $y = \frac{4}{X^2}$, 4 »

---

27 If $y = 2 + a$, a varies inversely as X and $a = 5$ when $X = 2$, **find :**

1. The relation between y and X
2. The value of y when $X = 5$

(El-Sharkia 17) « $y = 2 + \frac{10}{X}$, 4 »

---

28 If $X = \ell + 9$ and $\ell \propto y$, then find the relation between ℓ and y known that : $X = 24$ when $y = 5$, then find the value of y when $\ell = 12$ « $\ell = 3y$, 4 »

Geometric Application

29 If (h) the height of a right circular cylinder (its volume is constant) varies inversely as the square of radius length (r) and h = 27 cm. when r = 10.5 cm. , find h when r = 15.75 cm.

(Port Said 20) « 12 cm. »

Life Applications

30 A car moves with a uniform velocity where the distance varies directly with the time (t). If the car covered a distance of 150 km. in 6 hours , find the distance covered by that car in 10 hours.

(El-Kalyoubia 13 – El-Dakahlia 24) « 250 km. »

31 If the weight of a body on the moon (W) is directly proportional with its weight on the ground (R) If the body weighs 84 kg. on the ground and its weight on the moon is 14 kg. What will its weight be on the moon if its weight on the ground is 144 kg.?

« 24 kg. »

32 If the number of hours (n) needed for carrying out a work varies inversely as the number of workers (X) who carry out this work.
If the work is carried out by 6 workers within 4 hours , what is the needed time for carrying out the work by 8 workers ?

(El-Sharkia 11) « 3 hours »

33 If the distance covered by a bicycle (d) varies directly with the square of the time (t) , $d = \frac{81}{16}$ km. when $t = \frac{1}{4}$ hour , find the value of t when d = 144 km.

(Assiut 12) « $1\frac{1}{3}$ hour »

● Remember ● Understand ● Apply ● Problem Solving

34 If the value of speed v that water passes through a hose nuzzle inversely changes with the square of the hose nuzzle radius length r and v = 5 cm./s. when r = 3 cm., find v when r = 2.5 cm.

« 7.2 cm./s. »

35 If the weight of a body varies inversely as the square of its distance from the centre of the earth. If a satellite of weight 500 w. kg. is projected up to the space , what will its weight be when it becomes at a distance of 640 km. far from the surface of the earth to the nearest one (kg.) (Consider the radius length of the earth 6390 km.)

« 413 w.kg. »

For excellent pupils

36 If $x \propto y$ and $z \propto l$, **then prove that :** $(x + y)(z + l) \propto (x - y)(z - l)$

37 If $(a + b) \propto \frac{a}{b}$, $(a^2 - a b + b^2) \propto \frac{b}{a}$, **then prove that :** $a^3 + b^3 = \text{constant}$

UNIT THREE

Statistics

Exercises of the unit :

9. Collecting data.
10. Dispersion.

Scan the QR code to solve an interactive test on each lesson

Exercise 9 Collecting data

Remember Understand Apply Problem Solving

1 Choose the correct answer from those given :

1 .... is a secondary resource of collecting data. (El-Fayoum 12)

(a) Personal interview (b) Questionnaires

(c) Data base of the employees (d) Observing and measuring

2 .... is a primary resource of collecting data.

(a) Central agency for statistics (b) Data of the school pupils in the previous year

(c) Questionnaires (d) Data of the employees in one of the companies

3 The method of mass population is suitable for ....

(a) searching the formation of the sand of the Western Desert.

(b) examining the sweetness of water for one of the wells.

(c) finding out the ratio of existing a metal in one of the mines.

(d) getting the number of the students who had the full mark in maths exam in a class.

4 The method of samples is suitable for all the following except ....

(a) examining a patient's blood. (b) knowing population.

(c) checking the production of a factory. (d) finding out the ratio of existing gas somewhere.

5 Selecting a sample of layers of a statistical society is called .... sample. (El-Beheira 17)

(a) random (b) class (layer) (c) deliberate (d) bunch

6 A factory has 125 workers , 75 of them are technicians and 50 are engineers , it is wanted to take a sample of layers of size 50 individuals such that it represents each layer according to its size , then the number of engineers of the sample equals ....

(a) 30 (b) 20 (c) 25 (d) 15 (El-Monofia 16)

2 **Which of the following statistical data is primary and which of them is secondary ?**

1. A survey for the pupils in your class about the place to which will be the next trip.
2. If you count the number of seats existing in each class of your school.
3. If you make an investigation about the number of the successful pupils in each school subject in your school in the first session last year from the registered notebooks in the school.
4. If you go to a government authority in your governorate to collect data about the babies registered in each health office through March last year.
5. Searching the internet sites for the results of one team of sports teams in the league in Egypt in the year 2022 – 2023

3 Compare between the methods of mass population and samples , showing the advantages and disadvantages of each of them.

4 **Mention the suitable method (mass population or samples) for collecting data in each of the following statistical societies :**

1. The educational level of a class formed from 25 students.
2. The range of validity of drinking water in a well for drinking.
3. The ratio of oil existing in an exploratory location.
4. The range of spread of a disease in one of the crops.
5. The counting of factories in one of the industrial cities.

5 200 employees were surveyed about their favourite food during break time. Every one was given a digit number from 1 to 200 , then a sample representing 10% was selected to be interviewed about their favourite food :

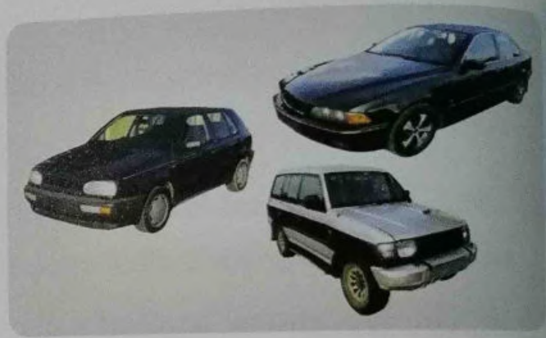
(a) Hot drinks. (b) Light meals. (c) Soft drinks.

Determine using your calculator the digits of target employees in this sample.

6 The administration of a hotel wanted to conduct a survey to 300 customers on the service level produced. Every customer got a digit from 201 to 500 , 10% of them were selected as a random sample to ask them about the service level.

Determine using the calculator the digits of the marked customers in this sample.

7 At a faculty , there are 4000 university students in the first grade , 3000 in the second grade , 2000 in the third grade and 1000 in the fourth grade. If we want to draw a layer sample of 500 students , where each layer is represented in this sample according to its size, calculate the number of students in each layer in the sample. « 200 , 150 , 100 , 50 »



8 One of the factories of cars produces 3 models of cars in the year , their numbers are :

300 cars from the first model.

500 cars from the second model.

200 cars from the third model.

The directorate of the factory wanted to select a sample of 5% of production to represent each model according to its size.

- Determine the number of the selected sample.
- Determine the number of each model in the sample. « 50 , 15 , 25 , 10 »

9 It is wanted to select a random layer sample to represent each layer due to its size from a society consisting of 5000 individuals and it is divided into two layers.
The number of the first layer is 1500 individuals.
If the number of the second layer in the sample is 140 individuals , calculate the number of individuals in the sample. « 200 »

10 **There is a need to draw a random layer sample to represent all the layers according to their sizes from a society of a total 40000 values divided into three layers as follows :**

Number of the layer	1	2	3
Number of values in the layer	12000	20000	8000

If the number of values in the first layer is 240 , calculate the size of the whole sample. « 800 »

From the school book

Exercise 10

Dispersion

Remember Understand Apply Problem Solving

Interactive test

1 Choose the correct answer from those given :

1 .... is one of the measures of the dispersions.

(New Vally 20 – El-Kalyoubia 22 – Cairo 23 – El-Menia 24)

(a) The median (b) The arithmetic mean
(c) The standard deviation (d) The mode

2 The simplest and easiest method of measuring dispersion is ....

(Ismailia 20 – Damietta 22 – Suez 24)

(a) the range. (b) the standard deviation.
(c) the arithmetic mean. (d) the mode.

3 The difference between the greatest value and the smallest value in a set of individuals is called .... *(El-Sharkia 18 – Souhag 18 – Port Said 19 – Cairo 24)*

(a) the range. (b) the arithmetic mean.
(c) the median. (d) the standard deviation.

4 The positive square root of the average of squares of deviations of the values from their mean is called ....

(Port Said 18 – Kafr El-Sheikh 18 – El-Fayoum 19 – El-Kalyoubia 20 – El-Kalyoubia 24)

(a) the range. (b) the arithmetic mean.
(c) the standard deviation. (d) the mode.

5 The mean of the values : 7 , 3 , 6 , 9 and 5 equals ....

(Alex. 17 – North Sinai 17 – El-Fayoum 18)

(a) 3 (b) 6 (c) 4 (d) 12

6 The range of the set of values : 23 , 22 , 15 , 18 and 17 is .... (Qena 23 – El-Dakahlia)

(a) 8 (b) 18 (c) 19 (d) 23

7 If 67 is the greatest value of a set and if the range equals 27 , then the smallest value of this set equals .... (El-Menia)

(a) 67 (b) 40 (c) 27 (d) 94

8 The most repeated value in a set of values represents .... (Damietta 13 – Luxor)

(a) the median. (b) the range.

(c) the mode. (d) the mean.

9 If the mean of the numbers : $3k-3$, $3k-1$, $2k+1$, $2k+3$ and $2k+5$ is 13 , then $k =$ .... (Alexandria)

(a) -5 (b) 10 (c) 5 (d) $\frac{1}{5}$

10 If the range of the values : $6+k$, $6-k$, $6+5k$ and $6-2k$ is 14 where $k \in \mathbb{N}$, then $k =$ .... (El-Sharkia 20)

(a) 1 (b) 2 (c) 3 (d) 4

11 If the range of the values 2 , 7 , a , 6 is 8 where $a > 0$, then $a =$ .... (El-Sharkia 14)

(a) 4 (b) 9 (c) -1 (d) 10

12 Which of the following values of a makes the range of the numbers : 53 , a , 58 , 57 , 60 and 55 equal to 9 ? (El-Dakahlia 16)

(a) 63 (b) 61 (c) 51 (d) 50

13 $\frac{\text{Sum of values}}{\text{Number of these values}} =$ .... (Suez 19 – Alex. 20)

(a) range. (b) standard deviation. (c) mean. (d) mode.

14 If $2x + 2y = 10$, $x \in \mathbb{R}^+$, $y \in \mathbb{R}^+$, then the arithmetic mean of the values x and y is .... (Suez 16)

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) 5 (d) 2

15 The set which has more dispersion of the following sets is .... (El-Kalyoubia 15)

(a) 28 , 17 , 30 , 36 , 20 (b) 20 , 19 , 29 , 37 , 43

(c) 31 , 35 , 26 , 37 , 41 (d) 25 , 39 , 19 , 5 , 27

16 The commonest measure of dispersion and the most accurate is the .... (Damietta 14 – El-Menia 18)

(a) range. (b) mean. (c) standard deviation. (d) median.

17 If all individuals are equal in values , then .... (Luxor 20 – Souhag 24)

(a) $x - \overline{x} > 0$ (b) $x - \overline{x} < 0$ (c) $\sigma = 0$ (d) $\overline{x} = 0$

18 The standard deviation of the values 5 , 5 , 5 , 5 equals .... (El-Dakahlia 22)

(a) 0 (b) 5 (c) 6 (d) 2

19 If the range of seven values is zero , then the standard deviation of these values equals .... (Souhag 23)

(a) 7 (b) $\sqrt{7}$ (c) zero (d) 1

20 If the standard deviation of a set of data : $X + 2$, 5 , $y - 2$ equal zero , then $X + y =$ .... (New Valley 22)

(a) 4 (b) 5 (c) 9 (d) 10

21 If the standard deviation of the values : $X + 6$, $y + 7$, $X + y$ is zero , then $X - y =$ .... (Luxor 24)

(a) 1 (b) – 1 (c) 0 (d) 13

22 If $\Sigma (X - \overline{X})^2 = 48$ of a set of values and the number of these values is 12 , then $\sigma =$ .... (Cairo 17 – El-Monofia 19)

(a) – 4 (b) – 2 (c) 2 (d) 4

2 **Calculate the standard deviation for the next data :**

1. 16 , 32 , 5 , 20 , 27 (El-Gharbia 18 – El-Monofia 19 – Port Said 20 – El-Kalyobia 23 – El-Beheira 24) « 9.3 »
2. 72 , 53 , 61 , 70 , 59 (Luxor 19 – Damietta 20) « 7.1 »
3. 15 , – 12 , – 9 , 27 , – 6 « 15.3 »
4. 22 , 20 , 20 , 20 , 18 (Luxor 22 – Ismailia 24) « 1.3 »

3 **Which of the following sets has more dispersion , using the standard deviation ?**

Set (A) : 7 , 8 , 9 , 10 , 11 *Set (B) :* 21 , 20 , 11 , 19 *Set (C) :* 29 , 30 , 30 , 35

4 **Calculate the mean and standard deviation of each of the following data :**

1. 73 , 54 , 62 , 71 , 60 (Assiut 17 – Qena 20) « 64 , 7.07 »
2. 13 , 14 , 17 , 19 , 22 (to the nearest 3 decimals digits) (El-Sharkia 17) « 17 , 3.286 »
3. 65 , 61 , 70 , 64 , 70 , 76 , 70 « 68 , 4.6 »
4. 23 , 12 , 17 , 13 , 15 , 16 , 8 , 9 , 37 , 10 « 16 , 8.2 »

5 The following values represent marks of five pupils in a test : 8 , 9 , 6 , 12 , 10 **Calculate :**

1. The mean of the marks. (El-Dakahlia 17) « 9 »
2. The standard deviation of the marks. « 2 »

6 **The opposite table shows the temperature in some cities :**

1. Calculate the mean and standard deviation of the maximum temperature.
2. Calculate the mean and standard deviation of the minimum temperature.

City	Max.	Min.
Ismailia	25	11
Suez	26	12
El-Arish	24	10
Nakhl	24	6
Taba	22	7
El-Tur	26	16
Hurghada	27	15
Rafah	26	11

« 25 , 1.5 , 11 , 3.2 »

7 The following frequency distribution shows the number of children of some families in a new city : (El-Beheira 16 – Alex. 19 – El-Monofia 20)

Number of children	zero	1	2	3	4
Number of families	8	16	50	20	6

Calculate the mean and the standard deviation of the number of children. « 2 , 1 »

8 The following are the frequency distribution for a number of defective units found in 100 boxes of manufactured units : (El-Beheira 14 – El-Beheira 17 – Souhag 18)

Number of defective units	zero	1	2	3	4	5
Number of boxes	3	16	17	25	20	19

Find the standard deviation of the defective units. « 1.4 »

9 The following frequency distribution shows the number of goals which have been scored by 30 players from 5 penalty kicks for each player during a training :

Number of scored goals	0	1	2	3	4	5
Number of players	2	4	5	8	7	4

Find the mean and standard deviation of the number of scored goals. « 2.9 , 1.4 »

10 The following frequency distribution shows the ages of 10 children :

Age in years	5	8	9	10	12	Total
Number of children	1	2	3	3	1	10

Calculate the standard deviation of the ages in years.

(Qena 19 – Cairo 20 – Alex. 22 – Souhag 24) « 1.7 years »

11 The following table shows the frequency distribution of the number of students who won in an art competition from a school having 20 classes :

Number of students	0	1	2	3	4	5	Total
Number of classes	1	3	5	6	3	2	20

Find the mean and the standard deviation of the number of students. « 2.65 , 1.3 »

12 **Calculate the mean and the standard deviation for the following frequency distribution :**

(Qena 16 – El-Gharbia 17)

Sets	0 –	4 –	8 –	12 –	16 – 20	Total
Frequency	3	4	7	2	9	25

« 11.6 , 5.7 »

13 **The following table represents the daily wages of a set of workers in a factory :**

(Kafr El-Sheikh 20)

Sets of wages	20 –	30 –	40 –	50 –	60 –	70 –
Number of workers	10	12	8	6	3	1

Find the mean and standard deviation of the wages.

« 40.75 , 13.4 »

14 **The following distribution table shows the amount of gasoline that a set of cars consumes :**

Number of kilometres per litre	5 –	7 –	9 –	11 –	13 –	15 – 17	Total
Number of cars	3	6	10	12	5	4	40

Find the standard deviation of the number of kilometres per litre.

« 2.7 »

For excellent pupils

15 **The two frequency tables represent the marks of the students of two classes A and B of third preparatory in an exam :**

Class A

Sets of marks	0 –	10 –	20 –	30 –	40 – 50	Total
Number of students	2	5	11	15	7	40

Class B

Sets of marks	0 –	10 –	20 –	30 –	40 – 50	Total
Number of students	2	3	18	7	10	40

1. Represent both of distributions using the frequency polygon in one figure.
2. Find the mean and standard deviation for both frequency distributions.
3. Which class is more homogeneous in getting marks ?

« 30 , 10.7 , 30 , 11 »

TIMSS Problems

Accumulative basic skills

Choose the correct answer from the given ones :

1 $\{3\} \subset$ .... *(Alex. 16)*

(a) $(3, 7)$ (b) $]3, 7]$ (c) $]3, 7[$ (d) $\{3, 7\}$

2 $[2, 7] - \{2, 7\} =$ .... *(Matrouh 17)*

(a) $[1, 6]$ (b) $\varnothing$ (c) $]2, 7[$ (d) $\{0\}$

3 The next number in the pattern : $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}$ is .... *(New Valley 20)*

(a) $\sqrt{50}$ (b) $\sqrt{75}$ (c) $\sqrt{60}$ (d) $\sqrt{90}$

4 $2^{2017} = 2^{2016} +$ .... *(Luxor 17)*

(a) 1 (b) 2 (c) 2016 (d) 2^{2016}

5 If $[-1, x] \cap [y, 5] = [2, 3]$, then $x^y =$ .... *(Damietta 24)*

(a) 8 (b) $\frac{1}{5}$ (c) 9 (d) -1

6 When the side length of a square increases by the ratio 10% , then its area increases by the ratio .... %

(a) 10 (b) 15 (c) 20 (d) 21

7 The ratio between the area of a square shaped region of side length x cm. to the area of another square shaped region of side length $2x$ cm. is .... *(Beni Suef 17)*

(a) 1 : 2 (b) x : 4 (c) 1 : 4 (d) 4 : 1

8 If F is an odd number , then the next odd number directly is .... *(South Sinai 19 – Qena 22)*

(a) F^2 (b) $F^2 + F$ (c) $F + 1$ (d) $F + 2$

9 If M represents a negative number , which of the following represents a positive number ? (Kafr El-Sheikh 17 – El-Menia 24)

(a) M^3 (b) M^2 (c) $2M$ (d) $\frac{M}{2}$

10 Half of the number 2^{20} is .... (Damietta 17)

(a) 2^{10} (b) 1^{20} (c) 2^{19} (d) 1^{10}

11 If $(x-3)^{zero} = 1$, then $x \in$ .... (El-Monofia 18)

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\mathbb{R} - \{4\}$ (d) $\mathbb{R} - \{1\}$

12 $\left(\frac{\sqrt{5}+1}{2}\right)^{1000} \left(\frac{\sqrt{5}-1}{2}\right)^{1000} =$ .... (El-Monofia 18)

(a) zero (b) 1 (c) $\frac{5^{1000}-1}{4}$ (d) 4^{1000}

13 $3^x + 3^x + 3^x =$ .... (Suez 16)

(a) 9^x (b) 3^{3x} (c) 3^{x+1} (d) 3^{x+3}

14 $2^5 + 2^5 + 2^5 + 2^5 =$ .... (Luxor 16)

(a) 2^7 (b) 2^6 (c) 2^4 (d) 2^{20}

15 If $x - y = 5$, $x + y = \frac{1}{5}$, then $x^2 - y^2 =$ .... (Kafr El-Sheikh 17 – Aswan 20 – El-Menia 24)

(a) $\frac{1}{25}$ (b) 1 (c) 25 (d) 5

16 If $x + y = y\,x = 5$, then $x^2 y + y^2 x =$ .... (Aswan 16 – Ismailia 20)

(a) 10 (b) 15 (c) 20 (d) 25

17 If $(x - y)^2 = 20$, $x^2 + y^2 = 10$, then $xy =$ .... (Alex. 16)

(a) 10 (b) 5 (c) -5 (d) 20

18 If $1 < x < 3$, $x \in \mathbb{R}$, then $(3x - 1) \in$ .... (Suez 16 – Giza 20)

(a) $[2, 8[$ (b) $[2, 8]$ (c) $]2, 8[$ (d) $\{2, 8\}$

19 The S.S. of the inequality : $5 - 3x > 11$ in $\mathbb{R}$ is .... (Kafr El-Sheikh 17)

(a) $]-\infty, -2[$ (b) $]-2, \infty[$ (c) $]-\infty, -2]$ (d) $[-2, 2]$

20 The sum of the two square roots of $2\frac{1}{4}$ is .... (El-Monofia 17 – North Sinai 19)

(a) zero (b) $\frac{3}{2}$ (c) 3 (d) $\frac{9}{4}$

21 Four times the number $2^8 =$ .... (Alex. 17 – Souhag 19)

(a) 2^{32} (b) 8^8 (c) 2^{10} (d) 4^8

22 If $x = \sqrt{3} + \sqrt{2}$, $y = \frac{1}{\sqrt{3}+\sqrt{2}}$, then $(x + y)^2 =$ .... (El-Gharbia 17)

(a) 8 (b) zero (c) 9 (d) 12

23 If $2^{x} = \frac{1}{8}$, then $x =$ .... (El-Monofia 17 – Red Sea 23 – El-Menia 24)

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 3 (d) – 3

24 A book consists of 56 pages. How many pages the number 5 appears in the pages serial of this book ?

(a) 6 (b) 7 (c) 12 (d) 13

25 If we put on one side of a road of length 12 km. some light poles from the begining to the end of the road, where the distance between each two consecutive poles is $\frac{1}{2}$ km. , then the number of poles is ....

(a) 12 (b) 24 (c) 25 (d) 23

26 The decimal that lies between 0.07 and 0.08 is .... (Alex. 17)

(a) 0.00075 (b) 0.0075 (c) 0.075 (d) – 0.75

27 The square of double the number $\frac{1}{2}$ is ....

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) 1 (d) 2

28 If three times a number = 45 , then $\frac{1}{5}$ of this number = ... (El-Menia 19)

(a) 15 (b) 5 (c) 3 (d) 9

29 If $\frac{5}{4} + \frac{5}{x} = \frac{5}{2}$, then $x =$ .... (El-Monofia 20)

(a) 2 (b) 4 (c) 5 (d) $\frac{5}{2}$

30 $]-1 , 3] \cap \{-3 , -1\} =$ .... (Assiut 18 – El-Gharbia 24)

(a) $\varnothing$ (b) $\{-3\}$ (c) $\{-1\}$ (d) $\{3\}$

31 $[2 , 7] -]2 , 7[=$ .... (Beni Suef 1

(a) $\varnothing$ (b) $\{2\}$ (c) $\{7\}$ (d) $\{2 , 7\}$

32 $\mathbb{Z}^{-} \cup \mathbb{N} =$ .... (Luxor 17 – Alex. 1

(a) $\varnothing$ (b) $\mathbb{N}$ (c) $\mathbb{Z}$ (d) $\mathbb{R}$

33 The expression : $(x-2)^2 - x^2$ is of the .... degree. (Kafr El-Sheikh 2

(a) first (b) second (c) third (d) fourth

34 The solution set of the equation : $x - 1 = |-1|$ in $\mathbb{N}$ is .... (Suez 1

(a) (1 , 2) (b) 2 (c) $\{2\}$ (d) $\{-2\}$

35 If $17x + 8 = 11$, then $17x + 11 =$ .... (Ismailia 1

(a) 8 (b) 11 (c) 14 (d) 17

36 The sum of integers in this interval $[-5 , 5[=$ ....

(a) zero (b) 10

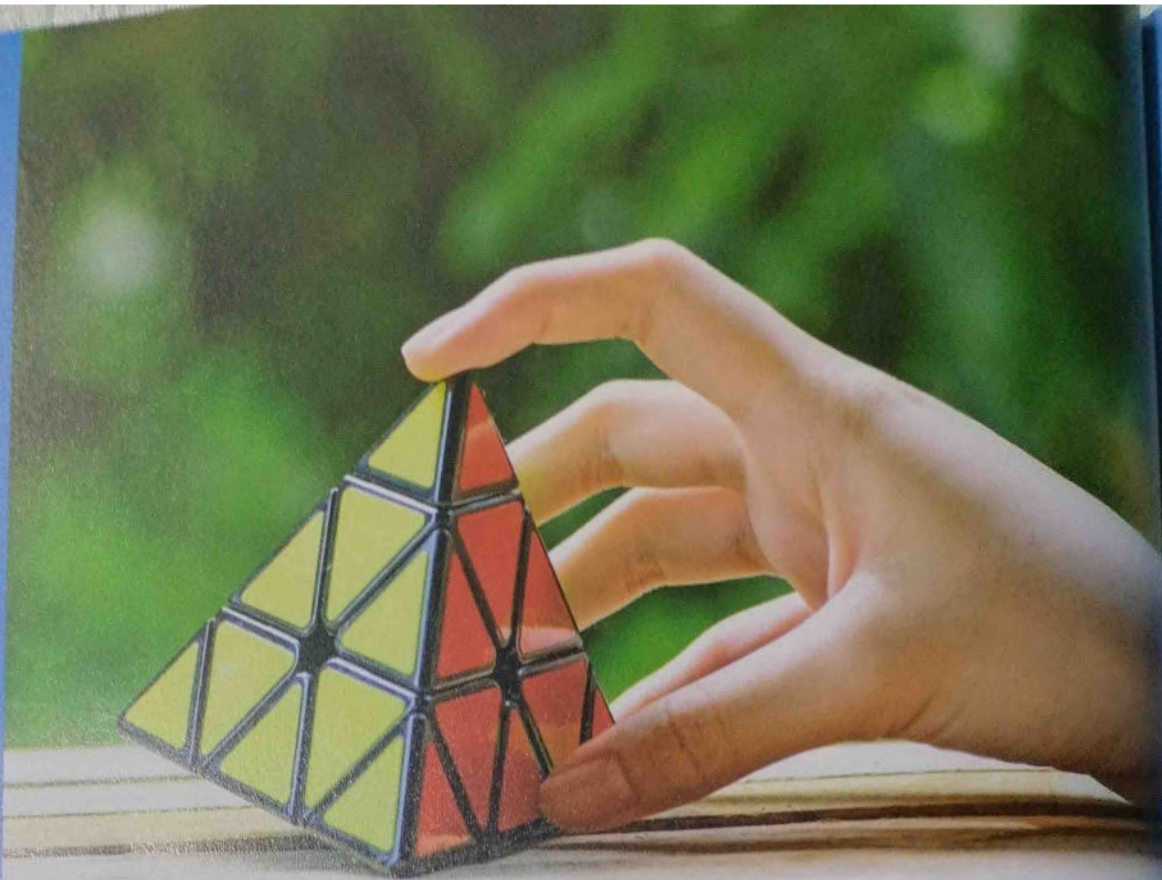
Second

Trigonometry and Geometry

Second

Trigonometry and Geometry

UNIT FOUR



Trigonometry

Exercises of the unit :

1. The main trigonometrical ratios of the acute angle.
2. The main trigonometrical ratios of some angles.

Scan
the QR code
to solve an interactive test on each lesson

From the school book

Exercise **1**

The main trigonometrical ratios of the acute angle

Remember Understand Apply Problem Solving

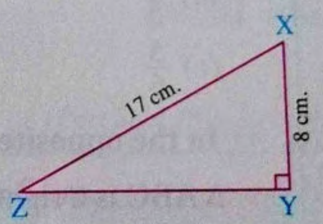
Interactive test

1 Complete the following :

In the opposite figure :

XYZ is a right-angled triangle at Y

in which XY = 8 cm. , XZ = 17 cm.



1 sin X = ... , sin Z = ...

2 cos X = ... , cos Z = ...

3 tan X = ... , tan Z = ...

2 Choose the correct answer from those given :

1 For any acute angle A : sin A – cos A tan A = ... *(New Valley 22 – Port Said 24)*

(a) zero (b) 1 (c) – 1 (d) 2

2 If X , y are the measures of two complementary angles and $\sin X = \frac{3}{5}$

, then cos y = ... *(Giza 17 – El-Beheira 18 – Giza 20 – Port Said 24)*

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{5}{3}$

3 For any two acute angles A and B , if sin A = cos B , then m (∠ A) + m (∠ B) = ...

(a) 30° (b) 60° (c) 90° (d) 180°

4 If sin 70° = cos X where X is the measure of an acute angle , then X = ...

(El-Kalyoubia 18 – El-Monofia 23)

(a) 60° (b) 45° (c) 10° (d) 20°

5 In Δ ABC , if m (∠ A) = 85° and sin B = cos B , then m (∠ C) = ...

(El-Beheira 17 – El-Dakahlia 19 – Matrouh 22 – El-Beheira 24)

(a) 30° (b) 45° (c) 50° (d) 60°

6 In Δ ABC , if m (∠ B) = 90° , then sin A + cos C = ... *(El-Monofia 17)*

(a) 2 sin A (b) 2 sin C (c) 2 sin B (d) 2 cos A

7 Δ ABC is a right-angled triangle at A , then cosine angle B : sine angle C equals .... *(El-Sharkia 18)*

(a) $\frac{3}{5}$ (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) 1

8 DEF is a right-angled triangle at E , which of the following relations is false ? *(El-Dakahlia 16)*

(a) tan D × tan F = 1 (b) sin D = cos F (c) cos D = sin F (d) cos D = sin E

9 ABC is a right-angled triangle at B , where 3 AC = 5 BC , then tan A = ... *(El-Sharkia 20)*

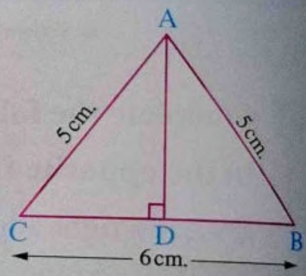
(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

10 **In the opposite figure :**

cos B = ... *(El-Gharbia 12)*

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$

(c) $\frac{5}{6}$ (d) $\frac{5}{4}$



11 **In the opposite figure :**

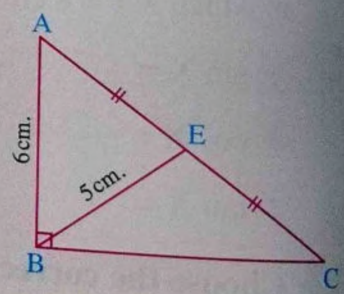
Δ ABC is a right-angled triangle at B

, $\overline{BE}$ is a median , BE = 5 cm.

, AB = 6 cm. , then sin C = ...

(a) $\frac{5}{6}$ (b) $\frac{3}{5}$

(c) $\frac{6}{5}$ (d) $\frac{5}{3}$



(Aswan 16)

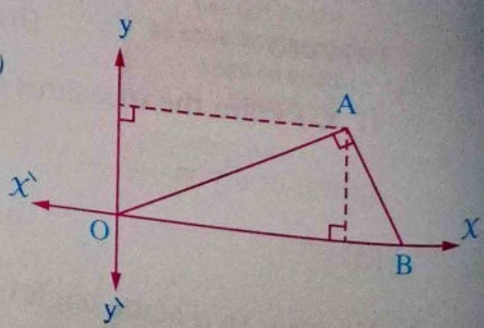
12 **In the opposite figure :**

ABO is a right-angled triangle

, A (6 , 3) , then tan (∠ ABO) = ... *(El-Gharbia 22)*

(a) $\frac{1}{2}$ (b) 2

(c) $\frac{\sqrt{5}}{5}$ (d) $\frac{2\sqrt{5}}{5}$

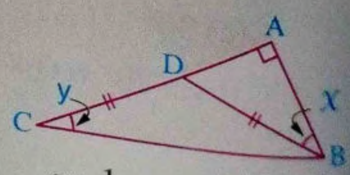


13 **In the opposite figure :**

If tan $X = \frac{3}{4}$

, then tan y = ...

(a) 3 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

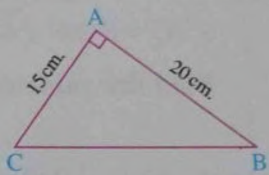


3 If the ratio between the measures of two supplementary angles is 3 : 5 , find the degree measure of each one. *(Aswan 15 – El-Gharbia 19 – Luxor 22)* « $67^\circ\ 30'$, $112^\circ\ 30'$ »

4 If the ratio between the measures of two complementary angles is 3 : 4 , find the degree measure of the greater angle in measure. « $51^\circ\ 25'\ 43''$ »

5 If the ratio between the measures of the interior angles of a triangle is 3 : 4 : 7 , find the degree measure of each angle. *(El-Beheira 13)* « $38^\circ\ 34'\ 17''$, $51^\circ\ 25'\ 43''$, 90° »

6 **In the opposite figure :**



ABC is a triangle in which : $m(\angle A) = 90^\circ$

, AC = 15 cm. and AB = 20 cm.

Prove that : $\cos C \cos B - \sin C \sin B = \text{zero}$

(El-Beheira 17 – El-Kalyoubia 18 – El-Menia 19 – Giza 20)

7 XYZ is a right-angled triangle at Z where : XZ = 7 cm. and XY = 25 cm.

Find the value of each of the following :

1. $\tan X \times \tan Y$
2. $\sin^2 X + \sin^2 Y$ *(Port Said 18)* « 1 , 1 »

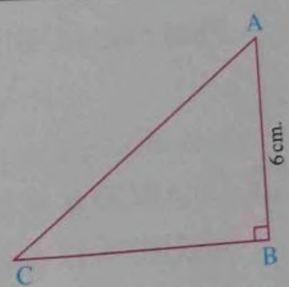
8 ABC is a right-angled triangle at B in which : BC = 4 cm. and AC = 5 cm.

Deduce that : $\sin^2 A - \cos^2 A = 2 \sin^2 A - 1$

9 ABC is a right-angled triangle at B , if AB : AC = 3 : 5 , find the main trigonometrical ratios of $\angle A$ *(Aswan 13)* « $\frac{4}{5}$, $\frac{3}{5}$, $\frac{4}{3}$ »

10 ABC is a right-angled triangle at B , if $2\,AB = \sqrt{3}\,AC$, find the main trigonometrical ratios of the angle C *(Alexandria 15 – El-Dakahlia 18 – Aswan 19)* « $\frac{\sqrt{3}}{2}$, $\frac{1}{2}$, $\sqrt{3}$ »

11 **In the opposite figure :**



ABC is a right-angled triangle at B

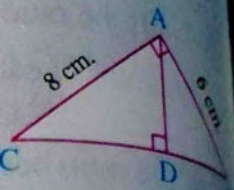
, AB = 6 cm. , $\tan C = \frac{3}{4}$, **find :**

1. The length of each of $\overline{BC}$ and $\overline{AC}$
2. $\sin A + \cos A$ *(Ismailia 12 – El-Monofia 16 – Matrouh 24)* « 8 cm. , 10 cm. , $\frac{7}{5}$ »

12 **In the opposite figure :**

$m(\angle BAC) = 90°$, $\overline{AD} \perp \overline{BC}$

, AB = 6 cm. , AC = 8 cm.



Find : 1 $\tan(\angle BAD)$

2 $\cos(\angle DAC) + \cos(\angle DAB)$

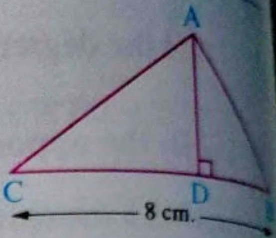
(El-Gharbia 16) « $\frac{3}{4}$, $\frac{7}{5}$ »

---

13 **In the opposite figure :**

ΔABC is an acute-angled triangle

, BC = 8 cm. , $\overline{AD} \perp \overline{BC}$



Find the value of : AB cos B + AC cos C

(El-Sharkia 17) « 8 cm. »

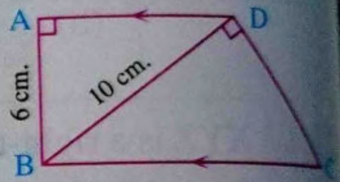
---

14 **In the opposite figure :**

ABCD is a trapezium in which $\angle A$ is right

, $\overline{AD} // \overline{BC}$, $m(\angle BDC) = 90°$

, AB = 6 cm. , BD = 10 cm.



Find : $\tan(\angle ADB)$ and the length of $\overline{DC}$

(El-Dakahlia 17 – El-Menia 24) « $\frac{3}{4}$, 7.5 cm. »

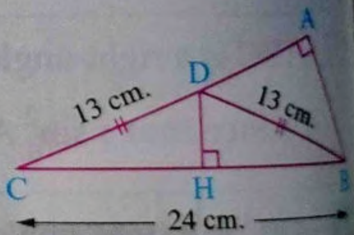
---

15 **In the opposite figure :**

ΔABC is right-angled at A , $D \in \overline{AC}$

, where BD = DC = 13 cm. , $\overline{DH} \perp \overline{BC}$

, BC = 24 cm.



Find the value of :

1 $\tan(\angle DCB)$ 2 $\cos(\angle ABC)$

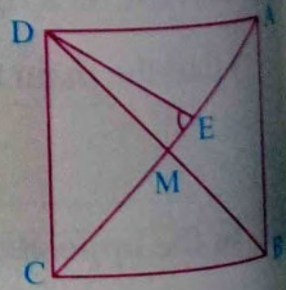
(El-Sharkia 23) « $\frac{5}{12}$, $\frac{5}{13}$ »

---

16 **In the opposite figure :**

ABCD is a square its diagonals intersect at M

, $E \in \overline{AC}$, CE = 5 cm. , AE = 3 cm.



Find : $\tan(\angle DEC)$

(El-Dakahlia 23) « 4 »

---

17 ABCD is an isosceles trapezoid in which : $\overline{AD} // \overline{BC}$, AD = 4 cm. , AB = 5 cm. and BC = 12 cm.

Prove that : $\dfrac{5 \tan B \cos C}{\sin^2 C + \cos^2 B} = 3$

18 ABCD is a trapezoid in which : $\overline{AD} \parallel \overline{BC}$, $m(\angle B) = 90°$, $AB = 3$ cm. , $AD = 6$ cm. and $BC = 10$ cm.

Prove that : $\cos(\angle DCB) - \tan(\angle ACB) = \frac{1}{2}$

(El-Monofia 17 – Matrouh 18 – Giza 20 – Kafr El-Sheikh 22)

---

19 ABC is an isosceles triangle in which : $AB = AC$ and $\sin \frac{A}{2} = \frac{4}{5}$

Find $\cos B$ without using the calculator. *(Red Sea 13)* « $\frac{4}{5}$ »

---

20 If ΔABC is a right-angled triangle at C , **prove that :** $\sin B + \cos B > 1$

---

21 ABC is a right-angled triangle at B and $\sin A = 0.6$

Find : The value of $\sin A \cos C + \cos A \sin C$ *(Kafr El-Sheikh 13)* « 1 »

---

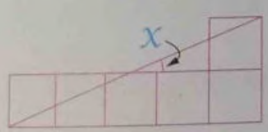
22 ABC is a right-angled triangle at B and $7 \tan A - 24 = 0$

Find : The value of $1 - \tan A \sin C$ « $\frac{1}{25}$ »

---

23 **If the following figures are formed from congruent squares , then find the required under each figure :**

1



Find : $\tan X$

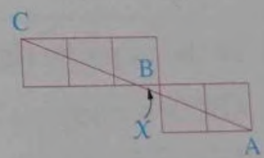
2

Find : $\tan X$

3

Find : $\cos X$

4



If A , B and C are collinear.

Find : $\tan X$

For excellent pupils

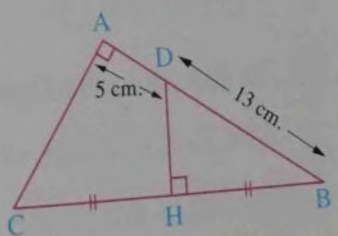
24 **In the opposite figure :**

$m(\angle A) = 90°$, $\overline{DH} \perp \overline{BC}$

where H is the midpoint of $\overline{BC}$

, $AD = 5$ cm. and $BD = 13$ cm.

Find with proof $\tan B$



(Damietta 17) « $\frac{2}{3}$ »

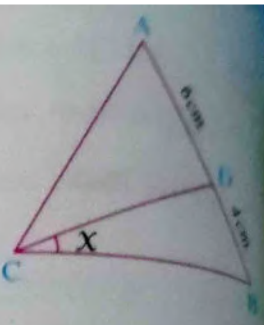
25 **In the opposite figure :**

ABC is an equilateral triangle , $D \in \overline{AB}$

where : AD = 6 cm. , DB = 4 cm. ,

if $k \tan X = \sqrt{3}$

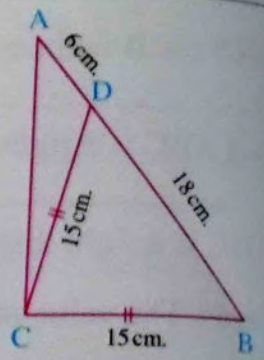
, find the value of : k



« 4 »

26 **From the opposite figure :**

Find : tan (∠ BAC)



« $\frac{4}{5}$ »

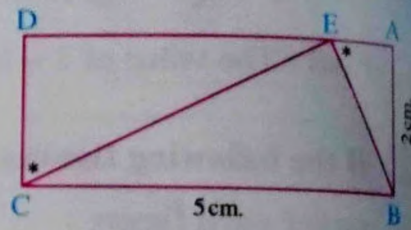
27 **In the opposite figure :**

ABCD is a rectangle in which :

AE < ED , AB = 2 cm. , BC = 5 cm.

, m (∠ AEB) = m (∠ ECD)

Find : tan (∠ CED)



« $\frac{1}{2}$ »

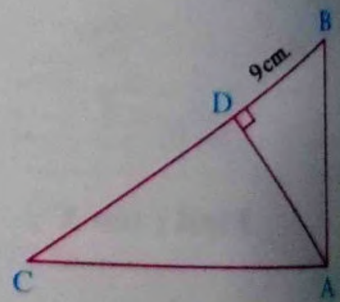
28 **In the opposite figure :**

ABC is a triangle , $D \in \overline{BC}$ where :

$\overline{AD} \perp \overline{BC}$, BD = 9 cm.

If $\sin (\angle BAD) = \cos (\angle CAD) = \frac{3}{5}$

, find the area of Δ ABC



« 150 cm^2 »

29 In any right-angled triangle ABC at B

, prove that : $\sin^2 A + \sin^2 C = 1$

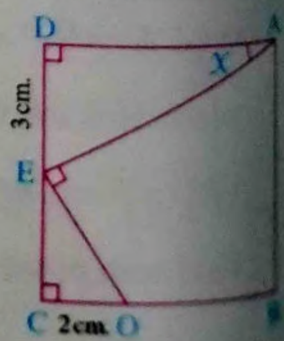
30 **In the opposite figure :**

ABCD is a square , $E \in \overline{DC}$,

$O \in \overline{BC}$, $\overline{AE} \perp \overline{EO}$

, DE = 3 cm. , CO = 2 cm.

Find : tan X



From the school book

Exercise

2 The main trigonometrical ratios of some angles

● Remember ● Understand ● Apply ● Problem Solving

Interactive test

1 Without using the calculator , find each of the following :

1. $\sin 45^\circ - \cos 45^\circ$
2. $\cos 60^\circ + \sin 30^\circ$
3. $\sin 30^\circ + \cos 60^\circ - \tan 45^\circ$
4. $\sin 60^\circ + \cos 30^\circ + \tan 60^\circ$
5. $\sin^2 45^\circ + \cos^2 45^\circ$
6. $4 \cos 30^\circ \tan 60^\circ$
7. $\tan^2 60^\circ - 2 \sin 45^\circ \cos 45^\circ$ *(North Sinai 17)*
8. $\sin^2 60^\circ - \tan 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$ *(Assiut 17)*
9. $2 \sin 30^\circ \cos 60^\circ + \sqrt{2} \sin 45^\circ$
10. $(\cos 30^\circ - \cos 60^\circ)(\sin 30^\circ + \sin 60^\circ)$ *(Ismailia 17)*
11. $\dfrac{\sin 30^\circ}{\cos 60^\circ} - \cos 30^\circ \sin 60^\circ$ *(El-Gharbia 17 – El-Gharbia 22)*
12. $\dfrac{\cos^2 60^\circ + \cos^2 30^\circ + \tan^2 45^\circ}{\sin 60^\circ \tan 60^\circ - \sin 30^\circ}$

2 Without using the calculator , prove each of the following :

1. 📖 $\sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ$ *(Giza 19 – North Sinai 20 – Alex. 22 – suez 23)*
2. 📖 $\cos 60^\circ = 2 \cos^2 30^\circ - 1$ *(South Sinai 20 – El-Kalyoubia 22 – Red sea 23 – South Sinai 24)*
3. $2 \cos^2 30^\circ - 1 = 1 - 2 \sin^2 30^\circ$ *(El-Sharkia 15)*

4 $\cos 60° = \cos^2 30° - \sin^2 30°$ (Alex. 17 – El-Fayoum 18 – South Sinai ...)

5 $\tan 60° = \frac{2 \tan 30°}{1 - \tan^2 30°}$ (Damietta 19 – Alex. 20 – New Valley 22 – Ismailia ...)

6 $\cos^2 60° = 5 \sin^2 30° - \tan^2 45°$ (El-Menia 14 – Suez 17 – Giza ...)

7 $\sin^3 30° = 9 \cos^3 60° - \tan^2 45°$ (Alex. ...)

8 $\frac{\sin 30° \cos 45° + \sin 45° \cos 30°}{\sin 45° \cos 60° + \cos 45° \sin 60°} = \tan^2 45°$

9 $\sin 30° = \sqrt{\frac{1 - \cos 60°}{2}}$

3 Choose the correct answer from those given :

1 If $\sin \theta = 0.6214$, then $\theta \simeq$ .... (Port Said ...)

(a) 55° 38̀ (b) 38° 25̀ (c) 83° 52̀ (d) 48° 52̀

2 If $\cos X = \frac{1}{2}$ where X is an acute angle , then m $(\angle X)$ = ... (Cairo ...)

(a) 90° (b) 60° (c) 45° (d) 30°

3 If $\sin X = \frac{1}{2}$ where X is an acute angle , then m $(\angle X)$ = ... (Damietta ...)

(a) 90° (b) 60° (c) 45° (d) 30°

4 If $\tan X = \frac{1}{\sqrt{3}}$ where X is the measure of an acute angle , then $\tan 2X$ = ...

(a) $\frac{2}{\sqrt{3}}$ (b) $2\sqrt{3}$ (c) $\sqrt{3}$ (d) 3

5 If $\cos X = \frac{\sqrt{3}}{2}$ where X is the measure of an acute angle , then $\sin 2X$ = ... (El-Gharbia 18 – Red Sea ...)

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

6 If $2 \sin X = \tan 60°$ where X is an acute angle , then X = ... (Souhag ...)

(a) 30° (b) 45° (c) 60° (d) 40°

7 If X is the measure of an acute angle , $2 \sin X - 1 = 0$, then X = ... (El-Dakahlia 18 – Damietta ...)

(a) 60° (b) 90° (c) 45° (d) 30°

8 If $\tan 3X = \sqrt{3}$ where $3X$ is the measure of an acute angle , then X = ... (Ismailia 15 – North Sinai 20 – El-Menia ...)

(a) 20° (b) 30° (c) 45° (d) 60°

9 If $\sin 2X = \frac{\sqrt{3}}{2}$, then X = ... (where $2X$ is the measure of an acute angle) (Giza ...)

(a) 20° (b) 30° (c) 45° (d) 60°

10 If $\cos \frac{X}{2} = \frac{1}{2}$ where $\frac{X}{2}$ is an acute angle , then $m(\angle X) =$ ....

(a) $30°$ (b) $45°$ (c) $60°$ (d) $120°$

11 If $\cos(X + 10°) = \frac{1}{2}$ where $(X + 10°)$ is the measure of an acute angle , then $X =$ ....

(El-Fayoum 11)

(a) $30°$ (b) $40°$ (c) $50°$ (d) $70°$

12 If $\tan(2X - 5°) = 1$ where X is the measure of an acute angle , then $X =$ ....

(El-Gharbia 16 – Luxor 20)

(a) $45°$ (b) $35°$ (c) $25°$ (d) $15°$

13 If $\sin(X + 5°) = \frac{1}{2}$ where $(X + 5°)$ is the measure of an acute angle , then $\tan(X + 20°) =$ ....

(El-Dakahlia 11)

(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

14 If C is an acute angle and $\sin C = \cos C$, then $\tan C =$ .... *(Luxor 24)*

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{3}$

15 If $2 \sin X = \tan X$ where X is an acute angle , then $m(\angle X) =$ ....

(El-Monofia 22 – El-Kalyobia 23 – El-Gharbia 24)

(a) $60°$ (b) $45°$ (c) $30°$ (d) $15°$

16 If X and y are complementary angles where $X : y = 1 : 2$, then $\sin X + \cos y =$ ....

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

(El-Beheira 15)

17 In ΔABC , if $m(\angle A) : m(\angle B) : m(\angle C) = 3 : 4 : 5$, then $\cos B =$ ....

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$

(El-Gharbia 16)

18 The tangent of an acute angle of the right isosceles triangle is equal to ....

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $\frac{\sqrt{2}}{2}$

(El-Dakahlia 16)

19 ΔABC is right-angled at A , if $\tan B = 1$, then $\tan C - \sin C \cos C =$ ....

(a) zero (b) 1 (c) 2 (d) $\frac{1}{2}$

(Red Sea 16)

20 If the straight line : $y = X \sin 30° + c$ passes through the point $(4 , 6)$, then $c =$ ....

(a) 4 (b) 6 (c) 8 (d) 2

(El-Monofia 16)

4 **Find the value of X in each of the following :** (Souhag 17) « 6 »

1. $X \sin^2 45° = \tan^2 60°$ (South Sinai 16 – Alex. 19 – Assiut 20 – Matrouh 23 – El-Kalyoubia 24) « 3 »
2. $X \sin 30° \cos^2 45° = \sin^2 60°$ « $\frac{\sqrt{3}}{2}$ »
3. $X \sin 45° \cos 45° \tan 60° = \tan^2 45° - \cos^2 60°$
4. $4\,X = \cos^2 30° \tan^2 30° \tan^2 45°$ (Alex. 17 – El-Fayoum 19 – Suez 20 – Luxor 23 – El-Menia 24) « $\frac{1}{16}$ »

5 **Find the value of X in each of the following :**

1. $\tan X = 4 \sin 30° \cos 60°$ where X is an acute angle. (El-Gharbia 19 – Giza 20 – Damietta 22 – Sohag 23) « 45° »
2. $\sin X = \sin 60° \cos 30° - \cos 60° \sin 30°$ where $0° < X < 90°$ (Cairo 17 – Luxor 24) « 30° »
3. $2 \sin X = \sin 30° \cos 60° + \cos 30° \sin 60°$ where X is an acute angle. (Giza 24) « 30° »
4. $6 \sin X \cos 45° \sin 45° = 1 - \cos^2 60°$ where $0° < X < 90°$ (Aswan 13) « 14° 28' 39" »
5. $\cos X = \dfrac{\sin 60° \sin 30°}{\tan 45° \sin^2 45°}$ where X is an acute angle. (El-Dakahlia 18) « 30° »
6. $\cos (3\,X + 6°) = \sin 30°$ where $(3\,X + 6°)$ is an acute angle. « 18° »
7. $\sqrt{3} \sin X \tan 30° = \tan 45° \cos 2\,X$ where X is an acute angle. (El-Monofia 20) « 30° »

6 **Find E in each of the following where E is the measure of an acute angle :**

1. $\sin^2 45° = \cos E \tan 30°$ (Damietta 16 – El-Monofia 17 – Beni Suef 19 – Souhag 23) « 30° »
2. $\sin E \sin^2 60° = 3 \sin^2 45° \cos^2 45° \cos 60°$ (Beni Suef 18) « 30° »
3. $3 \tan E - 4 \sin^2 30° = 8 \cos^2 60°$ « 45° »

7 If $\tan X = \dfrac{1}{\sqrt{3}}$, X is an acute angle , **find :** $\sin X \tan \left(\dfrac{3\,X}{2}\right) + \cos 2\,X$ (Damietta 13) « 1 »

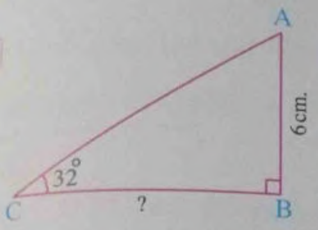
8 If $\sin X = \tan 30° \sin 60°$ where X is the measure of an acute angle , then find without using the calculator the value of : $4 \cos X \sin X$ (El-Kalyoubia 20) « $\sqrt{3}$ »

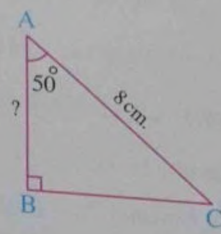
9 If $\dfrac{\cos 5\,X}{\sin X} = 1$ (where $5\,X$ is the measure of an acute angle)

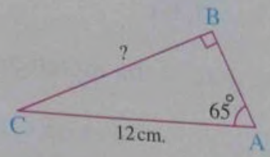
Find the value of : $\sin 2\,X$ (El-Gharbia 22) « $\frac{1}{2}$ »

10 **Find the value of X if :** $\cos X \tan X + \sin 30° = 1$, where $\angle X$ is acute (El-Sharkia 24) « 30° »

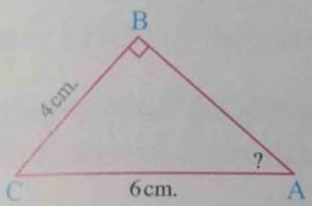
11 **Find the length of the side marked by the sign (?) in each of the following figures to the nearest two decimal digits :**

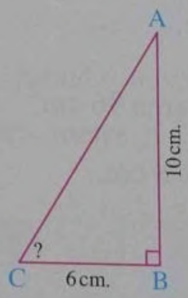
1


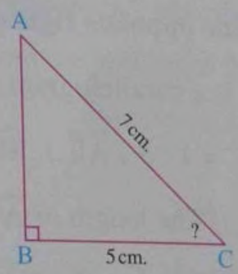
2


3


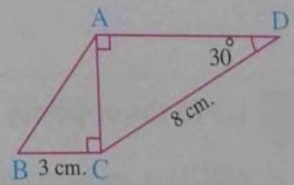
12 **Find in each of the following figures the measure of the angle marked by the sign (?) in degrees , minutes and seconds :**

1


2


3


13 **In the opposite figure :**



m (∠ D) = 30°

, m (∠ CAD) = m (∠ ACB) = 90°

, BC = 3 cm. , CD = 8 cm.

Find : 1 tan B 2 m (∠ BAD) *(El-Sharkia 18)* « $\frac{4}{3}$, 126° 52′ 12″ »

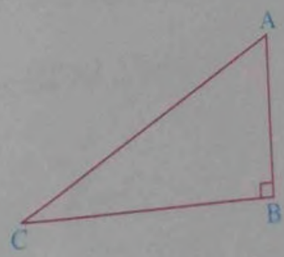
14 ABC is an isosceles triangle in which AB = AC = 7 cm. and BC = 10 cm.

Find : 1 m (∠ B) 2 The area of Δ ABC « 44° 24′ 55″ , $10\sqrt{6}$ cm.² »

15 ABC is an isosceles triangle in which AB = AC = 12.6 cm. and m (∠ C) = 84° 24′

Find the length of $\overline{BC}$ to the nearest one decimal number. « 2.5 cm. »

16 **In the opposite figure :**



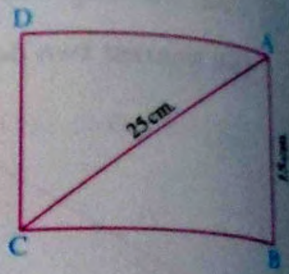
ABC is a right-angled triangle at B ,

m (∠ A) = 2 m (∠ C)

Find : The value of $\cos^2 A + \tan^2 C$

(El-Sharkia 13) « $\frac{7}{12}$ »

17 **In the opposite figure :**



ABCD is a rectangle in which : AB = 15 cm. and AC = 25 cm.

Find :

1 m (∠ ACB)

2 The area of the rectangle ABCD

(Alex. 16 – Qena 17 – El-Fayoum 20 – El-Behira 23 – Suez 24) « $36^\circ\ 52'\ 12''$, 300 cm² »

---

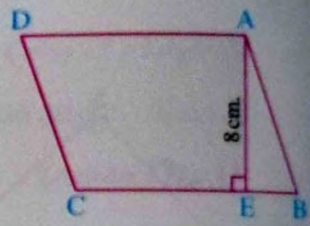
18 ABCD is a rectangle whose diagonal length AC = 24 cm. , m (∠ ACB) = 25°

Find : The length of $\overline{BC}$

« 21.8 cm. »

---

19 **In the opposite figure :**



ABCD is a parallelogram of surface area 96 cm².

BE : EC = 1 : 3 , $\overline{AE} \perp \overline{BC}$ and AE = 8 cm.

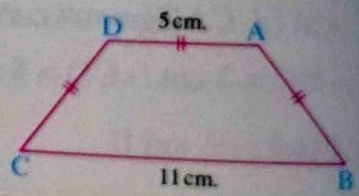
Find : 1 The length of $\overline{AD}$ 2 m (∠ B)

3 The length of $\overline{AB}$ to the nearest one decimal (Use more than one way)

« 12 cm. , $69^\circ\ 26'\ 38''$, 8.5 cm. »

---

20 **In the opposite figure :**



ABCD is an isosceles trapezium in which :

AB = AD = DC = 5 cm. , BC = 11 cm. **Find :**

1 m (∠ B) , m (∠ A)

2 The area of the trapezium ABCD

(Matrouh 13) « $53^\circ\ 7'\ 48''$, $126^\circ\ 52'\ 12''$, 32 cm² »

---

21 ABCD is a trapezium in which : $\overline{AD} \mathbin{//} \overline{BC}$ and m (∠ ABC) = 90°

If AB = 12 cm. , AD = 16 cm. and BC = 25 cm. , **find :**

1 The length of $\overline{DC}$ 2 m (∠ C)

3 sin (∠ DCB) – tan (∠ ACB)

« 15 cm. , $53^\circ\ 7'\ 48''$, $\frac{8}{25}$ »

Life Applications

22 A ladder $\overline{AB}$ is of length 6 metres, its upper edge A lies on a vertical wall and its other edge B on a horizontal floor. If C is the projection of the point A on the surface of the floor and its angle of slope on the surface of the floor was of measure 60° , then find the length of $\overline{AC}$ *(Kafr El-Sheikh 17 – Luxor 23 – El-Dakahlia 24)* « $3\sqrt{3}$ m. »

23 A person walks up an inclined plane which makes with the horizontal plane an angle of measure 22°. If this person walks 500 m. up the plane , calculate the height of this plane above the ground surface to the nearest metre. « 187 m. »

24 The wind broke the upper point of a tree to make an angle of measure 60° with the ground level , if the top of the tree meets the ground 4 metres away from the bottom of the tree , find the height of the tree to the nearest metre. *(El-Fayoum 14)* « 15 m. »

For excellent pupils

25 **Choose the correct answer from those given :**

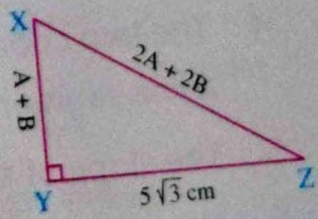
1 If the figure ABCD is a parallelogram , then : $\sin\left(\frac{A+B}{4}\right) =$ .... *(El-Dakahlia 23)*

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{2}}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\sqrt{2}$

2 If ABCDEF is a regular hexagon , m (∠ BAC) = X° , then $\sin X^\circ =$ .... *(Alex. 23)*

(a) $\frac{BC}{AB}$ (b) $\frac{BC}{AC}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

3 **From the opposite figure :**

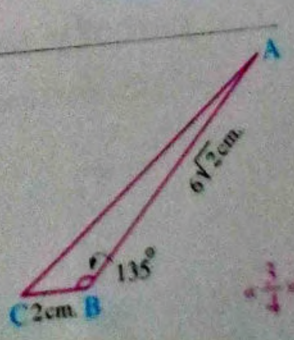


Perimeter of triangle XYZ = ... cm.

(a) $15+\sqrt{3}$ (b) $15-\sqrt{3}$
(c) $15+5\sqrt{3}$ (d) $3+\sqrt{15}$

(New Valley 23)

26 **In the opposite figure :**



« $\frac{3}{4}$ »

If m (∠ B) = 135°

, AB = $6\sqrt{2}$ cm.

, BC = 2 cm.

UNIT FIVE

Analytical geometry

Exercises of the unit :

3. Distance between two points.
4. The two coordinates of the midpoint of a line segment.
5. The slope of the straight line.
6. The equation of the straight line given its slope and the intercepted part of y-axis.

Scan the QR code to solve an interactive test on each lesson

From the school book

Exercise

3 Distance between two points

● Remember ● Understand ● Apply ● Problem Solving

Interactive test

1 Find the length of $\overline{AB}$ in each of the following cases :

1. A (1 , 2) , B (4 , 6)
2. A (2 , – 1) , B (5 , – 5)
3. A (– 2 , 7) , B (3 , – 5)
4. A (– 2 , 5) , B (3 , 0)
5. A (15 , 0) , B (6 , 0)
6. A (6 , 0) , B (0 , – 8)

2 Choose the correct answer from those given :

1 The distance between the two points (3 , a) and (– 1 , a) is .... length unit. *(Ismailia 17 – Port Said 24)*

(a) 16 (b) 9 (c) 5 (d) 4

2 The distance between the point $\left(\sqrt{3} , 1\right)$ and the origin point is .... *(Souhag 18)*

(a) 4 (b) 3 (c) 2 (d) 1

3 If the distance between the two points (a , 0) , (0 , 1) is one length unit , then a = ... *(El-Gharbia 20)*

(a) 1 (b) – 1 (c) 0 (d) 2

4 The radius length of the circle whose centre is (7 , 4) and passes through (3 , 1) equals .... length units. *(El-Menia 24)*

(a) 5 (b) – 5 (c) 2.5 (d) 25

5 If ABCD is a rectangle , A (– 1 , – 4) , C (5 , 4) , then the length of $\overline{BD}$ = ... length units. *(Assiut 23)*

(a) 4 (b) 6 (c) 8 (d) 10

6 If ABCD is a square and A (3 , 5) and B (4 , 2) , then the area of the square ABCD equals .... area unit.

(a) $\sqrt{10}$ (b) 10 (c) $4\sqrt{10}$ (d) 40

7 The distance between the point (– 5 , – 2) and y-axis is .... length unit. *(El-Gharbia 16)*

(a) – 5 (b) – 2 (c) 2 (d) 5

8 The distance between the point $(5 , \tan^2 60°)$ and the X-axis is .... length unit. *(Suez 17)*

(a) 5 (b) $\sqrt{5}$ (c) 3 (d) $\sqrt{3}$

9 The distance between the point $(\ell , -4)$ and y-axis is .... length unit, where $\ell \in \mathbb{R}$ *(Damietta 18)*

(a) 4 (b) ℓ (c) – 4 (d) $|\ell|$

10 The perpendicular distance between the two straight lines : y – 3 = 0 , y + 2 = 0 equals .... length units. *(Alex. 17 – El-Fayoum 17)*

(a) 5 (b) 1 (c) 2 (d) 3

11 A circle its centre is the origin and its radius length is 2 length unit , which of the following points belongs to the circle ?

(El-Gharbia 14 – Beni Suef 16 – El-Beheira 17 – Matrouh 24)

(a) (1 , 2) (b) (– 2 , 1) (c) $(\sqrt{3} , 1)$ (d) $(\sqrt{2} , 1)$

3 If A (3 , 1) , B (1 , 2) and C (5 , 4) , **prove that :** BC = 2 AB *(Luxor 16 – El-Dakahlia 22)*

4 **Prove that :** The points A (4 , 3) , B (1 , 1) and C (– 5 , – 3) are collinear.

(Assiut 14 – Kafr El-Sheikh 15 – El-Fayoum 17 – El-Monofia 23 – Red Sea 24)

5 If A (– 2 , 2) and B (1 , – 1) , then prove that the point C (3 , 4) lies on the axis of symmetry of $\overline{AB}$

6 **Show which of the following sets of points are collinear :**

1 A (1 , 4) , B (3 , – 2) and C (– 3 , 16)

2 A (7 , 0) , B (– 3 , 6) and C (22 , 9) *(Cairo 08)*

3 A (– 1 , 4) , B (3 , – 14) and C (– 5 , – 6)

7 Show the type of Δ ABC such that A (– 2 , 4) , B (3 , – 1) and C (4 , 5) according to its side lengths.

(Giza 17 – Damietta 19 – El-Beheira 20 – Damietta 22 – Souhag 23 – Giza 24)

8 **Show the type of each of the following triangles according to its angles if its vertices are :**

1. A (2 , 1) , B (4 , – 2) and C (7 , 5)
2. A (3 , 5) , B (– 1 , 1) and C (5 , – 5)
3. A (4 , 4) , B (3 , – 1) and C (– 2 , 4)
4. A (0 , 0) , B (6 , 0) and C (0 , 8)
5. A (1 , – 1) , B (2 , 1) and C (– 3 , – 2)

9 Prove that the triangle whose vertices are : A (5 , – 5) , B (– 1 , 7) and C (15 , 15) is right-angled at B , then find its area.

(Beni Suef 13 – El-Monofia 14 – Qena 16 – New valley 23) « 120 square units »

10 If the points A (5 , 0) , B $\left(7 , 2\sqrt{3}\right)$ and C $\left(3 , 2\sqrt{3}\right)$ are three points in a Cartesian coordinates plane , **prove that :** Δ ABC is equilateral and find its area. « $4\sqrt{3}$ square units »

11 **In each of the following , prove that the points A , B , C and D are vertices of a parallelogram where :**

1. A (– 1 , 1) , B (0 , 5) , C (5 , 6) and D (4 , 2) *(Suez 11)*
2. A (– 2 , 4) , B (5 , – 3) , C (7 , 1) and D (0 , 8) *(Souhag 08)*

12 **Prove that :** The points A (0 , 1) , B (4 , 5) , C (1 , 8) and D (– 3 , 4) are vertices of a rectangle and find its diagonal length. *(Souhag 09)* « $5\sqrt{2}$ length units»

13 **Prove that :** The points A (3 , 3) , B (0 , 3) , C (0 , 0) and D (3 , 0) in the Cartesian coordinates plane are vertices of a square and calculate the length of its diagonal and its area. *(Luxor 09)* « $3\sqrt{2}$ length units , 9 square units»

14 ABCD is a quadrilateral where A (5 , 3) , B (6 , – 2) , C (1 , – 1) and D (0 , 4)

Prove that : ABCD is a rhombus , then find its area. *(Qena 19)* « 24 square units »

15 **Prove that :** The points A (– 2 , 5) , B (3 , 3) and C (– 4 , 2) are non-collinear and if D (– 9 , 4) *(Port Said 17)*

Prove that : The figure ABCD is a parallelogram.

16 ABCD is a quadrilateral where A $(2, 4)$, B $(-3, 0)$, C $(-7, 5)$ and D $(-2, 9)$

(El-Beheira 17 – Cairo 19 – El-Monofia 2…)

Prove that : The figure ABCD is a square.

---

17 **Prove that :** The points A $(3, -1)$, B $(-4, 6)$ and C $(2, -2)$ are located on a circle whose centre is M $(-1, 2)$, then find the circumference of the circle where $\pi = 3.14$

(Cairo 15 – North Sinai 16 – El-Kalyoubia 18 – Alex. 19 – Aswan 20) « 31.4 length units »

---

18 If the distance between the point $(x, 5)$ and the point $(6, 1)$ equals $2\sqrt{5}$ length units.

, **find :** the value of x *(El-Monofia 18 – El-Kalyoubia 19 – Damietta 20 – Qena 22 – Giza 24)* « 4 or 8 »

---

19 **Find the value of a in each of the following cases :**

1. If the distance between the two points $(a, 7)$, $(-2, 3)$ equals 5 length unit.

   (Alex. 18 – El-Menia 19 – El-Fayoum 20 – Port Said 22 – Souhag 24) « 1 or −5 »

2. If the distance between the two points $(a, 7)$, $(3a - 1, -5)$ equals 13 length unit.

   « −2 or 3 »

---

20 If A $(x, 3)$, B $(3, 2)$ and C $(5, 1)$ and AB = BC, then find the value of x

(El-Beheira 15 – El-Beheira 17 – El-Beheira 19 – Matrouh 22) « 5 or 1 »

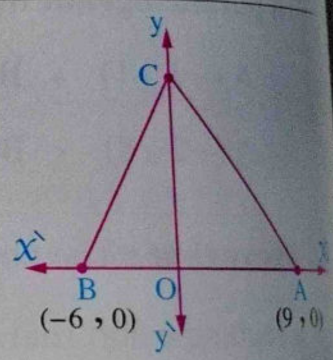
---

21 **In the opposite figure :**

If AB = AC

, **find :** the length of $\overline{CO}$

« 12 length units »



---

22 If the axis of symmetry of $\overline{CD}$ is passing through the point A $(6, m)$ where C $(3, 1)$, D $(-3, 7)$, then find the value of m

(El-Dakahlia 16 – El-Sharkia 22) « 10 »

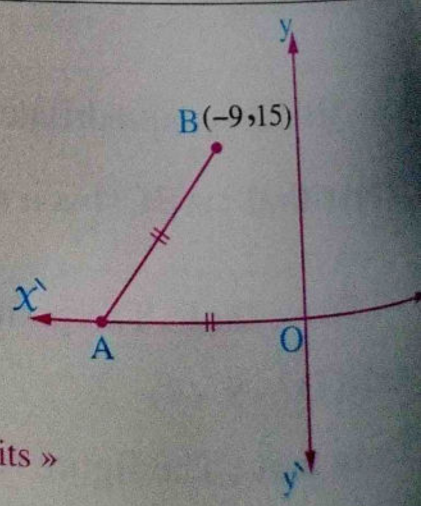
---

23 **In the opposite figure :**

If A $\in$ the x-axis

and AO = AB

, **find :** the length of $\overline{AB}$

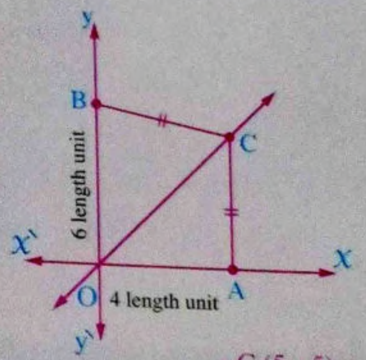


(El-Dakahlia 18) « 17 length units »

24 **In the opposite figure :**

$A \in \overleftrightarrow{XX'}$, $B \in \overleftrightarrow{yy'}$ where OA = 4 length unit , OB = 6 length unit , the straight line $\overleftrightarrow{OC}$ represents the function $f : f(X) = X$ and AC = BC

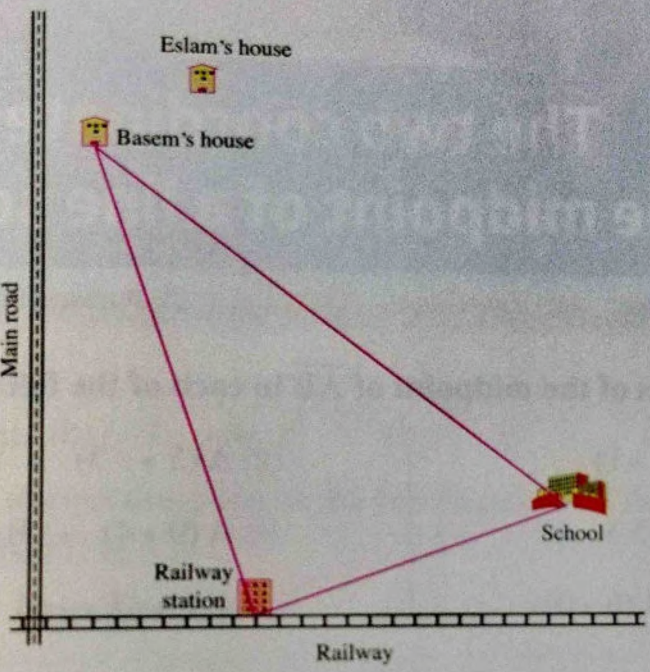
Find the coordinates of the point C



« C (5 , 5) »

Life Application

25



If the distance between Basem's house and the main road is 1 km. and the distance between Basem's house and the railway lines is 9 km.

Eslam's house is 3 km. away from the main road and 10 km. away from the railway lines.

The school is 10 km. away from the main road and 2 km. away from the railway lines.

The railway station is 4 km. away from the main road.

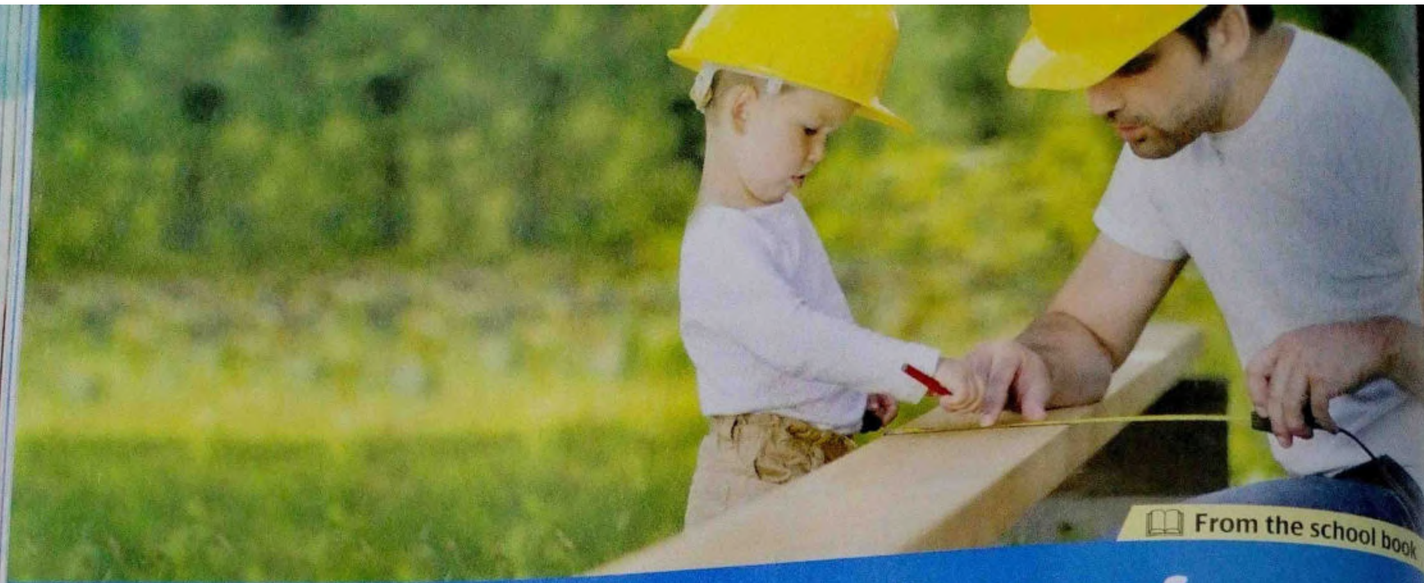
1 Which is nearer to school , Basem's house or Eslam's house ?

2 Is the way (school – railway station) perpendicular to the way (Basem's house – railway station) ? Mention the reason.

For excellent pupils

26 If the points A (4 , – 2) , B (X , 2) and C (3 , 5) are three points in the Cartesian coordinates plane , find the value of X which makes Δ ABC a right-angled triangle at B and find its area.

« 0 or 7 , 12 square units or 12.5 square units »



From the school book

Exercise **4** The two coordinates of the midpoint of a line segment

● Remember ● Understand ● Apply ● Problem Solving

Interactive test

1 **Find the coordinates of the midpoint of $\overline{AB}$ in each of the following cases :**

1 A (3 , 5) , B (7 , 1)

2 A (5 , – 3) , B (– 1 , 3)

3 A (– 5 , 4) , B (5 , – 4)

4 A (0 , 4) , B (8 , 0)

5 A (2 , 4) , B (6 , 0)

6 A (7 , – 6) , B (– 1 , 0)

2 If the point $(x , 0)$ is the midpoint of $\overline{AB}$ where A (1 , – 5) and B (2 , 5) ,
find the value of : x « $\frac{3}{2}$ »

3 If the point (5 , 3) is the midpoint of $\overline{AB}$ where A (15 , y) and
B (– 5 , – 2) , **find the value of :** y «8»

4 If C (6 , – 4) is the midpoint of $\overline{AB}$ where A (5 , – 3) , find the coordinates of the point B
(El-Dakahlia 18 – Beni Suef 19 – Cairo 19 – El-Kalyoubia 22 – Damietta 23 – Aswan 24) « (7 , – 5) »

5 **If C is the midpoint of $\overline{AB}$, then find x , y in each of the following cases :**

1 A (1 , 5) , B (3 , 7) , C (x , y) « 2 , 6 »

2 A (– 3 , y) , B (9 , 11) , C $(x , – 3)$ *(Aswan 18 – El-Kalyoubia 23)* « 3 , – 17 »

3 A $(x , – 6)$, B (9 , – 11) , C (– 3 , y) « – 15 , – 8.5 »

4 A $(x , 3)$, B (6 , y) , C (4 , 6) *(Cairo 15 – Matrouh 17)* « 2 , 9 »

6 Choose the correct answer from those given :

1 If the point of the origin is the midpoint of $\overline{AB}$ where A $(5 , -2)$, then the point B is .... (Port Said 24 – Souhag 24)

(a) $(2 , 5)$ (b) $(5 , -2)$ (c) $(-2 , -5)$ (d) $(-5 , 2)$

2 If C $(-3 , y)$ is the midpoint of $\overline{AB}$ where A $(x , -6)$ and B $(1 , -8)$, then $x + y =$ .... (Qena 18)

(a) -11 (b) 11 (c) -18 (d) -14

3 If $\overline{AB}$ is a diameter in a circle where A $(3 , -5)$ and B $(5 , 1)$, then the centre of the circle is .... (El-Fayoum 18 – Matrouh 19 – Port Said 23– El-Monofia 23)

(a) $(4 , -2)$ (b) $(4 , 2)$ (c) $(2 , -2)$ (d) $(8 , -2)$

4 If ABCD is a square where A $(3 , 4)$ and C $(5 , 6)$, then the midpoint of its diagonal is .... (El-Menia 18)

(a) $(8 , 10)$ (b) $(10 , 8)$ (c) $(4 , 5)$ (d) $(15 , 24)$

5 The point $(4 , 6)$ is the image of the point $(-2 , 2)$ by reflection in the point .... (El-Sharkia 24)

(a) the origin point. (b) $(-1 , -4)$ (c) $(1 , 4)$ (d) $(4 , 1)$

6 If M $(1 , 2)$ is the intersection point of the two diagonals of the parallelogram ABCD where A $(2 , 5)$, then C is ....

(a) $(0 , 2)$ (b) $(0 , -1)$ (c) $(-4 , 1)$ (d) $(-1 , 0)$

7 If $\left(\frac{1}{2} , \frac{5}{2}\right)$ is the midpoint of $\overline{AB}$ where A $(1 , -1)$ and B $(x , 6)$, then $x =$ ....

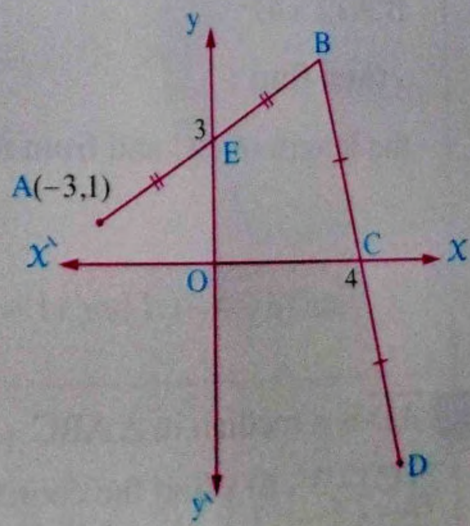
(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

8 If the x-axis bisects $\overline{AB}$ such that A $(3 , 2)$ and B $(-2 , y)$, then y = ... (El-Dakahlia 17)

(a) 3 (b) 2

(c) -2 (d) 4

9 **In the opposite figure :**



If E , C are the midpoints of $\overline{AB}$ and $\overline{BD}$ respectively , then the point D is ....

(a) $(5 , -5)$ (b) $(5 , -4)$

(c) $(6 , -5)$ (d) $(6 , -4)$

7 If A , B , C and D are four collinear points and AB = BC = CD , A $(1 , 3)$ and C $(5 , 1)$, find the points B and D

8 If A (1 , – 6) and B (9 , 2) , find the coordinates of the points which divide $\overline{AB}$ into four equal parts in length. *(Souhag 18 – Luxor 22)* « (5 , – 2) , (3 , – 4) , (7 , 0) »

9 If the origin point O is the midpoint of $\overline{AB}$ where A (X – 2 , y) and B (– 2 , 2) , **find :** (X , y) « (4 , – 2) »

10 Find the value of each of a and b that satisfies that (2 a – 3 , a – b) is the midpoint of the line segment whose terminals are (7 , – 1) and (3 , 7) *(El-Fayoum 12)* « 4 , 1 »

11 $\overline{AB}$ is a diameter in a circle M , if B (8 , 11) and M (5 , 7)

Find : 1 The coordinates of A 2 The circumference of the circle where (π = 3.14)

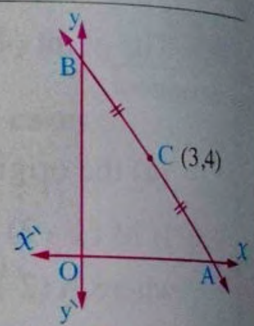
(El-Kalyoubia 16 – North Sinai 17 – Kafr El-Sheikh 18 – El-Gharbia 23) « A (2 , 3) , 31.4 length unit »

12 ABC is a triangle where A (1 , 3) , B (5 , 1) and C (3 , 7) , if D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$, by using the coordinates , **prove that :** DE = $\frac{1}{2}$ BC

13 **In the opposite figure :**

C (3 , 4) is the midpoint of $\overline{AB}$

Find : The perimeter of the triangle OAB



(Alex. 17 – El-Kalyoubia 20) « 24 length unit »

14 **In the opposite figure :**

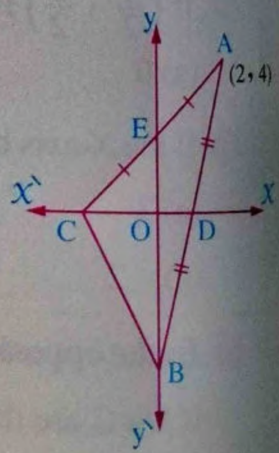
D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

If A (2 , 4)

, then find :

the length of $\overline{BC}$ and from it deduce the length of $\overline{DE}$



« 2$\sqrt{5}$ length unit , $\sqrt{5}$ length unit »

15 $\overline{AD}$ is a median in Δ ABC , M is the midpoint of $\overline{AD}$ where M (0 , 6) , B (3 , 2) , C (– 3 , 6) , find the coordinates of the point A *(Kafr El-Sheikh 17)* « A (0 , 8) »

16 If A (– 1 , – 1) , B (2 , 3) , C (6 , 0) and D (3 , – 4) are four points in the Cartesian coordinates plane , **prove that :** $\overline{AC}$ and $\overline{BD}$ bisect each other. *(Suez 19)*

17 **Prove that :** The points A (3 , − 2) , B (− 5 , 0) , C (0 , − 7) and D (8 , − 9) are the vertices of a parallelogram. *(El-Fayoum 19)*

18 If the points A (3 , 2) , B (4 , − 3) , C (− 1 , − 2) and D (− 2 , 3) are the vertices of a rhombus , **find :**

1. The coordinates of the point of intersection of the two diagonals.
2. The area of the rhombus ABCD *(Port Said 18 – Alex. 20 – El-Gharbia 24)* « (1 , 0) , 24 square unit »

19 ABCD is a parallelogram where A (3 , 2) , B (4 , − 5) and C (0 , − 3)
Find the coordinates of the intersection point of its diagonals , then find the coordinates of the point D
(El-Beheira 18 – Alex. 19 – El-Dakahlia 20 – El-Sharkia 23 – Red Sea 24) « $\left(1\frac{1}{2}, -\frac{1}{2}\right)$, (− 1 , 4) »

20 **Prove that :** The points A (6 , 0) , B (2 , − 4) and C (− 4 , 2) are the vertices of a right-angled triangle at B , then find the coordinates of D that make the figure ABCD a rectangle. *(Assiut 11 – Kafr El-Sheikh 14 – El-Beheira 19)* « D (0 , 6) »

21 **Prove that :** The points A (5 , 3) , B (3 , − 2) and C (− 2 , − 4) are the vertices of an obtuse-angled triangle at B , then find the coordinates of the point D that makes the figure ABCD a rhombus and find its area. « (0 , 1) , 21 square units »

22 **Prove that :** The points A (− 3 , 0) , B (3 , 4) and C (1 , − 6) are the vertices of an isosceles triangle of vertex A , then find the length of the drawn line segment from A perpendicular on $\overline{BC}$ *(El-Kalyoubia 12 – El-Monofia 16 – Qena 19)* « $\sqrt{26}$ length unit »

23 ABC is a triangle where A (1 , 1) , B (3 , 1) and C (1 , 3)
Prove that : Δ ABC is an isosceles triangle then find its area. *(El-Sharkia 18)* « 2 square unit »

24 ABCD is a parallelogram where A (3 , 4) , B (2 , − 1) and C (− 4 , − 3) , find the coordinates of D , take E $\in \overrightarrow{AD}$ where AE = 2 AD
What are the coordinates of the point E ? « (− 3 , 2) , (− 9 , 0) »

For excellent pupils

25 ABCD is a quadrilateral , X (2 , 3) , Y (m , 3) , Z (1 , − 1) and L (− 4 , n) are the midpoints of $\overline{AB}$, $\overline{AD}$, $\overline{BC}$ and $\overline{DC}$ respectively.
Find : The value of : m + n « − 4 »

26 ABCD is a trapezium in which BC = 2 AD and A (6 , 4) , B (4 , − 2) , C (− 2 , − 4)
Find the coordinates of D where $\overline{BC}$ // $\overline{AD}$ « (3 , 3) »
(**Hint** : Complete the parallelogram ABCE and use it to find D)

Exercise **5** The slope of the straight line

● Remember ● Understand ● Apply ● Problem Solving

Interactive test

1 **Choose the correct answer from those given :**

1 The slope of the straight line parallel to the X-axis is ....

(El-Kalyoubia 18 – Port Said 23 – Giza 24)

(a) -1 (b) zero (c) 1 (d) undefined.

2 The slope of the straight line parallel to the y-axis is .... *(El-Fayuom 23)*

(a) undefined. (b) zero (c) 1 (d) -1

3 If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$, then the slope of $\overleftrightarrow{CD} =$ ....

(Suez 23 – Beni Suef 24)

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{-3}{2}$ (d) $\frac{-2}{3}$

4 If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{1}{2}$, then the slope of $\overleftrightarrow{CD} =$ .... *(Cairo 19)*

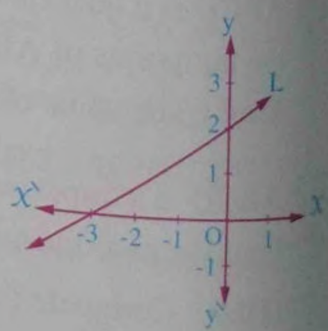
(a) 2 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) -2

5 **In the opposite figure :**

The slope of the straight line L equals .... *(Alex. 24)*

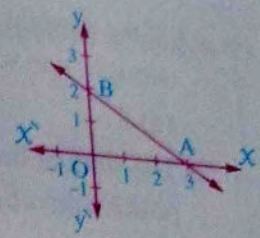
(a) $\frac{2}{3}$ (b) $\frac{-2}{3}$

(c) $\frac{3}{2}$ (d) $\frac{-3}{2}$



6 **In the opposite figure :**

The slope of $\overleftrightarrow{AB} = \dots\dots\dots$ *(Luxor 19)*



(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$

(c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

7 The slope of the straight line that makes with the positive direction of the X-axis a positive angle of measure θ equals $\dots\dots\dots$ *(Giza 17)*

(a) $\sin\theta$ (b) $\cos\theta$ (c) $\frac{\sin\theta}{\cos\theta}$ (d) $\sin\theta + \cos\theta$

8 If the slope of a straight line is more than zero , then the type of the positive angle which it makes with the positive direction of X-axis is $\dots\dots\dots$ *(Damietta 11)*

(a) zero. (b) acute. (c) right. (d) obtuse.

9 If m_1 and m_2 are the slopes of two perpendicular straight lines , then $\dots\dots\dots$ *(Qena 12)*

(a) $m_1 = m_2$ (b) $m_1 = -m_2$ (c) $m_1 m_2 = -1$ (d) $m_1 m_2 = 1$

10 If m_1 and m_2 are the slopes of two parallel straight lines , then $\dots\dots\dots$

(Qena 23 – Port Said 24)

(a) $m_1 - m_2 = 0$ (b) $m_1 + m_2 = 0$ (c) $m_1 m_2 = 0$ (d) $m_1 - m_2 \neq 0$

11 The slope of the straight line which is parallel to the straight line passing through the two points $(2 , 3)$, $(-2 , 3)$ is $\dots\dots\dots$ *(Port Said 18)*

(a) undefined. (b) zero. (c) -4 (d) -1

12 If the straight line L is perpendicular to the straight line which passes through the two points $(-1 , 2)$ and $(0 , 5)$, then the slope of the straight line L = $\dots\dots\dots$

(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

13 If m_1 and m_2 are the slopes of two perpendicular straight lines and $m_1 = 0.75$ *(El-Sharkia 13)*

, then $m_2 = \dots\dots\dots$

(a) $-\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $-\frac{4}{3}$ (d) $\frac{3}{4}$

14 If the two straight lines whose slopes are $-\frac{2}{3}$ and $\frac{k}{2}$ are parallel , *(Alex. 17 – Matrouh 19 – New Valley 24)*

then k = $\dots\dots\dots$

(a) $\frac{-3}{4}$ (b) $\frac{1}{3}$ (c) 3 (d) $\frac{-4}{3}$

15 If $\frac{-2}{3}$, $\frac{6}{k}$ are the slopes of two perpendicular straight lines , then k = $\dots\dots\dots$ *(Kafr El-Sheikh 19 – El-Menia 24)*

(a) 4 (b) -9 (c) -4 (d) 9

16 If the straight line which passes through the two points $(2 , 4)$, $(3 , k)$ makes angle of measure $45°$ with positive direction of X-axis , then k = $\dots\dots\dots$ *(El-Sharkia 24)*

(a) 3 (b) 1 (c) 5 (d) 6

17 If the straight line which passes through the two points $(X , 5)$ and $(2 , 3)$ is parallel to the straight line which passes through the two points $(3 , 4)$ and $(5 , 2)$, then $X =$ ....
(a) 2 (b) -2 (c) zero (d) 1

18 The straight line which passes through the two points $(-1 , -1)$ and $(4 , 4)$ makes with the positive direction of X-axis a positive angle of measure .... *(El-Monofia 15 – North Sinai 17)*
(a) 30° (b) 45° (c) 60° (d) 135°

19 If the straight line which passes through the two points $(k , 0)$ and $(0 , 4)$ is perpendicular to the straight line which makes a positive angle of measure 45° with the positive direction of X-axis , then $k =$ .... *(Aswan 13)*
(a) 4 (b) -4 (c) 1 (d) -1

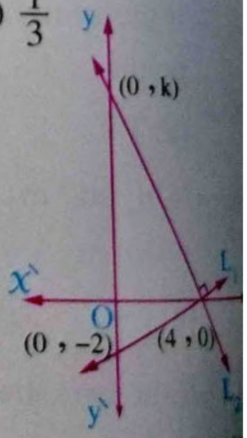
20 If the slope of the straight line L_1 is $\frac{a}{5}$ and the slope of the straight line L_2 is $\frac{-b}{3}$ where $a \neq 0$, $b \neq 0$ and $L_1 \perp L_2$, then $a\,b =$ .... *(El-Sharkia 19)*
(a) $\frac{3}{5}$ (b) $\frac{-3}{5}$ (c) 15 (d) -15

21 ABC is a right-angled triangle at B where $A = (1 , 5)$ and $B = (0 , 1)$, then the slope of $\overleftrightarrow{BC}$ equals ....
(a) -4 (b) $-\frac{1}{4}$ (c) $\frac{1}{4}$ (d) 4

22 ABCD is a parallelogram where $A(-1 , 4)$ and $B(0 , 1)$, then the slope of $\overleftrightarrow{DC} =$ ....
(a) -3 (b) $-\frac{1}{3}$ (c) $\frac{1}{3}$ (d) 3

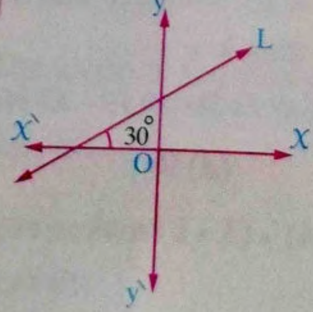
23 If ABCD is a square whose diagonals $\overline{AC}$ and $\overline{BD}$ where $A(3 , 5)$ and $C(5 , -1)$, then the slope of $\overleftrightarrow{BD} =$ ....
(a) -6 (b) -3 (c) $-\frac{1}{3}$ (d) $\frac{1}{3}$

24 **In the opposite figure :**

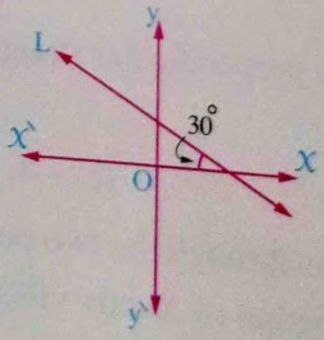


If $L_1 \perp L_2$
, then $k =$ ....
(a) 2 (b) 4
(c) 6 (d) 8

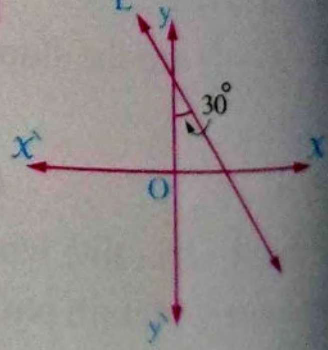
2 Write under each figure the slope of the straight line L :

1


The slope of L is ....

2


The slope of L is ....

3


The slope of L is ....

3 **Find the slope of the straight line which makes with the positive direction of X-axis a positive angle of measure :**

1 zero°
2 30°
3 45°
4 57°
5 60°
6 90°
7 86° 42̀
8 135°

4 **Using the calculator , find the measure of the positive angle which the straight line (whose slope is m) makes with the positive direction of X-axis in each of the following cases :**

1 $m = 0.3$
2 $m = 0.3673$
3 $m = 1.0246$
4 $m = \frac{4}{5}$

5 **Prove that :** The straight line which passes through the two points (4 , 2) and (5 , 6) is parallel to the straight line which passes through the two points (0 , 5) and (– 1 , 1)

6 **Prove that :** The straight line passing through the two points A (– 3 , 4) and C (– 3 , – 2) is perpendicular to the straight line passing through the two points B (1 , 2) and D (– 3 , 2) *(Aswan 23)*

7 **Prove that :** The straight line passing through the two points (2 , – 1) and (6 , 3) is parallel to the straight line that makes a positive angle of measure 45° with the positive direction of the X-axis. *(El-Menia 18 – Suez 20 – El-Fayoum 23)*

8 **Prove that :** The straight line which passes through the two points $\left(4 , 3\sqrt{3}\right)$ and $\left(5 , 2\sqrt{3}\right)$ is perpendicular to the straight line which makes a positive angle of measure 30° with the positive direction of X-axis. *(Alex. 22 – Ismailia 23 – El-Kalyoubia 24)*

9 In the Cartesian coordinates plane if A (1 , 5) , B (X – 1 , 3) , C (4 , 7) and D (2 , 1) are four points satisfying $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, **find the value of :** X « 6 »

10 If the triangle whose vertices are Y = (4 , 2) , X = (3 , 5) , Z = (– 5 , a) is right-angled at Y , **find the value of :** a *(El-Monofia 17 – Damietta 17 – Assiut 20 – El-Fayoum 22)* « – 1 »

11 If the straight line $\overleftrightarrow{AB}$ // the y-axis , where A (X , 7) and B (3 , 5) **, then find the value of :** X *(Luxor 19)* « 3 »

12 If the straight line $\overleftrightarrow{CD}$ // the X-axis , where C (4 , 2) and D (– 5 , y) **, find the value of :** y *(Damietta 22 – Alex. 24)* « 2 »

13 If the straight line L_1 passes through the two points (3 , 1) and (2 , k) and the straight line L_2 makes with the positive direction of the X-axis a positive angle whose measure is 45°, then find k if the two straight lines L_1 and L_2 are :

1 parallel 2 perpendicular *(Assiut 17 – Alex. 18 – Aswan 20 – Giza 22)* « 0 , 2 »

14 Find the measure of the positive angle which the straight line L makes with the positive direction of X-axis if the straight line L passes through the two points (4 , 3) and (2 , – 5) « 75° 57 50 »

15 Find the measure of the positive angle which the straight line L makes with the positive direction of X-axis if the straight line L passes through the two points (0 , 0) and (2 , – 2) « 135° »

16 Find the measure of the positive angle which the straight line L makes with the positive direction of X-axis if the straight line L is perpendicular to the straight lines which passes through the two points (– 2 , 5) and (4 , – 1) « 45° »

17 **Prove that :** The points A (1 , 1) , B (2 , 3) and C (0 , – 1) are collinear. *(Cairo 13)*

18 If the points (0 , 1) , (a , 3) and (2 , 5) are located on one straight line **, then find the value of :** a

(Qena 16 – Souhag 18 – Damietta 19 – Cairo 20 – El-Dakahlia 22 – Assiut 23 – El-Monofia 24) « 1 »

19 If A (1 , 7) , B (– 1 , 5) and C (4 , 2) **, prove that :** $C \notin \overleftrightarrow{AB}$

20 If A (– 1 , – 1) , B (2 , 3) and C (6 , 0) **, prove that :** the triangle ABC is a right-angled triangle at B *(Kafr El-Sheikh 17 – Aswan 19 – Matrouh 22 – Ismailia 24)*

21 **Prove that :** The points A (– 1 , 1) , B (0 , 5) , C (4 , 2) and D (5 , 6) are the vertices of a parallelogram. *(Giza 23)*

22 Prove by using the slope that the points A (– 1 , 3) , B (5 , 1) , C (6 , 4) and D (0 , 6) are the vertices of the rectangle ABCD *(North Sinai 18 – Ismailia 22 – Alex. 23)*

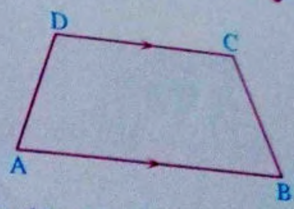
23 **Prove that :** The points A (1 , 3) , B (6 , 4) , C (7 , 9) and D (2 , 8) are the vertices of the rhombus ABCD

24 **Prove that :** The points A (– 1 , – 1) , B (2 , 3) , C (6 , 0) and D (3 , – 4) are the vertices of a square.

25 **In the drawn figure :**

ABCD is a trapezoid where $\overline{AB} // \overline{CD}$, A (9 , – 2) , B (3 , 2) , C (X , – X) and D (4 , – 3)

Find the coordinates of the point C



(Alex. 14 – Suez 19) « (1 , – 1) »

26 **Prove that :** The points A (4 , 3) , B (7 , 0) and C (1 , – 2) are the vertices of a triangle and if the point D (1 , 2) , then prove that the figure ABCD is a trapezoid and find the ratio between AD and BC « 1 : 2 »

For excellent pupils

27 Find the slope of the straight line which makes with the positive direction of X-axis a positive acute angle whose sine = $\frac{3}{5}$ « $\frac{3}{4}$ »

28 If the points A (1 , 1) , B (3 , 3) , C (0 , – 3 X) and D (X , y) are the vertices of the rectangle ABCD , **find the value of each of :** X and y *(El-Sharkia 24 – Luxor 24)* « – 2 , 4 »

29 ABCD is a rhombus in which : A (3 , 2) , B (4 , k) and C (– 1 , – 2) *(Ismailia 13)*

Find : 1 The value of k « – 3 »

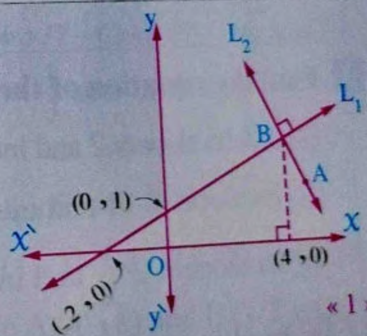
2 The length of $\overline{BD}$ « $6\sqrt{2}$ length unit »

30 **In the opposite figure :**

If $\overleftrightarrow{L_1} \perp \overleftrightarrow{L_2}$

, A $\in L_2$ where A (5 m , m)

, **find :** the value of m



« 1 »

Wonders of numbers

The two digits 8 , 5

↘ 8 × 5 = 40

↘ 88 × 5 = 440

↘ 888 × 5 = 4440

↘ 8888 × 5 = 44440

Try it yourself !

From the school book

Exercise 6 The equation of the straight line given its slope and the intercepted part of y-axis

● Remember ● Understand ○ Apply ● Problem Solving

Interactive test

1 Find the slope and the intercepted part of y-axis by each of the following straight lines :

1. $y = 5x - 3$
2. $2y = 4 - x$
3. $2x - 3y - 6 = 0$ *(Alex. 23)*
4. $\frac{y-2}{x} = \frac{1}{2}$ *(Beni Suef 24)*
5. $\frac{x}{2} + 3y = 6$
6. $\frac{x}{2} + \frac{y}{3} = 1$ *(Matrouh 19 – El-Kalyoubia 20 – Alex. 24)*

2 Find the equation of the straight line if :

1. Its slope = 2 and intercepts from the positive part of y-axis 7 units. *(Damietta 19 – Suez 20)*
2. Its slope = – 1 and intercepts from the positive part of y-axis 3 units.
3. Its slope = $2\frac{1}{2}$ and intercepts from the negative part of y-axis one unit.
4. Its slope = $-\frac{3}{4}$ and intercepts from the negative part of y-axis $2\frac{1}{2}$ units.
5. Its slope = zero and intercepts from the negative part of y-axis 2 units.

3 Find the equation of the straight line :

1. Passing through the point (3 , 2) and makes with the positive direction of x-axis a positive angle of measure 45° *(El-Sharkia 17 – Damietta 22)*
2. Which cuts a part of length 3 units from the negative part of y-axis and is parallel to the line whose equation is : $2x - 3y = 6$ *(El-Beheira 11 – Souhag 24)*
3. Which is perpendicular to the straight line : $3x - 4y + 7 = 0$ and intercepts from the positive part of y-axis a part of length 6 units.

4 Which intercepts a positive part from y-axis of length 5 units and perpendicular to the straight line which passes through the two points $(-2, 1)$ and $(2, 7)$

5 Which intercepts from the positive parts of the coordinate axes «x-axis and y-axis» two parts of lengths 4 and 9 length unit respectively. *(Luxor 17 – Kafr El-Sheikh 18 – El-Kalyoubia 19)*

6 Which passes through the point $(2, -1)$ and its slope equals 2 *(El-Kalyoubia 11)*

7 Passing through the point $(-2, 3)$ and perpendicular to the straight line whose equation is : $y = \frac{1}{2}x - 5$ *(El-Dakahlia 13)*

8 Passing through the point $(3, -5)$ and it is parallel to the straight line : $x + 2y - 7 = 0$ *(Giza 23 – El-Beheira 24)*

9 Which passes through the point $(3, -1)$ and is parallel to the straight line passing through the two points $(1, 5)$ and $(-2, 1)$ *(El-Sharkia 23)*

10 Passing through the point $(1, 2)$ and perpendicular to the straight line passing through the two points $A(2, -3)$ and $B(5, -4)$ *(El-Gharbia 14 – Luxor 18 – Suez 19 – Port Said 20 – Aswan 24)*

11 Passing through the point $(2, -2)$ and perpendicular to the straight line which makes a positive angle of measure $45°$ with the positive direction of x-axis. *(Luxor 11)*

12 Which passes through the two points $(2, -1)$ and $(1, 1)$ *(El-Kalyoubia 16 – El-Beheira 22 – Luxor 23 – El-Gharbia 24)*

13 Which passes through the two points $(4, 2)$ and $(-2, -1)$, then prove that it passes through the origin point. *(El-Beheira 17 – Cairo 19 – Port Said 22)*

14 Whose slope equals the slope of the straight line : $\frac{y-1}{x} = \frac{1}{3}$ and intercepts a negative part of y-axis of 3 length units. *(Damietta 18 – Suez 19)*

15 Which is perpendicular to $\overline{AB}$ from the point A where $A(-3, 6)$ and $B(2, 1)$

16 Which is perpendicular to $\overline{AB}$ from its midpoint where $A(1, 3)$ and $B(3, 5)$ *(Qena 18)*

17 Passing through the midpoint of the line segment $\overline{AB}$ where $A(4, 8)$ and $B(-2, 4)$ and parallel to the straight line whose equation is $2y = 4x - 5$

18 Passing through the midpoint of the line segment $\overline{AB}$ where $A(3, 6)$ and $B(-1, 4)$ and perpendicular to the straight line whose equation is $2y - 4x + 11 = 0$ *(Cairo 09)*

19 Passing through the point $(2, 3)$ and intercepts from the positive part of x-axis a part of length 4 units. *(El-Sharkia 18)*

4 Choose the correct answer from those given :

1 The straight line whose equation is : $3y = 2x - 6$, its slope = ... *(El-Sharkia 19)*

(a) 2 (b) $\frac{3}{2}$ (c) 6 (d) $\frac{2}{3}$

2 The slope of the straight line which is perpendicular to the straight line whose equation $3x - 4y - 15 = 0$ is ....

(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$ (c) $\frac{4}{3}$ (d) $-\frac{4}{3}$

(Damietta 19 – El-Fayoum 22)

3 The slope of the straight line : $x - 5 = 0$ is ....

(a) 5 (b) $\frac{1}{5}$ (c) undefined. (d) zero

(El-Monofia 11)

4 The straight line whose equation is : $3x - 3y + 5 = 0$ makes a positive angle with the positive direction of x-axis , its measure = ...

(a) $30°$ (b) $45°$ (c) $60°$ (d) $90°$

5 The straight line whose equation is : $2x - 3y - 6 = 0$ intercepts from the negative part of y-axis a part of length .... units. (El-Fayoum 13 – Cairo 14 – Qena 17 – El-Kalyoubia 18)

(a) -6 (b) -2 (c) $\frac{2}{3}$ (d) 2

6 The straight line whose equation is : $2x + 5y - 10 = 0$ cuts from the positive part of x-axis a part of length .... units. (El-Dakahlia 11)

(a) $\frac{2}{5}$ (b) 2 (c) $\frac{5}{2}$ (d) 5

7 The equation of the straight line passing through the origin point and its slope = 1 is .... (El-Kalyoubia 19 – Matrouh 24)

(a) $y = x$ (b) $y = -x$ (c) $y = 2x$ (d) $y = 0$

8 The equation of the straight line which passes through the origin point and makes with the positive direction of x-axis an angle of measure $60°$ is .... (El-Sharkia 19)

(a) $x = \sqrt{3}\,y$ (b) $y = \sqrt{3}\,x + 2$ (c) $y = 3x$ (d) $y = \sqrt{3}\,x$

9 The equation of the straight line which its slope = $\frac{1}{2}$ and cuts the y-axis at the point $(0 , 3)$ is .... (El-Monofia 19)

(a) $2y = \frac{1}{2}x + 6$ (b) $y = \frac{1}{2}x$ (c) $y = \frac{1}{2}x + 3$ (d) $2y = \frac{1}{2}x + 3$

10 The equation of the straight line which passes through the point $(2 , -3)$ and is parallel to x-axis is .... (Kafr El-Sheikh 19 – El-Menia 22)

(a) $x = 2$ (b) $y = 3$ (c) $x = -2$ (d) $y = -3$

11 The equation of the straight line which passes through the point $(-5 , 3)$ and is parallel to y-axis is .... (Beni Suef 19)

(a) $x = -5$ (b) $y = -5$ (c) $y = 3$ (d) $x = 3$

12 The equation of the straight line which intercepts a part of length 4 units from the positive part of y-axis and is parallel to the straight line : $y = 3x + 5$ is ....

(a) $y = 3x + 4$ (b) $y = 4x + 3$ (c) $y = 3x - 4$ (d) $y = -3x + 4$

13 The two straight lines : $y = 3x - 5$ and $2y = 6x + 5$ are .... *(Giza 09)*

(a) parallel. (b) coincident.

(c) intersecting and not perpendicular. (d) perpendicular.

14 If the two straight lines : $3x - 4y - 3 = 0$ and $ky + 4x - 8 = 0$ are perpendicular , then k = ... *(Giza 16 – Red Sea 19 – Alex. 23)*

(a) – 4 (b) – 3 (c) 3 (d) 4

15 If the two straight lines : $x + y = 5$ and $kx + 2y = 0$ are parallel , then k = ...

(El-Dakahlia 15 – Souhag 16 – Qena 17 – El-Menia 19 – Giza 23)

(a) – 2 (b) – 1 (c) 1 (d) 2

16 If the straight line whose equation is : $y = kx + 5$ is parallel to x-axis , then k = ... *(El-Gharbia 18)*

(a) 0 (b) 1 (c) 2 (d) 3

17 The two straight lines : $y = ax + b$ and $y = cx + d$ are perpendicular , then .... $= -1$ *(El-Gharbia 08 – Souhag 16)*

(a) $a \times d$ (b) $b \times c$ (c) $a \times c$ (d) $b \times d$

18 The straight line passing through the two points (5 , 4) and (1 , 5) is perpendicular to the straight line ....

(a) $4x = 3 - 4y$ (b) $5y + x = 4$ (c) $y = 4x$ (d) $x + 2y = 4$

19 The slope of the straight line whose equation is : $3y = ax - 5$ and passes through the point (20 , 5) is ....

(a) – 1 (b) 1 (c) – 2 (d) $\frac{1}{3}$

20 If the straight line whose equation is : $ax + (2 - a)y = 5$ is parallel to the straight line which passes through (1 , 4) , (3 , 5) , then a = ... *(El-Dakahlia 19 – Kafr El-Sheikh 20)*

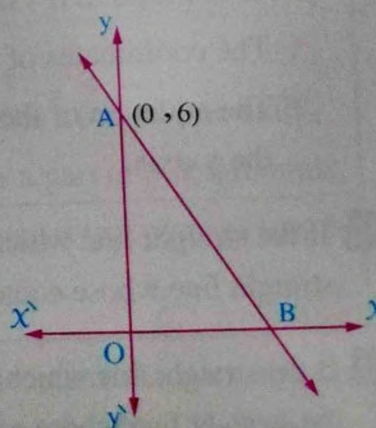
(a) 3 (b) – 2 (c) 6 (d) 4

21 The area of the triangle in square units which is bounded by the straight lines $3x - 4y = 12$, $x = 0$, $y = 0$ equals .... *(El-Kalyoubia 15 – El-Fayoum 20)*

(a) 6 (b) 7 (c) 12 (d) – 6

22 **In the opposite figure :**

If the area of Δ AOB = 9 square unit , then the equation of $\overleftrightarrow{AB}$ is .... *(El-Monofia 17)*



(a) $y = 2x + 6$

(b) $y = 6 - 2x$

(c) $y = 2x - 6$

(d) $y = \frac{1}{2}x - 6$

23 **In the opposite figure :**

OABC square of side length 4 cm.

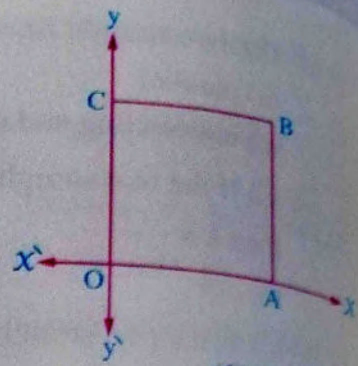
, then the equation of $\overleftrightarrow{AC}$ is ....

(a) $y = X + 4$

(b) $y = X - 4$

(c) $y = -X + 4$

(d) $X = 4y + 4$



(El-Sharkia 23)

24 **In the opposite figure :**

The equation of $\overleftrightarrow{OA}$ is

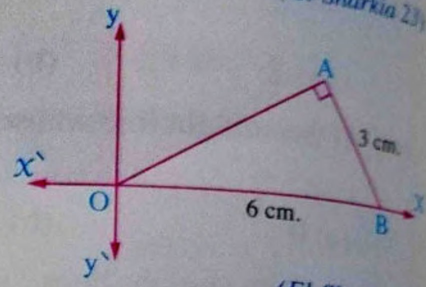
y = ...

(a) $\sqrt{3}\,X$

(b) $\frac{1}{2}X$

(c) $\frac{1}{\sqrt{3}}X$

(d) $\frac{1}{3}X$



(El-Sharkia 24)

5 **Prove that :** The straight line which passes through the two points A (3 , 1) and B (1 , 2) is parallel to the straight line : $2X + 4y - 3 = 0$ (El-Sharkia 17)

6 **Prove that :** The straight line whose equation is $\sqrt{3}\,X + y = 5$ is perpendicular to the straight line that makes with the positive direction of the X-axis a positive angle of measure 30° (Beni Suef 23)

7 Find the equations of the two straight lines which pass through the point (– 3 , 2) and parallel to the two axes.

8 Find the measure of the positive angle which is made by the straight line : $3X - 2y + 6 = 0$ with the positive direction of the X-axis , then find the coordinates of its intersection point with the y-axis.

9 **If the straight line whose equation is :** $2X - 3y - 6 = 0$ cuts the X-axis at the point A and the y-axis at the point B , **find :** (El-Sharkia 13)

1 The coordinates of the two points A and B

2 The equation of the straight line passing through the midpoint of $\overline{AB}$ and parallel to the y-axis.

10 If the straight line which passes through the two points (2 , – 1) and (5 , 1) is parallel to the straight line whose equation is : $aX + 3y + 5 = 0$, **find the value of :** a (El-Gharbia 18) « – 2 »

11 If the straight line which passes through the two po... the straight line wh...

12 If $A(2, -3)$ and $B(5, y)$, find the value of y if the straight line $\overleftrightarrow{AB}$ is parallel to the straight line $L: 3y - 4x + 1 = 0$ « 1 »

13 If the straight line : $y - (2k - 1)x = 7$ and the straight line which makes with the positive direction of the x-axis a positive angle of measure 45° are parallel
, then find the value of : k *(El-Sharkia 16)* « 1 »

14 Find the equation of the axis of symmetry of $\overline{XY}$, where $X(3, -2)$ and $Y(-5, 6)$ *(El-Dakahlia 12 – Port Said 14)*

15 $A(5, -6)$, $B(3, 7)$ and $C(1, -3)$, find the equation of the straight line which passes through the point A and the midpoint of $\overline{BC}$ *(Port Said 19 – El-Fayoum 20)*

16 ABC is a triangle whose vertices are $A = (0, 6)$, $B = (5, -1)$ and $C = (-2, 1)$
Find the equation of the straight line passing through the vertex A and perpendicular to $\overline{BC}$

17 ABC is a triangle in which $A(1, 2)$, $B(5, -2)$ and $C(3, 4)$, D is the midpoint of $\overline{AB}$ and $\overline{DE} \parallel \overleftrightarrow{BC}$ and intersects $\overline{AC}$ at E

Find : 1 The length of $\overline{DE}$

2 The equation of $\overleftrightarrow{DE}$ *(Alexandria 15 – Matrouh 18 – El-Monofia 22)*

18 ABCD is a square in which : $A(5, 4)$ and $C(-1, 6)$
Find the equation of $\overleftrightarrow{BD}$ *(El-Monofia 15 – El-Gharbia 22)*

19 ABCD is a rhombus, M is the point of intersection of its two diagonals where $A(1, 3)$ and $C(6, 0)$, find the equation of the straight line which passes through the two points B and D *(Aswan 09)*

20 Find the equation of the straight line passing through two points $A(2, 3)$ and $B(-1, -3)$
Show that for any point $C(2k + 1, 4k + 1)$, then $C \in \overleftrightarrow{AB}$ *(El-Dakahlia 14)*

21 **Draw the straight line in each of the following cases :**

1 The slope $= -\frac{1}{2}$ and intercepts from the positive part of y-axis a part of one unit.

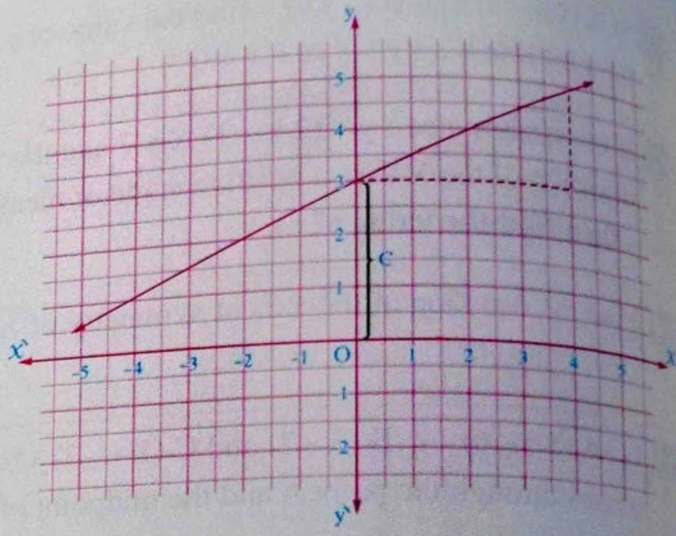
2 The slope $= 2$ and intercepts from the negative part of y-axis a part of 3 length units.

3 Intercepts from the positive parts of the two axes (x-axis, y-axis) two parts of lengths 2 and 3 length units respectively.

22 Find the slope of the straight line : $y - 2x - 3 = 0$, then find the length of the intercepted part from y-axis, also draw this line. *(Helwan 11)*

23 **From the opposite graph , find :**

1. The slope of the straight line (m)
2. The intercepted part of y-axis (c)
3. The equation of the straight line given (m) and (c)
4. The length of the intercepted part of X-axis.
5. The area of the triangle bounded by the straight line and the two axes.



24 **The opposite table represents a linear relation :**

1. Find the equation of the straight line.
2. Find the length of the intercepted part from y-axis.
3. Find the value of a

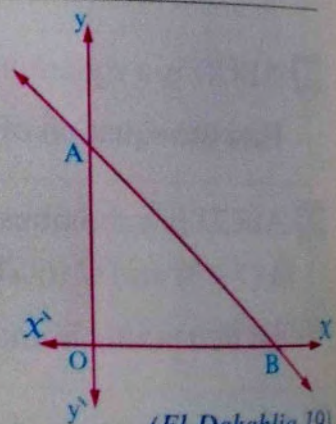
X	1	2	3
$y = f(X)$	1	3	a

(El-Kalyoubia 13 – Alexandria 15)

25 The opposite figure represents $\overleftrightarrow{AB}$ whose equation is $y = kX + c$ and cuts from the two axes two equal parts and passes through the point (2 , 3)

Find : 1. The values of k , c

2. The area of the triangle ABO



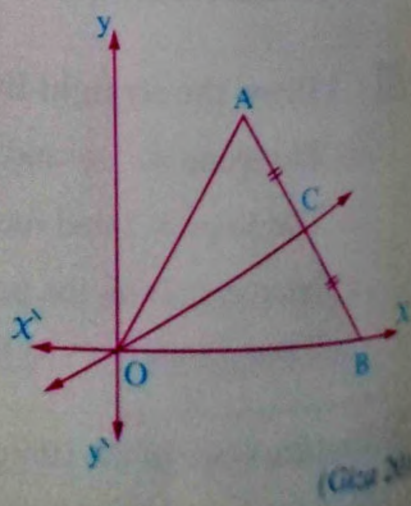
(El-Dakahlia 19)

26 **In the opposite figure :**

ABO is an equilateral triangle ,

C is the midpoint of $\overline{AB}$

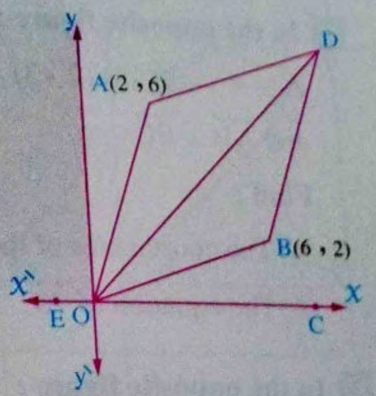
Find the equation of the straight line $\overleftrightarrow{OC}$



(Giza 20)

27 **In the opposite figure :**

The points A (2 , 6) , O (0 , 0) , B (6 , 2) and D are the vertices of a rhombus.



Find :

1. The coordinates of the point D
2. The equation of $\overleftrightarrow{OD}$
3. m (∠ DOE)

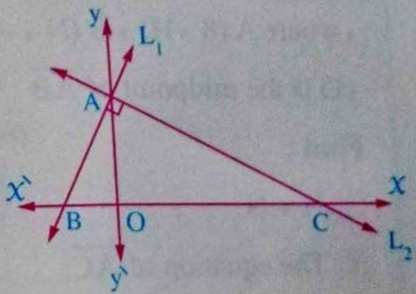
(El-Sharkia 14)

28 **In the opposite figure :**

If $L_1 \perp L_2$

and the equation of L_1 is : $2x - y + 2 = 0$

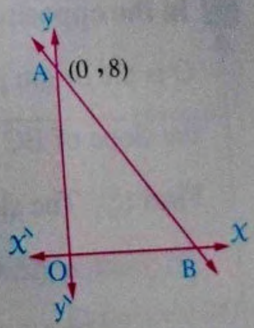
, find the equation of the straight line L_2



29 **In the opposite figure :**

$\overleftrightarrow{AB}$ cuts y-axis at the point A (0 , 8) and cuts x-axis at the point B

If $\tan (\angle ABO) = \frac{4}{3}$, **find :**



1. **First :** m (∠ BAO)

   Second : The coordinates of B

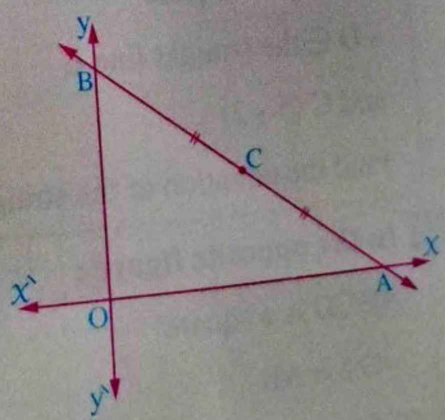
2. **First :** The slope of $\overleftrightarrow{AB}$

   Second : The equation of the straight line passing through the point O and perpendicular to $\overleftrightarrow{AB}$

(El-Sharkia 13)

30 **In the opposite figure :**

The point C is the midpoint of $\overline{AB}$ where C (4 , 3) :



1. **Find the coordinates of each of :**

   O , A and B

2. **Find the length of each of :**

   $\overline{OA}$, $\overline{OB}$, $\overline{CA}$, $\overline{CB}$ and $\overline{CO}$

3. **Find the slope of each of :**

   $\overleftrightarrow{AB}$, $\overleftrightarrow{OC}$, $\overleftrightarrow{OA}$ and $\overleftrightarrow{OB}$

4. **Find the equation of each of :** $\overleftrightarrow{AB}$ and $\overleftrightarrow{CO}$

31 **In the opposite figure :**

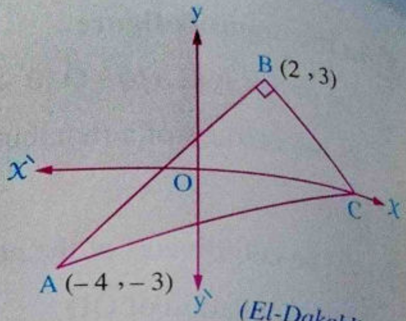
A (– 4 , – 3) , B (2 , 3)

and $\overline{AB} \perp \overline{BC}$

Find :

1 The coordinates of the point C

2 The equation of $\overleftrightarrow{AC}$



(El-Dakahlia 23)

32 **In the opposite figure :**

Δ ABC is right-angled at C

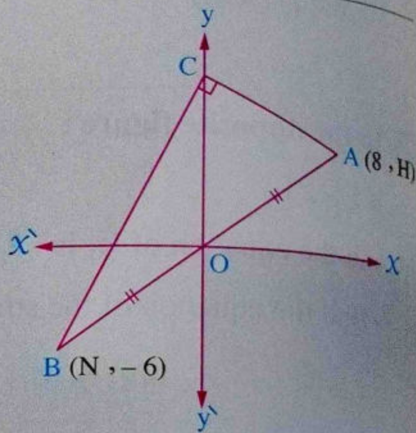
, where A (8 , H) , B (N , – 6)

, O is the midpoint of $\overline{AB}$

Find :

1 H + N

2 The equation of $\overleftrightarrow{AC}$



(El-Sharkia 23)

33 **In the opposite figure :**

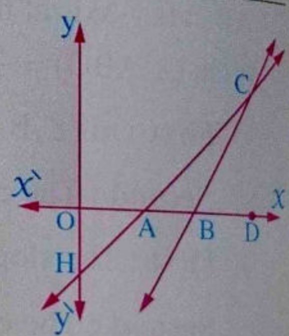
O is the origin point , A $\in$ x-axis , B $\in$ x-axis , D $\in$ x-axis ,

the slope of $\overleftrightarrow{BC} = \sqrt{3}$, the equation of $\overleftrightarrow{AC}$ is : $x - y = 3$

Find : 1 The slope of $\overleftrightarrow{AC}$ and the length of $\overline{OH}$

2 m ($\angle$ CBD) and m ($\angle$ CAD)

3 m ($\angle$ ACB)



(El-Sharkia 16)

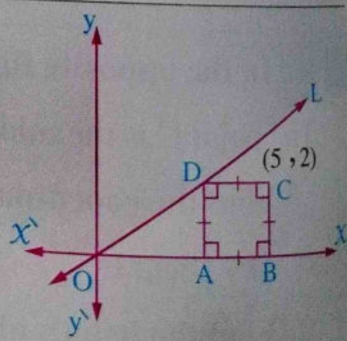
34 **In the opposite figure :**

ABCD is a square

, D $\in$ the straight line L

and C (5 , 2)

Find the equation of the straight line L

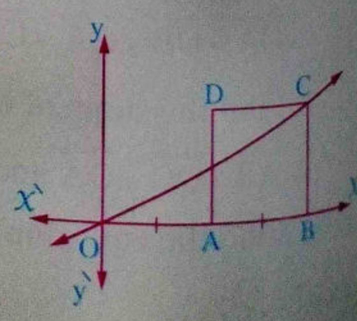


35 **In the opposite figure :**

ABCD is a square

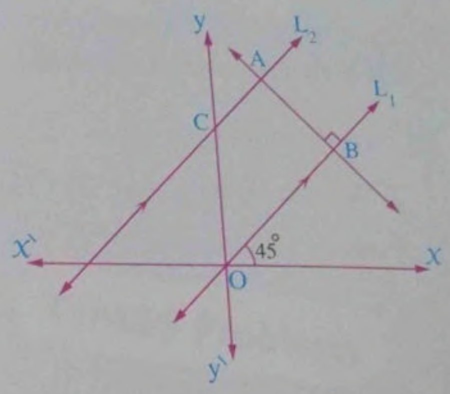
, OA = AB

Find the equation of $\overleftrightarrow{OC}$



36 **In the opposite figure :**

L_1 and L_2 are parallel lines , L_1 makes with the positive direction of X-axis an angle of measure 45° and passes through the origin point O , $A \in L_2$ where A (1 , 5) , $\overleftrightarrow{AB} \perp L_1$, L_2 intersects the y-axis at the point C



Find : 1 The equation of the straight line L_1

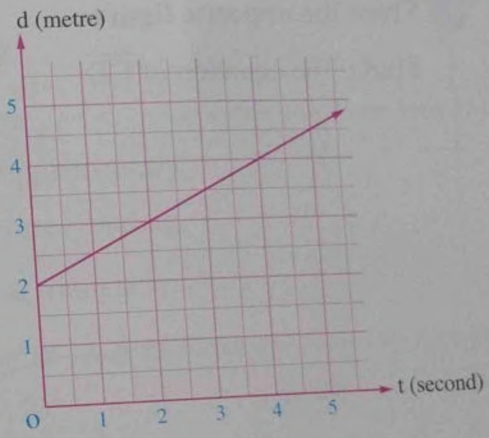
2 The equation of the straight line L_2

3 The length of $\overline{AB}$

(El-Sharkia 15)

Life Applications

37 The opposite graph represents the motion of a particle moving with uniform velocity (v) where the distance (d) is measured in metre and the time (t) in seconds.



Find :

1 The distance at the beginning of the motion.

2 The velocity of the particle.

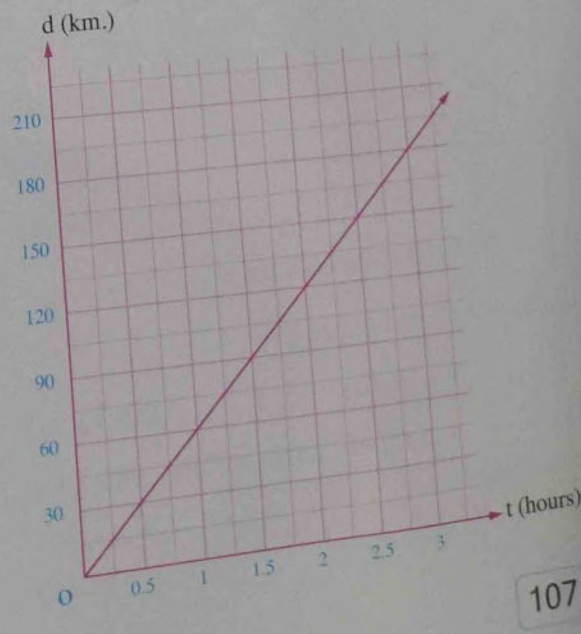
3 The equation of the straight line representing the motion of the particle.

4 The covered distance after 4 seconds from the beginning of the motion.

5 The time in which the particle covers a distance of 3.5 metres from the beginning of the motion.

(El-Dakahlia 24)

38 The opposite graph represents the relation between the distance the car covers (d in km.) and the time the car covers in (t in hour).



Find :

1 The covered distance after 90 minutes.

2 The time which the car took to cover a distance of 150 km.

3 The velocity of the car.

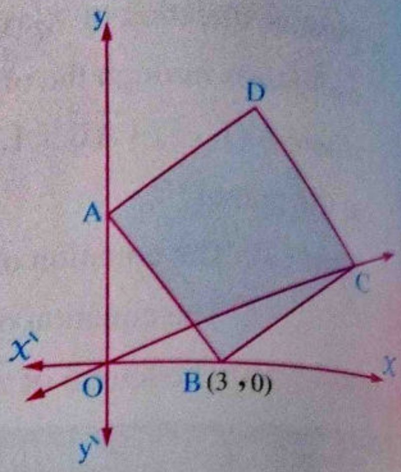
4 The equation of the straight line which represents the relation between the distance (d) and the time (t).

For excellent pupils

39 **In the opposite figure :**

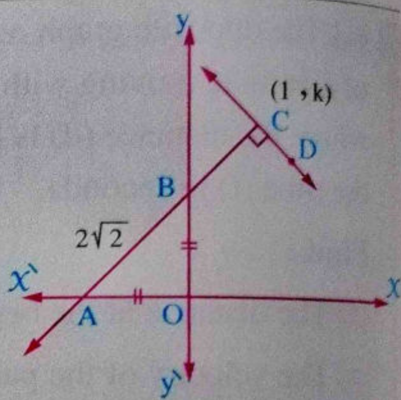
If the area of the square ABCD = 25 square units

Find : The equation of $\overleftrightarrow{CO}$



40 **From the opposite figure :**

Find : The equation of $\overleftrightarrow{CD}$



Accumulative basic skills

Choose the correct answer from those given :

1 The number of diagonals of the hexagon is .... *(Qena 20)*

(a) 6 (b) 3 (c) 12 (d) 9

2 The two angles of base of an isosceles triangle are .... *(Alexandria 16 – North Sinai 17)*

(a) congruent. (b) supplementary.

(c) vertically opposite angles. (d) corresponding.

3 The measure of an exterior angle of an equilateral triangle is ....

(Alex. 17 – Beni Suef 18 – Kafr El-Sheikh 19 – Cairo 20 – Giza 24)

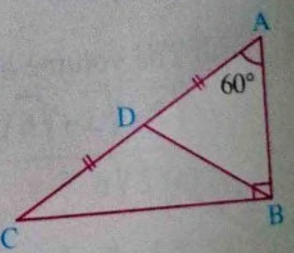
(a) $60°$ (b) $150°$ (c) $120°$ (d) $30°$

4 The number of axes of symmetry of the isosceles triangle equals ....

(El-Sharkia 22 – Alex. 23 – Port Said 24)

(a) 0 (b) 1 (c) 2 (d) 3

5 **In the opposite figure :**



If $m(\angle ABC) = 90°$, $m(\angle A) = 60°$

and $\overline{BD}$ is a median in ΔABC , then $m(\angle DBC) =$ ....

(a) $20°$ (b) $30°$

(c) $60°$ (d) $45°$

6 The triangle whose side lengths are 5 cm. , 5 cm. , ... is an isosceles triangle.

(El-Menia 17 – El-Monofia 24)

(a) 9 cm. (b) 10 cm. (c) 11 cm. (d) 12 cm.

7 The triangle whose side lengths are 5 cm. , 12 cm. and 13 cm., its area = ... cm^2. *(Matrouh 18)*

(a) 30 (b) 32.5 (c) 78 (d) 144

8 In any triangle , the sum of the lengths of any two sides is .... the length of the third side. *(El-Fayoum 18 – El-Menia 19)*

(a) greater than (b) smaller than (c) equal to (d) half

9 The point of concurrence of the medians of the triangle divides the median in the ratio of .... from the base. *(El-Fayoum 18)*

(a) 1 : 3 (b) 2 : 1 (c) 3 : 1 (d) 1 : 2

10 The sum of the measures of the accumulative angles at a point equals .... *(El-Fayoum 19 – Ismailia 22 – Damietta 24)*

(a) 90° (b) 180° (c) 270° (d) 360°

11 If ABCD is a square , then m (∠ CAB) = ... *(El-Beheira 18 – Kafr El-Sheikh 23)*

(a) 90° (b) 45° (c) 60° (d) 30°

12 If the lengths of the diagonals of a rhombus are 6 cm. , 10 cm. , then its area equals .... cm.^{2} *(Kafr El-Sheikh 17)*

(a) 30 (b) 60 (c) 15 (d) 10

13 The image of the point (– 4 , 5) by the translation (2 , – 3) is .... *(Kafr El-Sheikh 17)*

(a) (– 2 , – 2) (b) (2 , – 2) (c) (2 , 2) (d) (– 2 , 2)

14 The image of the point (– 2 , 5) by reflection in X-axis is .... *(Ismailia 16)*

(a) (– 2 , – 5) (b) (2 , 5) (c) (2 , – 5) (d) (5 , – 2)

15 The quadrilateral whose diagonals are equal in length and perpendicular is the .... *(Beni Suef 20 – Alex. 24)*

(a) square. (b) rhombus. (c) rectangle. (d) parallelogram.

16 The volume of the cuboid whose dimensions are $\sqrt{2}$, $\sqrt{3}$, $\sqrt{6}$ centimetres equals .... cm.^{3} *(South Sinai 16 – Alex. 24)*

(a) $2\sqrt{6}$ (b) $3\sqrt{6}$ (c) $3\sqrt{2}$ (d) 6

17 If 3 , 7 , ℓ are lengths of sides of a triangle , then ℓ may be equal to .... *(Souhag 23)*

(a) 3 (b) 4 (c) 7 (d) 10

18 Δ ABC is a triangle , m (∠ B) = 3 m (∠ A) = 90° , then m (∠ C) = ... *(Aswan 16)*

(a) 30° (b) 45° (c) 60° (d) 90°

19 ABC is a triangle , if m (∠ B) > m (∠ C) , then .... *(Suez 16 – Damietta 24)*

(a) AC – AB < 0 (b) AC – AB ≤ 0 (c) BC ≤ AB (d) AC – AB > 0

20 The circumference of the circle with diameter length 14 cm. is .... cm. (where $\pi = \frac{22}{7}$) *(El-Fayoum 17)*

(a) 7 (b) 22 (c) 44 (d) 14

21 If m (∠ X) = m (∠ Y) , ∠ X , ∠ Y are complementary , then m (∠ X) = ... *(North Sinai 17)*

(a) 90° (b) 60° (c) 45° (d) 30°

22 If $\overleftrightarrow{XY}$ is the axis of symmetry of $\overline{AB}$, then XA .... XB *(Suez 20)*

(a) > (b) < (c) = (d) ⊥

23 ABCD is a parallelogram in which m (∠ A) + m (∠ C) = 200° , then m (∠ B) = ... *(Alex. 18 – Suez 19 – Damietta 22 – El-Behera 23)*

(a) 50° (b) 80° (c) 100° (d) 160°

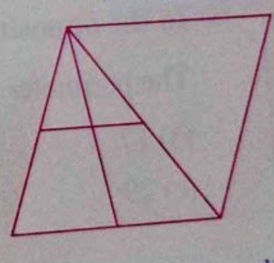
24 If ABCD is a parallelogram , then AB + CD = ... *(Suez 18)*

(a) 2 AC (b) 2 BC (c) 2 BD (d) 2 CD

25 If $L_1 // L_2$, $L_3 \perp L_1$, $L_4 \perp L_2$, then .... *(El-Beheira 17)*

(a) $L_2 // L_3$ (b) $L_1 // L_4$ (c) $L_3 // L_4$ (d) $L_3 \perp L_4$

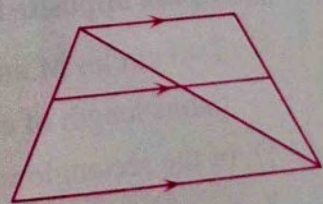
26 The number of triangles in the opposite figure = ... triangles. *(New Valley 16)*



(a) 5 (b) 6 (c) 7 (d) 8

(Luxor 17)

27 **In the opposite figure :**

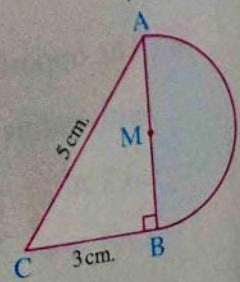


The number of trapeziums = ...

(a) 2 (b) 3 (c) 4 (d) 5

(Seuz 16)

28 **In the opposite figure :**

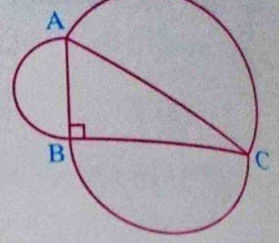


$\overline{AB}$ is a diameter of a circle , then the surface area of the shaded shape = ... cm^2

(a) 4 π (b) 16 π (c) 2 π (d) 9 π

29 **In the opposite figure :**

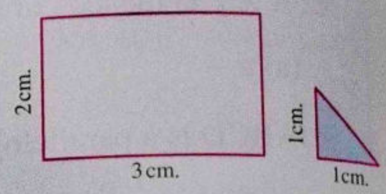
ABC is a right-angled triangle at B , what is the area of the semicircle drawn on the hypotenuse $\overline{AC}$ if the areas of the two semicircles drawn on $\overline{AB}$ and $\overline{BC}$ are 36 and 64 square units respectively ?



(a) 80 square units (b) 96 square units (c) 100 square units (d) 120 square units

30 **In the opposite figure :**

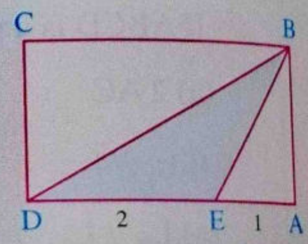
The number of the coloured right-angled triangles needed to cover the rectangle surface completely is ....



(a) 4 (b) 6 (c) 8 (d) 12

31 **In the opposite figure :**

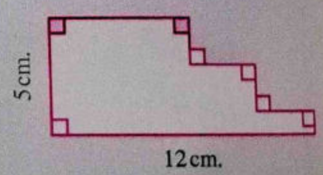
If AE : ED = 1 : 2 , then the ratio between the area of Δ BED and the rectangle ABCD is ....



(a) 1 : 2 (b) 1 : 3 (c) 2 : 3 (d) 2 : 5

32 **In the opposite figure :** *(Aswan 18)*

The perimeter of the figure = ... cm.

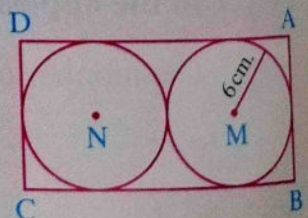


(a) 17 (b) 22

(c) 29 (d) 34

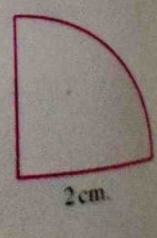
33 **In the opposite figure :**

Two circles M and N inside a rectangle , the radius length of each one is 6 cm. , then the area of the rectangle = ... cm^2.



(a) 288 (b) 252 (c) 216 (d) 144

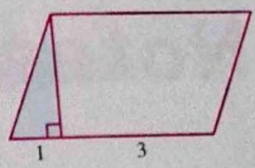
34 The opposite figure represents quarter a circle with radius 2 cm. long , then its perimeter = ... cm. *(Giza 19)*



(a) 2π (b) 5π

(c) $\pi + 4$ (d) $4\pi + 4$

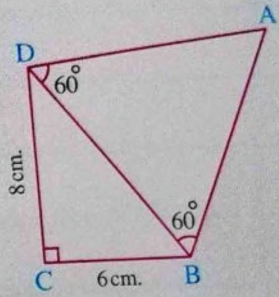
35 **In the opposite figure :**



If the base of the parallelogram is divided by the ratio 1 : 3 , then the ratio between the area of the coloured triangle and the area of the parallelogram is ....

(a) 1 : 3 (b) 1 : 6 (c) 1 : 8 (d) 1 : 9

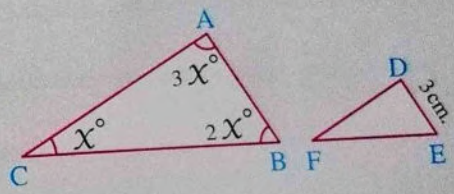
36 **In the opposite figure :**



The perimeter of the figure = ... cm.

(a) 44 (b) 34
(c) 24 (d) 14

37 **In the opposite figure :**



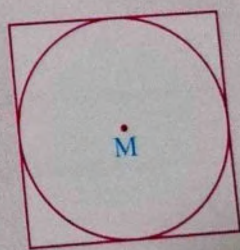
If $\Delta ABC \sim \Delta DEF$

, DE = 3 cm.

, then EF = ... cm. *(Luxor 16)*

(a) 3 (b) 9 (c) 4 (d) 6

38 **In the opposite figure :**

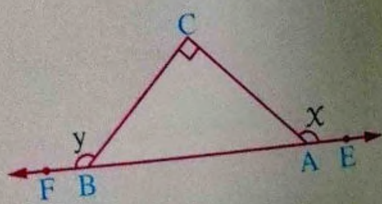


If the side length of the square = 10 cm.

, then the area of the circle = ... cm^2.

(a) 100π (b) 25π
(c) 50π (d) 40π

39 **In the opposite figure :**



If $A \in \overline{EF}$, $B \in \overline{EF}$, m $(\angle C) = 90°$

, then $x + y =$ ....

(a) 90° (b) 180°
(c) 270° (d) 360°

EL-MOASSER

By a group of supervisors

NOTEBOOK

- Accumulative Tests
- Important Questions
- Final Revision
- Final Examinations

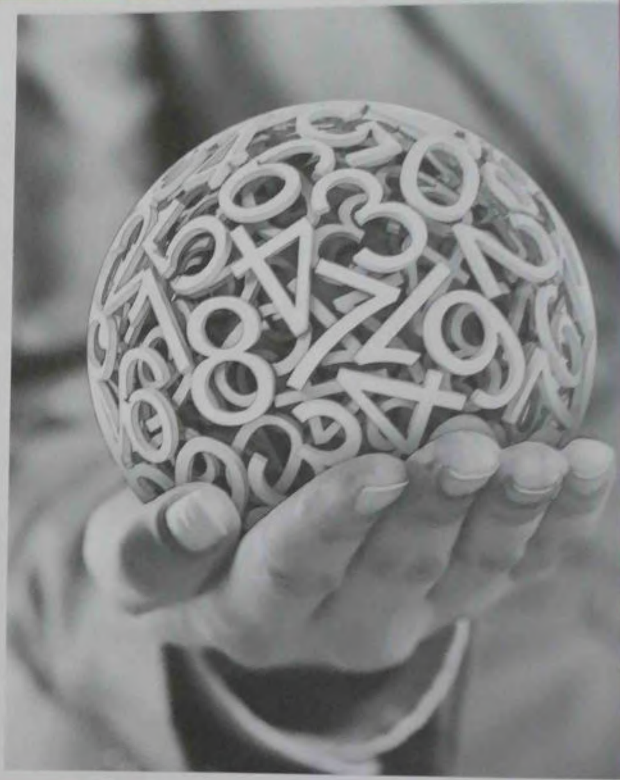
3rd PREP. 2025

FIRST TERM

Maths

Contents

First | Algebra and Statistics



- 9 Accumulative tests
- Important questions
- Final revision
- Final examinations :
  - School book examinations (2 models + model for the merge students)
  - 15 governorates' examinations
  - 5 examinations on Port Said specifications

Second | Trigonometry and Geometry

- 6 Accumulative tests
- Important questions
- Final revision
- Final examinations :
  - School book examinations (2 models + model for the merge students)
  - 15 governorates' examinations
  - 5 examinations on Port Said specifications

First

Algebra and Statistics

Accumulative Tests

on Algebra and Statistics

Accumulative test 1 on lesson 1 – unit 1

1 Choose the correct answer from those given :

1. If $(X+5, 8) = (1, 6y + X)$, then $X + y =$ .... « Alexandria 24 »

(a) 8 (b) -2 (c) -4 (d) 6

2. If $n(X^2) = 9$, $n(X \times Y) = 15$, then $n(Y) =$ .... « North Sinai 24 »

(a) 3 (b) 5 (c) 15 (d) 24

3. If $X \times Y = \{(3, 2)\}$, then $Y^2 =$ .... « Suez 24 »

(a) 1 (b) $\{(2, 2)\}$ (c) $(2, 2)$ (d) 4

4. If $(2^X, 27) = (32, y^3)$, then $\frac{X}{y} =$ .... « El-Gharbia 17 »

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{32}{27}$ (d) $\frac{27}{32}$

5. If $(X-3, 2-X)$ lies in the fourth quadrant, then $X =$ .... « El-Dakahlia 20 »

(a) 4 (b) 3 (c) 2 (d) 1

6. If the point $(k-2, 3k-2)$ is at a distance of 4 length units from the X-axis, then $k =$ .... « El-Sharkia 24 »

(a) 0 (b) 1 (c) 2 (d) 3

2 If $X = \{2\}$, $Y = \{3, 4, 5\}$, find :

1. $X \times Y$
2. $n(Y^2)$
3. X^2

« Cairo 20 »

3 If $(X-1, 29) = (4, y^3 + 2)$, then find the value of : $X + 2y$ « Red Sea 17 »

Accumulative test 2 till lesson 2 – unit 1

1 Choose the correct answer from those given :

1. If $X = \{3, 5, 7\}$ and R is a relation on X , then the relation which represents a function from the following relations is .... « Port Said 24 »

(a) $R = \{(3, 5), (5, 3), (3, 7)\}$ (b) $R = \{(3, 5), (5, 5), (7, 5)\}$

(c) $R = \{(3, 5), (5, 7)\}$ (d) $R = \{(3, 3), (3, 5), (3, 7)\}$

2. If $X = \{2\}$, then $X^2 =$ .... « El-Kalyoubia 20 »

(a) 4 (b) $\{4\}$ (c) $(2, 2)$ (d) $\{(2, 2)\}$

3. If $X = \{2, 1\}$, $Y = \{3, 5\}$, then $(3, 5) \in$ .... « Beni Suef 24 »

(a) $X \times Y$ (b) $Y \times X$ (c) X^2 (d) Y^2

4. The ordered pair (x^2, y^2) , where $x \neq 0$, $y \neq 0$ lies in the .... quadrant. « Qena 20 »

(a) first (b) second (c) third (d) fourth

5. If $a + b = ab = 5$, then $a^2 b + ab^2 =$ .... « Kafr El-Sheikh 18 »

(a) 25 (b) 20 (c) 15 (d) 10

6. If R is a function on X where $X = \{1, 3, 5\}$, and $R = \{(a, 3), (b, 1), (1, 5)\}$, then $a + b =$ .... « El-Dakahlia 18 »

(a) 4 (b) 6 (c) 8 (d) 2

---

2 If $X = \{1, 2, 3, 4\}$, $Y = \{2, 3\}$, $Z = \{7, 2\}$, **find :**

1. $(X \cap Y) \times Z$ 2. $(X - Y) \times Z$ « El-Sharkia 18 »

---

3 If $X = \{\frac{1}{2}, 1, 0, -\frac{1}{2}, -1\}$, $Y = \{1, 2, 0, -1, -2\}$ and R is a relation from X to Y , where "a R b" means "a is the multiplicative inverse of b" for each $a \in X$, $b \in Y$, write R and represent it by an arrow diagram and show if R is a function or not , and why ?

« El-Sharkia 19 »

Accumulative test 3 till lesson 3 – unit 1

1 Choose the correct answer from those given :

1. The function d : d $(X) = X^2 - (X - 3)^2$ is of the .... degree. « El-Dakahlia 20 »

(a) zero (b) first (c) second (d) third

2. The following functions are polynomial functions of the first degree except $f : f(X) =$ .... « Ismailia 18 »

(a) $\frac{3}{5} X + 2$ (b) $\sqrt{2} X + 1$ (c) $X + (X + 5)$ (d) $X\left(\frac{1}{X} + 1\right)$

3. If $(X - 3)^{zero} = 1$, then $X \in$ .... « El-Monofia 18 »

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\mathbb{R} - \{4\}$ (d) $\mathbb{R} - \{1\}$

4. If (5 , b – 7) lies on X-axis , then b = ... « Alexandria 18 »

(a) 2 (b) 5 (c) 7 (d) 12

5. The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = X(2X^3 + 5X)$ is polynomial of the .... degree. « Red Sea 16 »

(a) first (b) second (c) third (d) fourth

6. If $f(X) = X^3$, then $f(2) + f(-2) =$ .... « Port Said 24 »

(a) 8 (b) 4 (c) – 8 (d) zero

2 If $X = \{3, 5, 7\}$, $Y = \{X : X \in \mathbb{N} , 8 < X < 30\}$ and the set of the function $f : X \longrightarrow Y$ is as follows $f = \{(3, 9), (5, 15), (7, 21)\}$

1. Find the domain of the function f
2. Write the rule of the function f

« El-Dakahlia 19 »

3 If $f(X) = 3X + b$, $f(4) = 13$, **find the value of :** b

« Alexandria 17 »

Accumulative test 4 till lesson 4 – unit 1

1 Choose the correct answer from those given :

1 If $f(X) = 4$, then $\frac{f(4)}{f(8)} = \dots\dots$ « Damietta 23 »

(a) 4 (b) 1 (c) $\frac{1}{2}$ (d) 8

2 If $X - y = 5$, $X + y = 1$, then $X^2 - y^2 = \dots\dots$ « Red Sea 19 »

(a) $\frac{1}{25}$ (b) 1 (c) 5 (d) 25

3 If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 2X + 3 + c$ passes through the origin point , then $c = \dots\dots$ « El-Sharkia 24 »

(a) -2 (b) -3 (c) zero (d) 3

4 The linear function $f : f(X) = 2X - 1$ is represented by a straight line cutting the y-axis at the point $\dots\dots$ « Matrouh 20 »

(a) $(0 , 1)$ (b) $(0 , -1)$ (c) $(1 , 0)$ (d) $(-1 , 0)$

5 The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$ represents a linear function on condition $a \in \dots\dots$ « El-Gharbia 20 »

(a) $\mathbb{R}$ (b) $\mathbb{R}_+$ (c) $\mathbb{R} - \{0\}$ (d) $\mathbb{R}_-$

6 If the point $(a , 3)$ lies on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 4X - 5$, then $a = \dots\dots$ « New Valley 20 »

(a) 2 (b) 3 (c) 4 (d) 5

---

2 Graph the curve of the function $f : f(X) = (X - 3)^2$, where $X \in [1 , 5]$

and from the graph find :

1 The equation of the axis of symmetry of the curve.

2 The minimum value of the function. « Cairo 24 »

---

3 If $X = \{3\}$, $Y = \{4 , 5\}$, $Z = \{6 , 5\}$, **find :**

1 $(X \cap Y) \times Z$

2 $X \times (Y - Z)$

3 $n(X^2)$ « Souhag 24 »

Accumulative test 5 till lesson 1 – unit 2

1 **Choose the correct answer from those given :**

1 If a , X , b , 2 X are proportional , then a : b = ... « El-Monofia 24 »

(a) 2 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4

2 If $\frac{9}{a^2} = \frac{4}{b^2}$ (where a ≠ 0 , b ≠ 0) , then $\frac{a}{b}$ = ... « Port Said 17 »

(a) $\frac{2}{3}$ (b) $\pm \frac{3}{2}$ (c) $\pm \frac{2}{3}$ (d) $\pm \frac{4}{9}$

3 If $f(3X) = 6$, then $f(-2)$ = ... « El-Fayoum 19 »

(a) – 12 (b) – 3 (c) 6 (d) – 18

4 If n (X) = 3 , Y = {1 , 2} , then n (X × Y) = ... « Assiut 24 »

(a) 1 (b) 2 (c) 3 (d) 6

5 If $\frac{a}{b} = \frac{3}{5}$, 5 a – 2 b = 20 , then b = ... « El-Dakahlia 24 »

(a) 3 (b) 5 (c) 15 (d) 20

6 If $X^2 + y^2 = 6$, $X y = 5$, then $(X + y)^2$ = ... « El-Gharbia 20 »

(a) 16 (b) ± 16 (c) 11 (d) ± 11

2 If $f(X) = X^2 - \sqrt{2}\,X$, $g(X) = X + 1$

1 **Find :** $f(3) + 3\,g(\sqrt{2})$

2 **Prove that :** $f(\sqrt{2}) = g(-1)$

« Beni Suef 20 »

3 Find the number which if we add it to each term of the ratio 3 : 7 , it becomes 1 : 2

« El-Gharbia 24 »

Accumulative test 6 till lesson 2 – unit 2

1 Choose the correct answer from those given :

1 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = m$ (where $m \in \mathbb{R}^*$) , then $\frac{a\,c\,e}{b\,d\,f} =$ .... « El-Kalyoubia 18 »

(a) m (b) 3 m (c) m^3 (d) $3\,m^3$

2 If $\frac{a}{5} = \frac{b}{3} = \frac{c}{4} = \frac{a+b-c}{X}$, then the value of $X =$ .... « Suez 17 »

(a) 3 (b) 4 (c) 5 (d) 6

3 If $\frac{a}{2} = \frac{b}{3} = \frac{c}{5}$, then each ratio is equal to .... « El-Fayoum 19 »

(a) $\frac{a+b+c}{3}$ (b) $\frac{a+2\,b-c}{3}$ (c) $\frac{a-b+c}{10}$ (d) $\frac{a-b}{5}$

4 The ratio between the area of a square of side length ℓ and the area of a square of side length $3\,\ell$ equals .... « Qena 20 »

(a) 1 : 3 (b) 3 : 1 (c) 1 : 9 (d) 9 : 1

5 If $2\,a + 2\,b + c = 36$ and $a + b = 15$, then the value of $c =$ .... « Ismailia 16 »

(a) 3 (b) 6 (c) 10 (d) 21

6 If $X \times Y = \{(1\,,2)\,,(1\,,3)\,,(1\,,4)\}$, then $n(X) + n(Y^2) =$ .... « El-Dakahlia 17 »

(a) 3 (b) 4 (c) 6 (d) 10

2 If $\frac{a}{2\,X+y} = \frac{b}{3\,y-X} = \frac{c}{4\,X+5\,y}$

, prove that : $\frac{a+2\,b}{7} = \frac{4\,b+c}{17}$ « El-Kalyoubia 19 »

3 If $\frac{a}{4} = \frac{b}{3}$ **, find the value of :** $\frac{a\,b+a^2}{a\,b-b^2}$ « El-Sharkia 20 »

Accumulative test 7 till lesson 3 – unit 2

1 Choose the correct answer from those given :

1 If a , 2 , 4 , b are in continued proportion , then a + b = ... « El-Dakahlia 20 »

(a) 2 (b) 4 (c) 6 (d) 9

2 The middle proportional between 3 and 27 is .... « Ismailia 20 »

(a) 9 (b) – 9 (c) ± 9 (d) 1

3 If 7 , X , $\frac{1}{y}$ are in continued proportion , then X^2 y = ... « Port Said 18 »

(a) 5 (b) 9 (c) 7 (d) 12

4 If the point (2 , a – 1) lies on the straight line which represents the function $f : f(X) = 4X - 5$, then a = ... « El-Gharbia 17 »

(a) 4 (b) 1 (c) 3 (d) 2

5 If 2 , 6 , X + 15 are proportional quantities , then X = ... « Luxor 16 »

(a) 1 (b) 2 (c) 3 (d) 4

6 [2 , 7] –]2 , 7[= .... « Beni Suef 18 »

(a) ∅ (b) {2} (c) {7} (d) {2 , 7}

2 If a , b , c , d are in continued proportion

, prove that : $\frac{ab - cd}{b^2 - c^2} = \frac{a + c}{b}$ « El-Monofia 20 »

3 If a : b : c = 4 : 5 : 3

, prove that : $\frac{a - b + c}{a + b - c} = \frac{1}{3}$ « Kafr El-Sheikh 16 »

Accumulative test 8 till lesson 4 – unit 2

1 **Choose the correct answer from those given :**

1 If $\frac{X}{3} = \frac{y}{5}$, then $X \alpha$ .... « El-Sharkia 23 »

(a) y^2 (b) y (c) $\frac{1}{y}$ (d) 5 y

2 If y varies inversely with X and $X = \sqrt{3}$ when $y = \frac{2}{\sqrt{3}}$, then the constant proportional equals .... « New Valley 20 »

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 2 (d) 6

3 If $X^2 - 4 X y^2 + 4 y^4 = 0$, then $X \alpha$ .... « El-Sharkia 17 »

(a) y (b) y^2 (c) $\frac{1}{y}$ (d) $\frac{1}{y^2}$

4 If $1 < X < 3$, $X \in \mathbb{R}$, then $(3 X - 1) \in$ .... « El-Monofia 20 »

(a) $[2 , 8[$ (b) $[2 , 8]$ (c) $]2 , 8[$ (d) $\{2 , 8\}$

5 If $f(X) = 3$, then $f(5) + f(-5) =$ .... « Souhag 24 »

(a) 6 (b) 1 (c) zero (d) – 1

6 The third proportional of the two numbers 3 , 6 is .... « El-Menia 23 »

(a) $\frac{1}{2}$ (b) 9 (c) 2 (d) 12

---

2 If $y \propto X$ and y = 8 when $X = 4$, **find :**

1 The relation between y and X

2 The value of X when $y = \frac{1}{2}$ « El-Sharkia 23 »

---

3 If $\frac{3a - 2c}{3b - 2d} = \frac{a}{b}$, **prove that :**

a , b , c , d are proportional quantities. « El-Sharkia 18 »

Accumulative test 9 till lesson 2 – unit 3

1 Choose the correct answer from those given :

1 The relation which represents a direct variation between the two variables x and y is .... « Ismailia 23 »

(a) $x y = 7$ (b) $\frac{x}{5} = \frac{y}{2}$ (c) $y = x + 3$ (d) $\frac{x}{2} = \frac{4}{y}$

2 If $\sum(x - \overline{x})^2 = 36$ of a set of values and the number of these values = 9 , then $\sigma =$ .... « El-Monofia 23 »

(a) 2 (b) 4 (c) 18 (d) 27

3 If 18 is the greatest value of a set of individuals and the range = 6 , then the smallest value of this set is .... « El-Kalyoubia 23 »

(a) 8 (b) 12 (c) 14 (d) 36

4 The most common measure of dispersion and the most accurate is .... « Damietta 19 »

(a) the median. (b) the arithmetic mean.

(c) the mode. (d) the standard deviation.

5 If $17 x + 8 = 11$, then $17 x + 11 =$ .... « Ismailia 19 »

(a) 8 (b) 11 (c) 14 (d) 17

6 If all individuals are equal in values , then .... « El-Sharkia 16 »

(a) $\overline{x} = 0$ (b) $\sigma = 0$ (c) $x - \overline{x} > 0$ (d) $x - \overline{x} < 0$

2 The following frequency distribution shows the ages of 20 persons :

Ages in years	15	20	22	23	25	30	Total
Number of persons	2	3	5	5	1	4	20

Calculate the mean and the standard deviation of ages. « Damietta 17 »

3 If a , b , c , d are proportional

, **prove that :** $\frac{a - b}{c - d} = \frac{a}{c}$

« Kafr El-Sheikh 24 »

Important Questions

on Algebra and Statistics

Important questions on Unit One

First Multiple choice questions

1 If the point $(3, b-1)$ lies on the X-axis, then $b =$ .... *(Aswan 24)*

(a) 3 (b) -3 (c) -1 (d) 1

2 If $(3-X, X-1)$ lies in the fourth quadrant where $X \in \mathbb{Z}$, then $X =$ .... *(Ismailia 17)*

(a) 4 (b) 3 (c) 2 (d) zero

3 If $\left(125, \sqrt{y}\right) = (X^3, 4)$, then $X + y =$ .... *(El-Sharkia 23)*

(a) 15 (b) 21 (c) 7 (d) 10

4 If $(2a, b) = (3b, 2)$, then $\frac{a}{b} =$ .... *(Luxor 24)*

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) 2

5 If $n(X) = 2$, $n(X \times Y) = 6$, then $n(Y^2) =$ .... *(Giza 23)*

(a) 4 (b) 9 (c) 16 (d) 12

6 If $X = \{3\}$, then $X^2 =$ .... *(El-Sharkia 17)*

(a) 9 (b) $(3, 3)$ (c) $\{9\}$ (d) $\{(3, 3)\}$

7 If $X = \{1, 2\}$, $Y = \{3, 4\}$, then $(3, 4) \in$ .... *(Qena 23)*

(a) $X \times Y$ (b) $Y \times X$ (c) X^2 (d) Y^2

8 If $X \times Y = \{(2, 3)\}$, then $X^2 =$ .... *(El-Sharkia 18)*

(a) $\{(4, 9)\}$ (b) $\{(4, 3)\}$ (c) $\{(2, 2)\}$ (d) $\{(2, 9)\}$

9 If $f(X) = 4X + b$, $f(3) = 15$, then $b =$ .... *(El-Kalyoubia 18)*

(a) 156 (b) 3 (c) 4 (d) -3

10 If $f : f(X) = nX^2 + 2X^n - 3$, then the set of possible values of n that make f a function of the second degree is .... *(El-Dakahlia 16)*

(a) $\{2, 3\}$ (b) $\{1, -1\}$ (c) $\{2, 1, 0\}$ (d) $\{2, 1\}$

11 $f : f(X) = 5$ is represented by a straight line parallel to the X-axis and passing through the point .... *(Ismailia 16)*

(a) $(0, 5)$ (b) $(5, 0)$ (c) $(5, -5)$ (d) $(0, 0)$

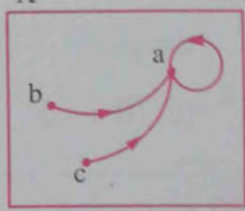
12 The straight line that represents the function $f : f(X) = X + 1$ cuts the y-axis at the point .... *(Port Said 24)*

(a) $(1, 0)$ (b) $(0, 1)$ (c) $(-1, 0)$ (d) $(0, -1)$

---

13 The opposite figure represents a function on X , its range is .... *(Cairo 11)*

X



(a) $\{a\}$ (b) $\{a, b, c\}$

(c) $\{a, b\}$ (d) $\{b, c\}$

---

14 The function $f : f(X) = 3$ is a polynomial function of the .... degree. *(Port Said 24)*

(a) third (b) second (c) first (d) zero

---

15 The function f where $f(X) = X^4 - 2X^3 + 7$ is a polynomial of the .... degree. *(Suez 15)*

(a) first (b) second (c) third (d) fourth

---

16 The function $f : f(X) = X^2 - (X^2 - 3X)$ is a polynomial of the .... degree. *(Aswan 13)*

(a) first (b) second (c) third (d) fourth

---

17 If $(2, b) \in$ the functions f where $f(X) = 3X - 6$, then b = ... *(Matrouh 17)*

(a) zero (b) 7 (c) 9 (d) 2

---

18 Which of the functions defined by the following rules is polynomial ? *(Matrouh 17)*

(a) $f(X) = X^3 + X^2 + 2$ (b) $f(X) = X^3 + \frac{1}{X} + 7$

(c) $f(X) = X^2 + \sqrt{X} + 8$ (d) $f(X) = X\left(X + \frac{1}{X} - 2\right)$

---

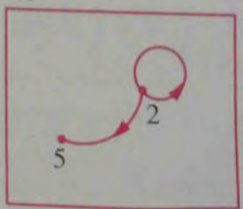
19 If $f(X) = 1$, then $f(1) + f(2) =$ .... *(El-Gharbia 24)*

(a) 1 (b) 2 (c) 3 (d) 4

---

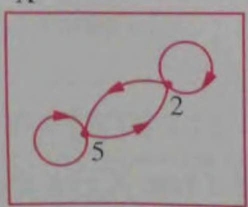
20 If $X = \{2, 5\}$, which of the following arrow diagrams represents a function on X ? *(Port Said 16)*

X



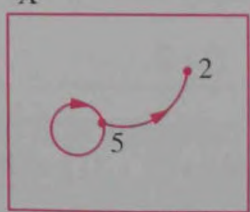
(a)

X



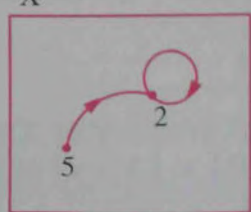
(b)

X



(c)

X



(d)

21 If $X = \{1, 3, 5\}$, $f : X \longrightarrow \mathbb{R}$ where $f(X) = 2X + 1$, then the set of images of the elements of the domain by the function f is .... *(Kafr El-Sheikh 17)*

(a) $\{3, 5, 11\}$ (b) $\{3, 7, 9\}$ (c) $\{1, 3, 11\}$ (d) $\{3, 11, 7\}$

22 The function $f : f(X) = 3X$ is represented by a straight line passing through the point .... *(Beni Suef 17)*

(a) $(0, -3)$ (b) $(0, 0)$ (c) $(3, 0)$ (d) $(3, 3)$

23 If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = X^{k-2} + 3$ and $f(2) = 11$, then $k =$ .... *(El-Sharkia 20)*

(a) 5 (b) 3 (c) 2 (d) -3

24 If $f(X + 2) = X - 2$, then $f(5) =$ .... *(El-Monofia 23)*

(a) 1 (b) 2 (c) 3 (d) 7

25 If $f(2X) = 4$, then $f(-X) =$ .... *(El-Dakahlia 09)*

(a) -2 (b) -4 (c) 4 (d) 2

26 If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 3X - a$ passes through the origin point, then $a =$ .... *(Damietta 24)*

(a) -3 (b) zero (c) 2 (d) 3

27 If $(k^2 - 4, k)$ lies on the negative part of y-axis, then $k =$ .... *(El-Sharkia 18)*

(a) ± 2 (b) 4 (c) -2 (d) 2

28 If the point (X, y) lies in the second quadrant, then the point $(-X, y^2)$ lies in the .... quadrant. *(El-Sharkia 23)*

(a) first (b) second (c) third (d) fourth

29 If X and Y are two non-empty sets, $n(X) = n(X \times Y)$, then $n(Y) =$ .... *(Damietta 18)*

(a) 1 (b) 2 (c) 3 (d) 4

30 If $\{2\} \times \{X, y\} = \{(2, 4), (2, 3)\}$, then $X - y =$ .... *(El-Sharkia 15)*

(a) 1 (b) -1 (c) ± 1 (d) zero

Second Essay questions

1 If $X = \{3 , 4\}$, $Y = \{4 , 5\}$ and $Z = \{5 , 6\}$, **find :**

1 $X \times (Y \cap Z)$

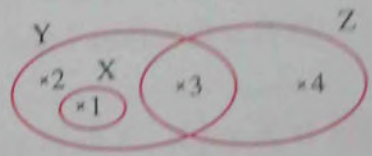
2 Y^2

3 $n(X^2)$

(El-Monofia 24)

---

2 **In the opposite figure :**



By using Venn diagram which represents the sets X , Y , Z , **find :**

1 $(X \cap Y) \times Z$

2 $(X \cup Y) \times (Z - Y)$

(El-Dakahlia 24)

---

3 If $X \times Y = \{(1 , 1) , (1 , 5) , (1 , 3) , (4 , 1) , (4 , 5) , (4 , 3)\}$

, **find :** 1 $Y \times X$ 2 X , X^2

(El-Sharkia 16)

---

4 If $X = \{1 , 2 , 3 , 5\}$, $Y = \{3 , 5 , 6\}$

, **find :** 1 $(Y \cap X) \times Y$ 2 $n(Y^2)$

(Kafr El-Sheikh 17)

---

5 If $X = \{-4 , -2 , 0 , 2 , 4\}$ and R is a relation on X where "a R b" means "a is the additive inverse of b" where $a \in X$ and $b \in X$, write R and represent it by an arrow diagram , and show if R is a function or not. *(El-Kalyoubia 24)*

---

6 If $X = \{2 , 3 , 5\}$, $Y = \{4 , 6 , 8 , 10\}$ and R is a relation from X to Y where "a R b" means "$a = \frac{1}{2}b$" for all $a \in X$, $b \in Y$, write R and represent it by an arrow diagram , show that R is a function and find its range. *(El-Monofia 24)*

---

7 If $X = \{1 , 3 , 4\}$, $Y = \{1 , 2 , 3\}$ and R is a relation from X to Y where "a R b" means "a + b = odd number" for each $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 If 2 a R 3 , **find :** the value of a

(Ismailia 17)

---

8 If $X = \{2 , 3 , 4\}$, $Y = \{6 , 9 , 12 , 15\}$ and R is a relation from X to Y where "a R b" means "3 a = b" for each $a \in X$ and $b \in Y$, write R and represent it by an arrow diagram , show that R is a function from X to Y

(El-Beheira 24)

9 If $X=\{0,1,2,3\}$, $Y=\{-1,0,1,4,9\}$ and $R: X \longrightarrow Y$ where "a R b" means "$a=\sqrt{b}$" for each $a \in X$ and $b \in Y$, write R and represent it by an arrow diagram. Is R a function or not ? giving reason. (El-Fayoum 16)

10 If $X=\{-1,1,2\}$, $Y=\{2,4,6,8\}$ and R is a relation from X to Y where "a R b" means "$b=2a+4$" for each $a \in X$ and $b \in Y$, write R and represent it by an arrow diagram. Is R a function ? and why ? (Alex. 19)

11 If $X=\{x: x \in \mathbb{N}, 0 \leq x \leq 2\}$ and R is a relation on X where "a R b" means "a + b is divisible by 3" for each $a \in X$, $b \in X$, write R and represent it by an arrow diagram and mention if R represents a function or not. (El-Gharbia 23)

12 If $X=\{1,3,5\}$ and R is a function on X where $R=\{(a,3),(b,3),(1,5)\}$

1. Find the range of the function.
2. Find the numerical value of : a + b
3. Represent R by an arrow diagram.

(New Valley 23)

13 If $f(x)=2x^2-5x+2$, **then prove that :** $f(2)=f\left(\frac{1}{2}\right)$ (Luxor 14)

14 If $f(x)=x^2-3x$, $g(x)=x-3$ (Luxor 23)

1. **Find :** $f\left(\sqrt{2}\right)+3g\left(\sqrt{2}\right)$
2. **Prove that :** $f(3)=g(3)=0$

15 If $f(x)=a$, $g(x)=x+1$, $f\left(\sqrt{2}\right)+g(2)=5$, find the value of a where f and g are two polynomial functions. (Beni Suef 24)

16 If $X=\{0,1,3\}$, $Y=\{1,2,3,4,5,7\}$ and the function $f: X \longrightarrow Y$ where $f(x)=5-x$

1. Find the range of f
2. Draw a Cartesian diagram for the function f

(New Valley 1

17 If the set of the function $f=\{(1,3),(2,5),(3,7),(4,9),(5,11)\}$, **write :**

1. The domain of the function f
2. The range of the function f
3. The rule of the function f

(Damietta

18 If the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 4X - a$ is represented graphically by a straight line intersecting the X-axis at the point $(2, b)$, **find :** a, b *(El-Menia 20)*

19 If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = aX - 3$ cuts the X-axis at the point $(3, 0)$, find the value of a, then find the value of $f(5)$ *(North Sinai 24)*

20 Represent graphically the function $f : f(X) = X^2 - 4X + 3$, taking $X \in [0, 4]$

, and from the graph find :

1. The minimum value of the function.
2. The equation of the axis of symmetry of the function. *(Giza 24)*

21 Represent graphically the function f where $f(X) = 4 - X^2$, $X \in \mathbb{R}$, consider $X \in [-3, 3]$ and from the graph deduce the coordinates of the vertex of the curve, the maximum value of the function and the equation of the symmetry axis. *(Giza 23 – Alex. 23)*

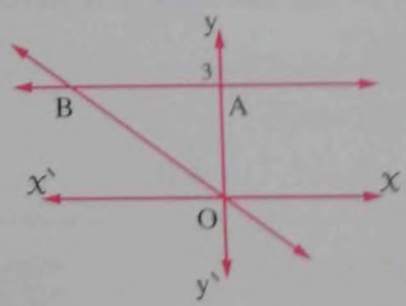
22 If $f(X) = a + X^2$, $\ell(X) = c$ are two polynomial functions, a, c are two constants,

$3f(2) + 3\ell(X) = 6$

, find the numerical value of : $2f(0) + 2\ell(7)$ *(El-Dakahlia 19)*

23 The opposite figure shows the straight line $\overleftrightarrow{AB}$ which represents the function f where $f(X) = 3$, if $\overleftrightarrow{OB}$ represents the function r where $r(X) = nX + k$ and the area of Δ AOB = 6 square units

, find : the value of each of k, n where O is the origin point.

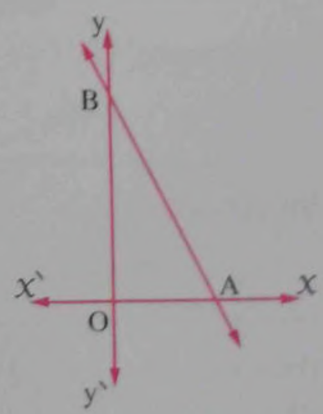


(El-Monofia 18)

24 The opposite figure represents the function f where $f(X) = 4 - 2X$

Find :

1. The coordinates of the two points A and B
2. The area of Δ AOB

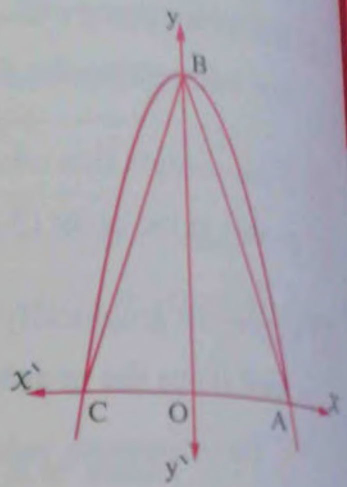


(Luxor 19)

25 The opposite figure represents the curve of the function f where $f(X) = 9 - X^2$

Find :

1. The coordinates of the two points A and C
2. The area of the triangle ABC



(Kafr El-Sheikh 18)

---

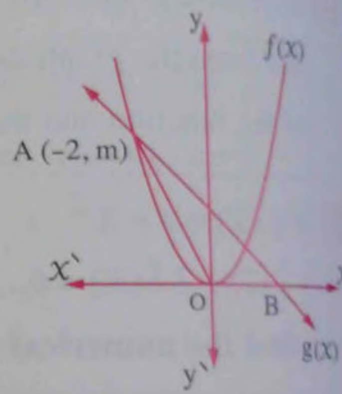
26 In the opposite figure :

The curve represents the quadratic function $f : f(X) = X^2$, $\overleftrightarrow{AB}$ represents the linear function $g : g(X) = k - X$

If A $(-2 , m)$

, **find :**

1. The values of k , m
2. The area of Δ AOB



(El-Dakahlia 24)

First Multiple choice questions

1 If $3a = 5b$, then $\frac{3a}{b} =$ .... *(El-Fayoum 17)*

(a) 3 (b) 5 (c) $\frac{3}{5}$ (d) $\frac{5}{8}$

2 If x, 3, y, 4 are proportional quantities, then $\frac{x}{y} =$ .... *(Assiut 24)*

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 3 (d) 4

3 If y varies inversely as x, then .... *(Giza 18)*

(a) $y = x$ (b) $y = m x$ (c) $x = m y$ (d) $y = \frac{m}{x}$

4 The relation which represents an inverse variation between y and x is .... *(Cairo 19)*

(a) $x y = 5$ (b) $y = x + 3$ (c) $\frac{x}{5} = \frac{y}{2}$ (d) $y = 2x$

5 The relation which represents a direct variation between y and x is .... *(El-Kalyoubia 23)*

(a) $x y = 7$ (b) $y = 4 - x$ (c) $\frac{x}{2} = \frac{y}{4}$ (d) $\frac{x}{4} = \frac{5}{y}$

6 If $y \propto x$ and $x = 3$ when $y = 2$, then the constant proportional equals .... *(Cairo 16)*

(a) 2 (b) 3 (c) $\frac{2}{3}$ (d) 6

7 If 2, 6 and $x + 15$ are proportional, then $x =$ .... *(Qena 24)*

(a) 1 (b) 2 (c) 3 (d) 4

8 If a, 2, 4 and b are in continued proportion, then $a + b =$ .... *(El-Monofia 16)*

(a) 2 (b) 4 (c) 6 (d) 9

9 The number if added to 1, 3 and 6, they become in continued proportion is .... *(Damietta 13)*

(a) 1 (b) 2 (c) 3 (d) 4

10 If 3, x and 12 are three proportional quantities, then $x =$ .... *(El-Gharbia 16)*

(a) 15 (b) -6 (c) 6 (d) ± 6

11 If $\frac{a}{2} = \frac{b}{5} = \frac{2a + b}{k}$, then $k =$ .... *(Giza 24)*

(a) 3 (b) 4 (c) 7 (d) 9

12 If $\frac{a}{b} = \frac{4}{3}$, then $3a - 4b + 5 =$ .... *(El-Beheira 23)*

(a) zero (b) 6 (c) 5 (d) -1

13 If $\frac{a}{b} = \frac{c}{d} = m$ where $m \in \mathbb{R}^*$, then $\frac{a\,c}{b\,d} =$ .... *(Suez 20)*

(a) m (b) m^2 (c) 2 m (d) $2\,m^2$

14 If $y^2 + 4x^2 = 4xy$, then .... *(El-Sharkia 20)*

(a) $y \propto x$ (b) $y \propto x^2$ (c) $y \propto \frac{1}{x}$ (d) $y \propto \frac{1}{x^2}$

15 Which of the following tables represents a direct variation between x and y ? *(Port Said 19)*

(a)

x	y
2	9
4	18

(b)

x	y
3	20
5	12

(c)

x	y
3	6
-2	-9

(d)

x	y
10	9
5	18

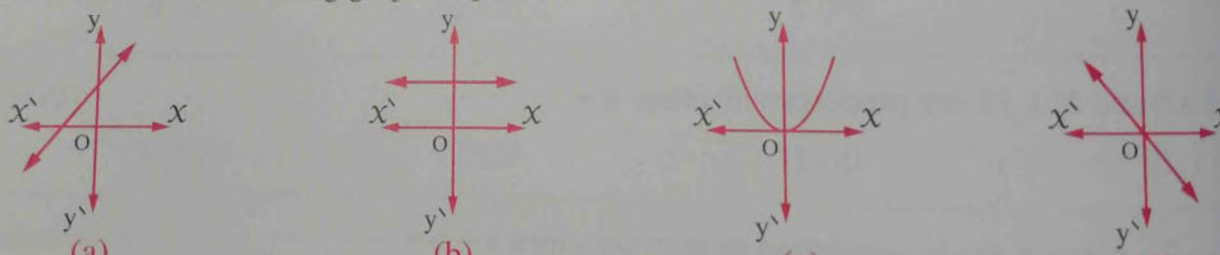
16 If $4x^2 = 9y^2$, then $\frac{x}{y} =$ .... *(Beni Suef 16)*

(a) $\frac{9}{4}$ (b) $\frac{3}{2}$ (c) $\pm\frac{2}{3}$ (d) $\pm\frac{3}{2}$

17 If $\frac{a}{b} = \frac{b}{c} = \frac{c}{3} = 2$, then $a =$ .... *(Port Said 24)*

(a) 3 (b) 6 (c) 12 (d) 24

18 Which of the following graphs represents a direct variation between x and y ? *(Port Said 24)*



(a) (b) (c) (d)

19 If $x^2 y^2 + \frac{1}{4} = xy$, then .... *(El-Monofia 16)*

(a) $x \propto y$ (b) $y \propto x$ (c) $2x \propto 5y$ (d) $y \propto \frac{1}{x}$

20 If $xy = 3$, then $y \propto$ .... *(El-Gharbia 24)*

(a) x^{-1} (b) x (c) $3x$ (d) x^2

21 The middle proportional between 3 and 27 is .... *(Port Said 24)*

(a) 9 (b) -9 (c) ± 9 (d) 81

22 If $\frac{a}{b} = \frac{2}{3}$, $\frac{a}{c} = \frac{4}{5}$, then $b : c =$ .... *(El-Gharbia 17)*

(a) 3 : 4 (b) 5 : 6 (c) 6 : 5 (d) 4 : 3

Second Essay questions

1 If $\frac{x-2y}{x+3y}=\frac{3}{5}$, **find the value of :** $x : y$ *(Giza 24)*

2 If $\frac{x}{y}=\frac{2}{3}$, **find the value of :** $\frac{3x+2y}{6y-x}$ *(Matrouh 23)*

3 If a , b , c and d are proportional quantities , **prove that :** $\frac{a+2c}{b+2d}=\frac{c-a}{d-b}$ *(Port Said 24)*

4 If a , b , c and d are in continued proportion , **prove that :** $\frac{a}{b+d}=\frac{c^3}{c^2 d+d^3}$ *(Kafr El-Sheikh 20)*

5 Find the number that if it is added to each of the numbers 3 , 5 , 8 and 12 , they become proportional. *(El-Gharbia 22)*

6 If $\frac{a}{b-a}=\frac{c}{d-c}$, **prove that :** a , b , c and d are proportional quantities. *(Giza 13)*

7 If $\frac{x}{4}=\frac{y}{5}=\frac{z}{3}$, **prove that :** $\frac{x-y+z}{x+y-z}=\frac{1}{3}$ *(El-Monofia 24)*

8 If $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$, **prove that :** $\sqrt{x^2+y^2}=2x+y-z$ *(Ismailia 23)*

9 If a : b : c = 1 : 2 : 3 , b + c = 25 , **find the value of each of :** a , b , c *(El-Kalyoubia 16)*

10 If $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}=\frac{2a-2b+5c}{3x}$, **find the value of :** x *(El-Monofia 23)*

11 Find the number which if added to each of the two terms of the ratio 5 : 11 , it becomes 4 : 7 *(Cairo 24)*

12 Find the positive number which if we add its square to each of the two terms of the ratio 7 : 11 , it becomes 4 : 5 *(El-Monofia 24)*

13 Two numbers , the ratio between them is 2 : 3 , if you add to the first 7 and subtract from the second 12 , the ratio between them becomes 5 : 3 Find the two numbers. *(Beni Suef 17)*

14 If $\frac{a}{2x-y}=\frac{b}{2y-x}$, **prove that :** $\frac{2a+b}{x}=\frac{a+2b}{y}$ *(Damietta 23)*

15 If $\frac{x+y}{7}=\frac{y+z}{5}=\frac{z+x}{8}$, **prove that :** $\frac{x+y+z}{x-z}=5$ *(Assiut 24)*

16 If $\frac{x}{2a+b}=\frac{y}{2b-c}=\frac{z}{2c-a}$, **prove that :** $\frac{2x+y}{4a+4b-c}=\frac{2x+2y+z}{3a+6b}$ *(El-Beheira 23)*

17 If $\frac{x+y}{3} = \frac{y+z}{8} = \frac{z+x}{6}$ **, prove that :** $\frac{x+y+z}{2x+3y+3z} = \frac{17}{50}$ *(Kafr El-Sheikh 20)*

18 If $\frac{x+y}{5} = \frac{y+z}{8} = \frac{z+x}{7}$ **, prove that :** $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ *(New Valley 17)*

19 If b is the middle proportional between a and c **, prove that :** $\frac{a^2+b^2}{b^2+c^2} = \frac{a}{c}$ *(Suez 24)*

20 If $y \propto x$ and $y = 20$ when $x = 4$ **, find :** *(Souhag 24)*

1. The relation between x and y
2. The value of x when $y = 40$

21 If $y \propto \frac{1}{x}$ and $y = 3$ when $x = 2$ **, find :** *(Qena 24)*

1. The relation between x and y
2. The value of y when $x = 1.5$

22 If $\frac{21x - y}{7x - z} = \frac{y}{z}$ **, prove that :** $y \propto z$ *(Assiut 24)*

23 If $x = z + 8$ where z varies inversely as y and $z = 2$ when $y = 3$, find the relation between y and x , then find y when $x = 3$ *(El-Dakahlia 20)*

24 If $y = 1 + b$ where b varies inversely as x^2 and $y = 5$ at $x = 2$, find the relation between y and x , then find y at $x = 4$ *(Kafr El-Sheikh 16)*

25 If $x^4 y^2 - 14 x^2 y + 49 = 0$ **, prove that :** $y \propto \frac{1}{x^2}$ *(Alex. 19)*

26 **From the data in the following table , answer the following questions :**

1. Show the type of variation between x and y
2. Find the constant of variation.
3. Find the value of y at $x = 3$
4. Find the value of x at $y = 2\frac{2}{5}$

x	2	4	6
y	6	3	2

(Damietta 16)

First Multiple choice questions

1 The simplest and easiest method of measuring dispersion is .... *(Ismailia 24)*

(a) the arithmetic mean. (b) the median. (c) the range. (d) the mode.

2 The range of the set of values : 7 , 3 , 6 , 5 , 9 is .... *(Giza 24)*

(a) 3 (b) 9 (c) 6 (d) 12

3 If $\sum (X - \overline{X})^2 = 36$ of a set of values and the number of these values is 9 , then $\sigma =$ .... *(El-Sharkia 20)*

(a) 2 (b) 18 (c) 27 (d) 4

4 If the standard deviation for some values = 3 and the number of these values = 2 , then $\sum (X - \overline{X})^2 =$ .... *(El-Sharkia 24)*

(a) 1 (b) 18 (c) 12 (d) 24

5 A factory has 125 workers , 75 of them are technicians and 50 are engineers , it is wanted to take a sample of layers of size 50 individuals such that it represents each layer according to its size , then the number of engineers of the sample equals .... *(El-Monofia 16)*

(a) 30 (b) 20 (c) 25 (d) 15

6 The most common value of a set of values is called .... *(El-Monofia 18)*

(a) the range. (b) the median. (c) the mean. (d) the mode.

7 .... is a secondary resource of collecting data. *(El-Fayoum 12)*

(a) Personal interview (b) Questionnaires

(c) Data base of the employees (d) Observing and measuring

8 Selecting a sample of layers of a statistical society is called .... sample. *(Alex. 14)*

(a) random (b) class (layer) (c) deliberate (d) bunch

9 The difference between the greatest value and the smallest value in a set of individuals is called .... *(Damietta 24)*

(a) the median. (b) the arithmetic mean.

(c) the range. (d) the standard deviation.

10 The mean of the values : 7 , 3 , 6 , 9 and 5 equals .... (North Sinai 17)

(a) 3 (b) 6 (c) 4 (d) 12

---

11 The set which has more dispersion of the following sets is .... (El-Kalyoubia 15)

(a) 28 , 17 , 30 , 36 , 20 (b) 20 , 19 , 29 , 37 , 43

(c) 31 , 35 , 26 , 37 , 41 (d) 25 , 39 , 19 , 5 , 27

---

12 If all individuals are equal in values , then .... (El-Gharbia 22)

(a) $x - \overline{x} > 0$ (b) $x - \overline{x} < 0$ (c) $\sigma = 0$ (d) $\overline{x} = 0$

---

13 The positive square root of the average of squares of deviations of the values from their mean is called .... (Qena 20)

(a) the range. (b) the median.

(c) the standard deviation. (d) the mode.

---

14 If the arithmetic mean of the values : a , 5 , 8 , 7 , 6 is 6 , then a = ... (Matrouh 20)

(a) 4 (b) 6 (c) 8 (d) 30

---

15 If the range of the values : 7 , k , 8 , 9 , 5 is 6 , then k = ... (El-Monofia 24)

(a) 3 (b) 4 (c) 6 (d) 12

---

16 If the standard deviation for the values : $x + 1$, y , 4 equals zero , then x y = ... (Qena 24)

(a) 4 (b) 12 (c) 16 (d) 20

Second Essay questions

1 **Calculate the mean and the standard deviation of the following data :**

12 , 13 , 16 , 18 , 21 (Damietta 19)

---

2 **The following frequency distribution shows the ages of 10 children :**

Age in years	5	8	9	10	12	Total
Number of children	1	2	3	3	1	10

Calculate the standard deviation of the ages in years. (Cairo 18)

---

3 **Calculate the mean and the standard deviation for the following frequency distribution :** (El-Gharbia 17)

Sets	0 –	4 –	8 –	12 –	16 – 20	Total
Frequency	3	4	7	2	9	[illegible]

Final Revision

on Algebra and Statistics

First Algebra

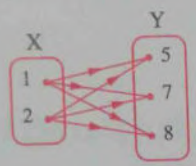
Remember: The Cartesian product of two finite sets and representing it

If $X = \{1, 2\}$, $Y = \{5, 7, 8\}$, then :

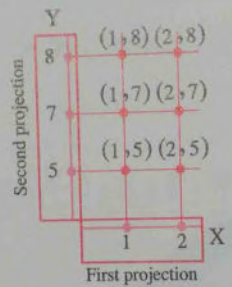
$X \times Y$

is the set of all ordered pairs whose first projection of each of them belongs to X and the second projection of each of them belongs to Y

i.e. $X \times Y = \{(1, 5), (1, 7), (1, 8), (2, 5), (2, 7), (2, 8)\}$



The arrow diagram

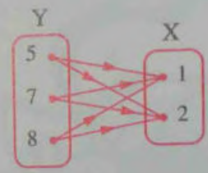


The graphical diagram (The Cartesian diagram)

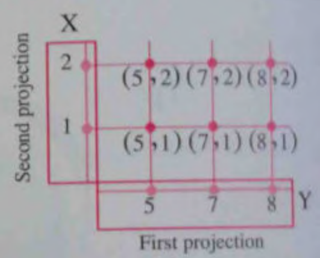
$Y \times X$

is the set of all ordered pairs whose first projection of each of them belongs to Y and the second projection of each of them belongs to X

i.e. $Y \times X = \{(5, 1), (5, 2), (7, 1), (7, 2), (8, 1), (8, 2)\}$



The arrow diagram

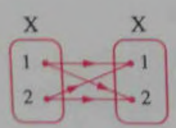


The graphical diagram (The Cartesian diagram)

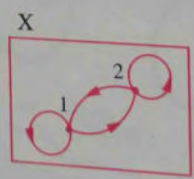
$X \times X$

is the set of all ordered pairs whose first projections and second projections belong to X

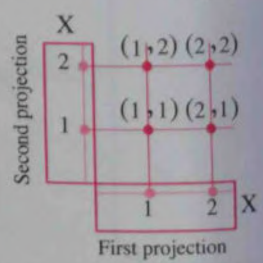
i.e. $X \times X = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$



The arrow diagram



The arrow diagram



The graphical diagram (The Cartesian diagram)

Remarks

(1) $X \times Y \neq Y \times X$, where $X \neq Y$

(2) $n(X \times Y) = n(Y \times X) = n(X) \times n(Y)$ where n is the number of elements

(3) $n(X \times X) = n(X^2) = [n(X)]^2$

(4) $X \times \varnothing = \varnothing \times X = \varnothing$

Remember **The relation and its representing**

- The relation from the set X to the set Y is a connecting joining some or all the elements of X with some or all the elements of Y
- If R is a relation from the set X to the set Y , then :

1. R is a set of ordered pairs where the first projection of each belongs to X and the second projection belongs to Y
2. $R \subset X \times Y$
3. The relation can be represented by an arrow diagram or by a Cartesian diagram (graphically)

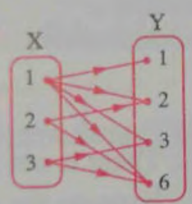
- If R is a relation from X to X , then R is a relation on X and $R \subset X \times X$

Example

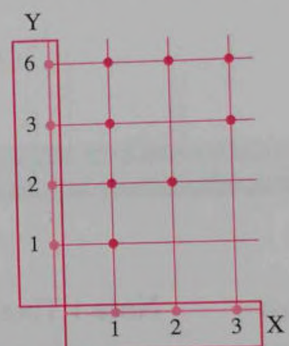
If $X = \{1 , 2 , 3\}$, $Y = \{1 , 2 , 3 , 6\}$ and R is a relation from X to Y where "a R b" means "a is a factor of b" for each $a \in X$, $b \in Y$, then write R and represent it by an arrow diagram and a Cartesian diagram.

Solution

$R = \{(1 , 1) , (1 , 2) , (1 , 3) , (1 , 6) , (2 , 2) , (2 , 6) , (3 , 3) , (3 , 6)\}$



The arrow diagram



The Cartesian diagram

Remember **The function**

- **A relation from X to Y is said to be a function if :**

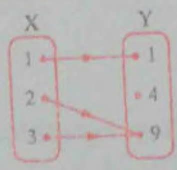
1. Each element of the set X appears only once as a first projection in one of the ordered pairs of the relation.
2. Each element of the set X has one and only one arrow going out of it to one element of Y in the arrow diagram which represents the relation.
3. Each vertical line has one and only one point lying on it of the points which represent the relation , in the Cartesian diagram which represents the relation.

- **If f is a function from the set X to the set Y is written as $f : X \longrightarrow Y$, then :**

1. X is called the domain of the function f
2. Y is called the codomain of the function f
3. The set of images of the elements of the set X by the function f is called the range of the function f which is a subset of the codomain Y

For example :

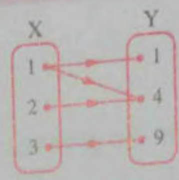
If $X = \{1, 2, 3\}$, $Y = \{1, 4, 9\}$, then the following diagrams show some of the relations from X to Y and we note which of the following relations represent a function from X to Y and which does not represent :



Note : Going out only one arrow from each element of the elements of X

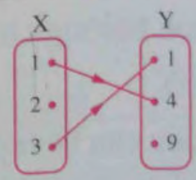
Then : The relation is a function from X to Y

- The domain $= \{1, 2, 3\}$
- The range $= \{1, 9\}$



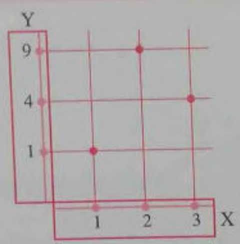
Note : Going out two arrows from the element 1 in X

Then : The relation is not a function from X to Y



Note : There are not arrows going out from the element 2 in X

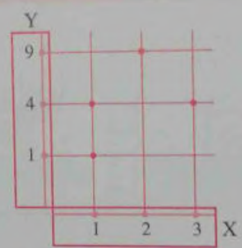
Then : The relation is not a function from X to Y



Note : Each vertical line has only one point lying on it

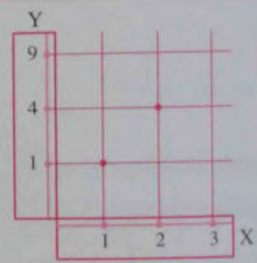
Then : The relation is a function from X to Y

- The domain $= \{1, 2, 3\}$
- The range $= \{1, 4, 9\}$



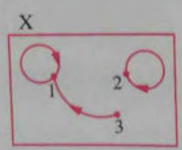
Note : There are two points lying on the vertical line at the element 1 in X

Then : The relation is not a function from X to Y



Note : There is not a point lying on the vertical line at the element 3 in X

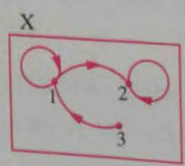
Then : The relation is not a function from X to Y



Note : Going out only one arrow from each element of the elements of X

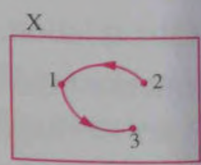
Then : The relation is a function on X

- The domain $= \{1, 2, 3\}$
- The range $= \{1, 2\}$



Note : Going out two arrows from the element 1 in X

Then : The relation is not a function on X



Note : There are not arrows going out from the element 3 in X

Then : The relation is not a function on X

Remember **The polynomial functions**

The polynomial function is a function whose rule is a term or an algebraic expression in condition that the following should be identified :

1. Each of the domain and the codomain of the function is the set of real numbers.
2. The power (The index) of the variable X in any of its terms is a natural number with noticing that : the degree of the function is the highest power of the variable X

For example :

- The function $f : f(X) = 3$ is a polynomial function of zero degree.
- The function $f : f(X) = 2X + 1$ is a polynomial function of the first degree.
- The function $f : f(X) = X^3 - 5X^2 + 1$ is a polynomial function of the third degree.

While :

The function $f : f(X) = \frac{1}{X^2} + X^2$ is not a polynomial function because : $\frac{1}{X^2} = X^{-2}$

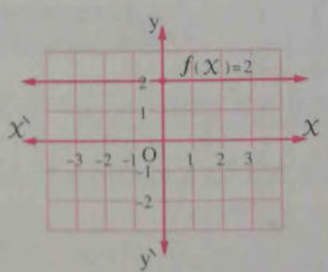
i.e. The index of the symbol X is not a natural number.

Remember **The graphical representation of the polynomial function**

1 The constant function

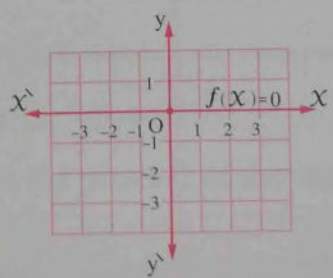
The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = b$, $b \in \mathbb{R}$ is represented by a straight line parallel to X-axis and intersects y-axis at the point $(0, b)$

$f : f(X) = 2$



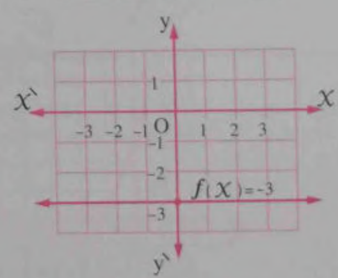
The straight line is above X-axis and passes through the point $(0, 2)$ (is of zero degree)

$f : f(X) = 0$



The straight line is coincident with X-axis and passes through the point $(0, 0)$ (has no degree)

$f : f(X) = -3$



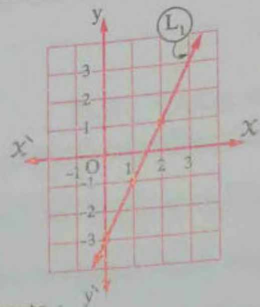
The straight line is below X-axis and passes through the point $(0, -3)$ (is of zero degree)

2 The linear function

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$, $a \in \mathbb{R} - \{0\}$, $b \in \mathbb{R}$ is called a linear function (function of the first degree) and is represented by a straight line intersecting y-axis at (0 , b) and X-axis at $\left(\frac{-b}{a}, 0\right)$

$f : f(X) = 2X - 3$

X	0	1	2
$f(X)$	-3	-1	1

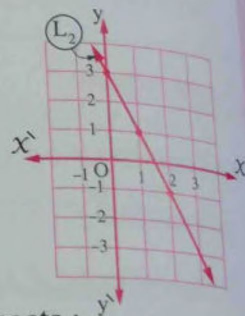


The straight line L_1 intersects :

- X-axis at $\left(1\frac{1}{2}, 0\right)$
- y-axis at (0 , − 3)

$f : f(X) = 3 - 2X$

X	0	1	2
$f(X)$	3	1	-1



The straight line L_2 intersects :

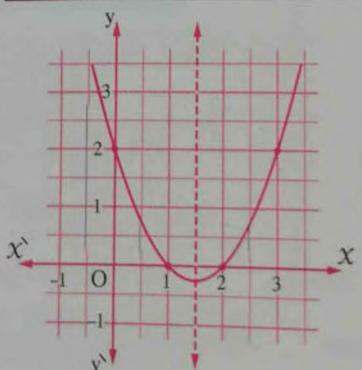
- X-axis at $\left(1\frac{1}{2}, 0\right)$
- y-axis at (0 , 3)

3 The quadratic function

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX^2 + bX + c$, a , b and $c \in \mathbb{R}$, $a \neq 0$ is called a quadratic function and it is a polynomial function of the second degree and it is represented by a curve whose vertex is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$f : f(X) = X^2 - 3X + 2 , X \in [0 , 3]$

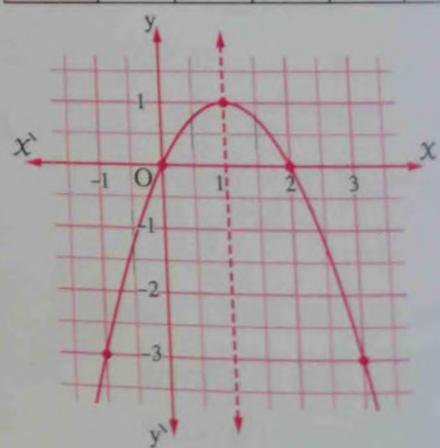
X	0	1	2	3
$f(X)$	2	0	0	2



- The vertex of the curve $= \left(\frac{3}{2}, -\frac{1}{4}\right)$
- The minimum value of the function $= -\frac{1}{4}$
- The equation of line of symmetry : $X = \frac{3}{2}$

$f : f(X) = 2X - X^2 , X \in [-1 , 3]$

X	-1	0	1	2	3
$f(X)$	-3	0	1	0	-3



- The vertex of the curve = (1 , 1)
- The maximum value of the function =
- The equation of line of symmetry : X =

Remember **The ratio and its properties**

- The ratio between the two real numbers a and b is written as a : b or $\frac{a}{b}$ and a is called the antecedent of the ratio , b is called the consequent and a , b are called the two terms of the ratio.
- The value of the ratio **does not change** if each of its terms is multiplied or divided by the same non-zero real number.
- The value of the ratio **changes** if we add or subtract (to or from) each of its two terms the same non-zero real number.
- If the ratio between two numbers is a : b , then : $\boxed{\text{The first number} = am}$, $\boxed{\text{the second number} = bm}$, $m \neq 0$

Example

Two numbers , their sum is 28 and the ratio between them is 3 : 4 , what are the two numbers ?

Solution

Let the two numbers be 3 m , 4 m $\quad \therefore 3\,m + 4\,m = 28 \quad \therefore 7\,m = 28 \quad \therefore m = \frac{28}{7} = 4$

$\therefore$ The two numbers are : 3×4 and 4×4 *i.e.* 12 and 16

Remember **The proportion**

- The proportion is the equality of two ratios or more.
- If $\frac{a}{b} = \frac{c}{d}$, then a , b , c and d are proportional quantities.
- If a , b , c and d are proportional quantities , then $\frac{a}{b} = \frac{c}{d}$

Remember **The properties of the proportion**

Property 1

If $\frac{a}{b} = \frac{c}{d}$, then $a \times d = b \times c$

i.e. the product of the extremes = the product of the means.

Example Find the fourth proportional of the quantities : 3 , 4 and 27

Solution

Let the fourth proportional be x $\quad \therefore$ The quantities : 3 , 4 , 27 and x are proportional

$\therefore \frac{3}{4} = \frac{27}{x} \quad \therefore 3 \times x = 4 \times 27 \quad \therefore x = \frac{4 \times 27}{3} = 36 \quad \therefore$ The fourth proportional = 36

Property 2

If $a \times d = b \times c$, then $\frac{a}{b} = \frac{c}{d}$

Also, each of the following proportions is correct : $\frac{a}{c} = \frac{b}{d}$, $\frac{d}{b} = \frac{c}{a}$, $\frac{b}{a} = \frac{d}{c}$

Example

If $\frac{x + 3y}{2x - y} = \frac{4}{3}$, then find the ratio $x : y$

Solution

$\because \frac{x + 3y}{2x - y} = \frac{4}{3}$ $\quad \therefore 3(x + 3y) = 4(2x - y)$ $\quad \therefore 3x + 9y = 8x - 4y$

$\therefore 13y = 5x$ $\quad \therefore x : y = 13 : 5$

Property 3

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

i.e. $\frac{\text{The antecedent of the first ratio}}{\text{The antecedent of the second ratio}} = \frac{\text{The consequent of the first ratio}}{\text{The consequent of the second ratio}}$

For example : If $\frac{a}{4} = \frac{b}{3}$, then $\frac{a}{b} = \frac{4}{3}$ or $\frac{b}{a} = \frac{3}{4}$

Property 4

If $\frac{a}{b} = \frac{c}{d}$, then $a = cm$, $b = dm$ where m is a constant $\neq 0$

Example

If $a : b = 3 : 5$, then find the ratio $20a - 7b : 15a + b$

Solution

$\because \frac{a}{b} = \frac{3}{5}$

$\therefore a = 3m$, $b = 5m$ where $m \neq 0$

Substituting by a and b in terms of m :

$\therefore \frac{20a - 7b}{15a + b} = \frac{60m - 35m}{45m + 5m} = \frac{25m}{50m} = \frac{1}{2}$

! Remark

If a , b , c and d are proportional quantities and we assume that : $\frac{a}{b} = \frac{c}{d} = m$, then $a = bm$, $c = dm$

For example : If $\frac{a}{b} = \frac{c}{d} = \frac{3}{4}$, then $a = \frac{3}{4} b$, $c = \frac{3}{4} d$

- **Generally :** If a , b , c , d , e , f , ... are proportional quantities and we assume that : $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ... = m$, then $a = bm$, $c = dm$, $e = fm$, ...

Example If a , b , c and d are proportional quantities , **prove that :**

1. $\frac{2a + 3c}{7a - 5c} = \frac{2b + 3d}{7b - 5d}$
2. $\frac{a + c}{b + d} = \frac{a^2 + c^2}{ab + cd}$

Solution Let $\frac{a}{b} = \frac{c}{d} = m$ $\therefore a = bm$, $c = dm$

1. L.H.S. $= \frac{2bm + 3dm}{7bm - 5dm} = \frac{m(2b + 3d)}{m(7b - 5d)} = \frac{2b + 3d}{7b - 5d} =$ R.H.S
2. $\because \frac{a + c}{b + d} = \frac{bm + dm}{b + d} = \frac{m(b + d)}{(b + d)} = m$ (1)

, $\frac{a^2 + c^2}{ab + cd} = \frac{(bm)^2 + (dm)^2}{bm \times b + dm \times d} = \frac{b^2 m^2 + d^2 m^2}{b^2 m + d^2 m} = \frac{m^2 (b^2 + d^2)}{m (b^2 + d^2)} = m$ (2)

From (1) and (2) , we deduce that : $\frac{a + c}{b + d} = \frac{a^2 + c^2}{ab + cd}$

Property 5

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ...$ and m_1 , m_2 , m_3 , ... are non-zero real numbers ,

then $\frac{m_1 a + m_2 c + m_3 e + ...}{m_1 b + m_2 d + m_3 f + ...} =$ one of the given ratios.

Example If $\frac{a + 3b}{X + 5y} = \frac{3b + 5c}{5y + 7z} = \frac{5c + a}{7z + X}$, **prove that :** $\frac{a}{3b} = \frac{X}{5y}$

Solution

Multiplying the two terms of 2^{nd} ratio by (–1) and adding the antecedents and consequents of the three ratios : $\therefore \frac{a + 3b - 3b - 5c + 5c + a}{X + 5y - 5y - 7z + 7z + X} = \frac{2a}{2X} = \frac{a}{X} =$ one of the given ratios. (1)

Multiplying the two terms of 3^{rd} ratio by (–1) and adding the antecedents and consequents of the three ratios : $\therefore \frac{a + 3b + 3b + 5c - 5c - a}{X + 5y + 5y + 7z - 7z - X} = \frac{6b}{10y} = \frac{3b}{5y} =$ one of the given ratios (2)

From (1) and (2) , we deduce that : $\frac{a}{X} = \frac{3b}{5y}$ $\therefore \frac{a}{3b} = \frac{X}{5y}$

Remember **The continued proportion**

- The quantities a , b and c are said to be in continued proportion if $\frac{a}{b} = \frac{b}{c}$

a is called the **first proportional** , c is called the **third proportional** and

b is called the **middle proportional (proportional mean)**

- $\because \frac{a}{b} = \frac{b}{c}$ $\quad\therefore b^2 = ac$ $\quad\therefore b = \pm\sqrt{ac}$

i.e.

The middle proportional between two quantities $= \pm\sqrt{\text{the product of the two quantities}}$

Notice that :

The two quantities a and c should be either positive together or negative together.

- If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$, then $\begin{cases} c = dm \\ b = dm^2 \\ a = dm^3 \end{cases}$

Example

If a , b , c and d are in continued proportion , **then prove that :** $\frac{2a + 3c}{2b + 3d} = \frac{a - c}{b - d}$

Solution

$\because$ a , b , c , d are in continued proportion

$\therefore \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm$, $b = dm^2$, $a = dm^3$

$\therefore \frac{2a + 3c}{2b + 3d} = \frac{2dm^3 + 3dm}{2dm^2 + 3d} = \frac{dm(2m^2 + 3)}{d(2m^2 + 3)} = m$ (1)

, $\frac{a - c}{b - d} = \frac{dm^3 - dm}{dm^2 - d} = \frac{dm(m^2 - 1)}{d(m^2 - 1)} = m$ (2)

From (1) and (2) , we deduce that : $\frac{2a + 3c}{2b + 3d} = \frac{a - c}{b - d}$

Remember **The direct variation and inverse variation**

Direct variation

- If y varies directly as x and is written as $y \propto x$, then :

1. $y = m\,x \left(i.e. \frac{y}{x} = m\right)$ where m is a constant $\neq 0$
2. $\frac{y_1}{y_2} = \frac{x_1}{x_2}$
3. The relation between x and y is represented graphically by a straight line passing through the origin point.

- To prove that $y \propto x$, we prove that : $y = m\,x$ where m is a constant $\neq 0$

For example :

If $y = 5\,x$, then $y \propto x$

Inverse variation

- If y varies inversely as x and is written as $y \propto \frac{1}{x}$, then :

1. $y = \frac{m}{x} \left(i.e.\ x y = m\right)$ where m is a constant $\neq 0$
2. $\frac{y_1}{y_2} = \frac{x_2}{x_1}$
3. The relation between x and y is not a linear relation.

- To prove that $y \propto \frac{1}{x}$, we prove that : $x\,y = m$ where m is a constant $\neq 0$

For example :

If $y = \frac{7}{x}$, then $x\,y = 7$, and then $y \propto \frac{1}{x}$

Example on direct variation

1. If $a \propto b$, $a = 5$ when $b = 2$, **find :** a when $b = 3$
2. If $a^2 + 4\,b^2 = 4\,a\,b$, **prove that :** $a \propto b$

Solution

1. $\because a \propto b \qquad \therefore \frac{a_1}{a_2} = \frac{b_1}{b_2}$

   $\therefore \frac{5}{a_2} = \frac{2}{3} \qquad \therefore a_2 = 7.5$

2. $\because a^2 + 4\,b^2 = 4\,a\,b \qquad \therefore a^2 - 4\,a\,b + 4\,b^2 = 0$

   $\therefore (a - 2\,b)^2 = 0 \qquad \therefore a - 2\,b = 0$

   $\therefore a = 2\,b \qquad \therefore a \propto b$

Example on inverse variation

If x and y are two real variables where : $x^2 y^2 + 25 = 10\,x\,y$

, prove that : x varies inversely as y

Solution

$\because x^2 y^2 - 10\,x\,y + 25 = 0$

$\therefore (x\,y - 5)^2 = 0 \qquad \therefore x\,y - 5 = 0$

$\therefore x\,y = 5 \qquad \therefore x \propto \frac{1}{y}$

Second Statistics

Remember: The resources of collecting data

Primary resources (field resources)

- These are the resources from which we get data directly.

Examples

* Questionnaires and survey.
* Observing and measuring.
* The personal interview.

Secondary resources (historical resources)

- These are the resources from which we get data that previously collected.

Examples

* Central agency for public mobilization and statistics.
* Mass-media.
* Internet.

Remember: The methods of collecting data

	Method of mass population	Method of samples
Definition	It is based on collecting the data related to the phenomenon under study from all individuals of the statistical society.	It is based on collecting the data related to the phenomenon under study from a representative sample of the society (Choosing a sample represented to the whole society)
Usages	• Elections • Census • Setting up a data base of all employees in an organization	• A sample of a patient's blood to make some clinical check up. • A sample of some products of a factory of find out if it matches the standard specifications.
Advantages	• Accuracy • Inclusiveness • Representing all the society individuals	• Saving time , effort and money. • It is the only method for collecting data about large unlimited societies. • It is the only method for collecting data about some limited societies in which mass population method leads to a great loss in it.
Disadvantages	• Sometimes it needs long time , great effort and a great cost.	• The results sometimes are not accurate specially if the sample doesn't represent the statistical society authentically.

Remember **The concept of the sample and the methods of collecting it**

The sample :
It is a small part from a large society that looks like the society and represents it well.

The methods of collecting the sample and its types

Biased selection

The type of the sample :
Not a random sample (deliberate sample)

Its usage :
It is used to select the individuals of the sample in a way to satisfy the objectives of the research.

Randomly selection

The type of the sample :
Simple random sample.

Its usage :
It is used for the homogeneous societies which are not naturally divided into groups or classes.

The type of the sample :
Layer random sample.

Its usage :
It is used in the statistical societies which are heterogeneous or made up of qualitative sets that are different in characteristics.

The number of individuals of the layer in the sample

$= \frac{\text{the total number of individuals in the layer}}{\text{the total number of individuals in the society}} \times$ the number of individuals of the sample

«approximating the result to the nearest unit»

Example

At a faculty , there are 4 000 university students in the first grade , 3 000 in the second grade , 2 000 in the third grade and 1 000 in the fourth grade. If we want to draw a layer sample of 500 students , where each layer is represented in this sample according to its size.

, calculate the number of students in each layer in the sample.

Solution

The total number of students = 4 000 + 3 000 + 2 000 + 1 000 = 10 000 students.

The number of the individuals of the first layer in the sample $= \frac{4\,000}{10\,000} \times 500 = 200$ students.

The number of the individuals of the second layer in the sample $= \frac{3\,000}{10\,000} \times 500 = 150$ students.

The number of the individuals of the third layer in the sample $= \frac{2\,000}{10\,000} \times 500 = 100$ students.

The number of the individuals of the fourth layer in the sample $= \frac{1\,000}{10\,000} \times 500 = 50$ students.

Remember **The dispersion and its measurements**

The dispersion :

It is a measure that expresses how much the sets are homogeneous.

Dispersion measurements

1 The range (the simplest measure of dispersion)

It is the difference between the greatest value and the smallest value in the set.

i.e. The range = the greatest value – the smallest value

For example :

The values of set X are : 55 , 53 , 57 , 56 and 54 , then the range = 57 – 53 = 4

The values of set Y are : 67 , 73 , 41 , 60 and 34 , then the range = 73 – 34 = 39

So the set Y is more divergent than the set X

2 The standard deviation

It is the most important , common and accurate measure of dispersion. We can calculate it by calculating the positive square root of the average of squares of deviations of the values from their mean.

The standard deviation of a set of values	The standard deviation of a frequency distribution
The standard deviation $\sigma = \sqrt{\dfrac{\sum (X - \overline{X})^2}{n}}$	The standard deviation $\sigma = \sqrt{\dfrac{\sum (X - \overline{X})^2 k}{\sum k}}$
Where : X denotes a value of the values , $\overline{X}$ denotes the mean of the values , n denotes the number of the values , $\sum$ denotes the summation operation.	Where : X represents the value or the centre of the set , k represents the frequence of the value or the set , $\sum k$ is the sum of frequences and $\overline{X}$ (the mean) $= \dfrac{\sum (X \times k)}{\sum k}$

Example on the standard deviation of a set of values

Calculate the standard deviation of the values : 55 , 53 , 57 , 56 and 54

Solution

1. We find the mean of the values $(\overline{X}) = \dfrac{\sum X}{n}$

$$= \frac{55 + 53 + 57 + 56 + 54}{5} = 55$$

2. We form the opposite table.

X	$X - \overline{X}$	$(X - \overline{X})^2$
55	55 – 55 = 0	0
53	53 – 55 = – 2	4
57	57 – 55 = 2	4
56	56 – 55 = 1	1
54	54 – 55 = – 1	1
Total		**10**

3. We calculate standard deviation by substituting in the law :

$$\text{The standard deviation } (\sigma) = \sqrt{\frac{\sum (X - \overline{X})^2}{n}} = \sqrt{\frac{10}{5}} = \sqrt{2} \simeq 1.4$$

Example on the standard deviation of a simple frequency distributionmial function

The following table shows the distribution of wages of 20 persons in pounds :

The wage	20	25	30	35	40	45	**Total**
Number of persons	2	3	5	5	1	4	**20**

Find the standard deviation of the wages.

Solution

1. **We find the mean of the wages $(\overline{X})$ by using the opposite table :**

The wage (X)	Number of persons (k)	$X \times k$
20	2	40
25	3	75
30	5	150
35	5	175
40	1	40
45	4	180
Total	**20**	**660**

$$\therefore \text{The mean } (\overline{X}) = \frac{\sum (X \times k)}{\sum k}$$

$$= \frac{660}{20} = 33 \text{ pounds.}$$

2. **We form the opposite table :**

X	k	$X - \overline{X}$	$(X - \overline{X})^2$	$(X - \overline{X})^2 \times k$
20	2	20 – 33 = – 13	169	338
25	3	25 – 33 = – 8	64	192
30	5	30 – 33 = – 3	9	45
35	5	35 – 33 = 2	4	20
40	1	40 – 33 = 7	49	49
45	4	45 – 33 = 12	144	576
Total	**20**			**1220**

3. **We calculate the standard deviation from the law :**

$$\text{The standard deviation } (\sigma) = \sqrt{\frac{\sum (X - \overline{X})^2 \times k}{\sum k}} = \sqrt{\frac{1220}{20}} = \sqrt{61} \simeq 7.8 \text{ pounds.}$$

Example on the standard deviation of a frequency distribution of sets

The following is the frequency distribution of weekly incentives of 100 workers in a factory :

Incentives in pounds	35 –	45 –	55 –	65 –	75 –	85 –	Total
Number of workers	10	14	20	28	20	8	100

Find the standard deviation of this distribution.

Remember that

The centre of the set = $\frac{\text{lower limit} + \text{upper limit}}{2}$

Solution

1 We find the mean ($\overline{X}$)

by using the following table :

Sets	Centres of sets (X)	Frequency (k)	$X \times k$
35 –	40	10	400
45 –	50	14	700
55 –	60	20	1200
65 –	70	28	1960
75 –	80	20	1600
85 –	90	8	720
Total		100	6580

$\therefore$ The mean ($\overline{X}$) = $\frac{\sum (X \times k)}{\sum k} = \frac{6580}{100} = 65.8$ pounds.

2 We form the following table :

X	k	$X - \overline{X}$	$(X - \overline{X})^2$	$(X - \overline{X})^2 \times k$
40	10	$40 - 65.8 = -25.8$	665.64	6656.4
50	14	$50 - 65.8 = -15.8$	249.64	3494.96
60	20	$60 - 65.8 = -5.8$	33.64	672.8
70	28	$70 - 65.8 = 4.2$	17.64	493.92
80	20	$80 - 65.8 = 14.2$	201.64	4032.8
90	8	$90 - 65.8 = 24.2$	585.64	4685.12
Total	100			20036

3 We calculate the standard deviation by using the law :

The standard deviation (σ) = $\sqrt{\frac{\sum (X - \overline{X})^2 \times k}{\sum k}} = \sqrt{\frac{20036}{100}} \simeq 14.15$ pounds.

Notice that :

- The values which are more homogeneous have less dispersion and their standard deviation is small.
- If the standard deviation equals zero that means the all values are equal , it is the perfect homogeneous case (the vanished dispersion)

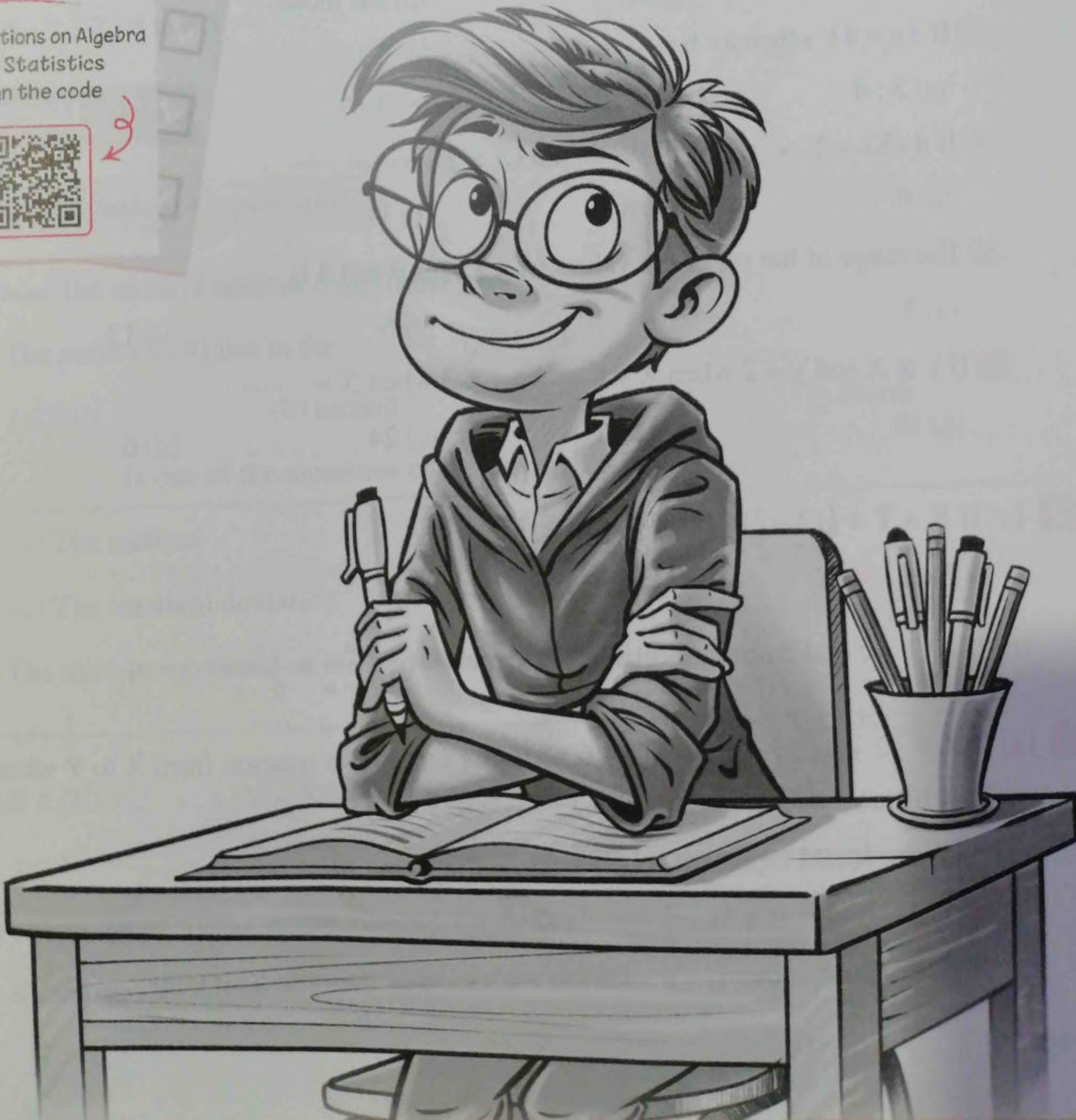
Final Examinations

on Algebra and Statistics

- School book examinations.
- Governorates' examinations.
- Examinations on Port Said specifications.

For more

Examinations on Algebra and Statistics scan the code



Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1 The point (– 3 , 4) lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

2 The positive square root of mean of the squares of deviations of values from its arithmetic mean is called ....

(a) the range. (b) the arithmetic mean.

(c) the standard deviation. (d) the mode.

3 If 3 a = 4 b , then a : b = ...

(a) 3 : 4 (b) 4 : 3 (c) 3 : 7 (d) 4 : 7

4 If n (X) = 2 , n (Y^2) = 9 , then n (X × Y) = ...

(a) 6 (b) 18 (c) 11 (d) 7

5 The range of the set of the values : 7 , 3 , 6 , 9 and 5 is ....

(a) 3 (b) 4 (c) 6 (d) 12

6 If y ∝ X and y = 2 when X = 8 , then y = 3 when X = ...

(a) 16 (b) 12 (c) 24 (d) 6

2 [a] If X × Y = {(2 , 2) , (2 , 5) , (2 , 7)}

, **find :** 1 Y 2 Y × X

[b] If a , b , c and d are proportional , **prove that :** $\frac{a}{b-a} = \frac{c}{d-c}$

3 [a] If X = {2 , 3 , 5} , Y = {4 , 6 , 8 , 10} and R is a relation from X to Y where "a R b" means "2 a = b" for all a ∈ X , b ∈ Y

1 Write R and represent it by an arrow diagram.

2 Show that R is a function.

[b] Find the number that if we add it to each term of the ratio 7 : 11 ...

4 [a] If $X = \{1, 3, 5\}$ and R is a function on X, where $R = \{(a, 3), (b, 1), (1, 5)\}$

, **find :**

1 The range of the function.

2 The value of $a + b$

[b] If $y \propto \frac{1}{X}$ and $y = 3$ when $X = 2$

, **find :**

1 The relation between X and y

2 The value of y when $X = 1.5$

---

5 [a] Represent graphically the function $f : f(X) = (X - 3)^2$, $X \in [0, 6]$, from the graph deduce the vertex of the curve , the minimum value of the function and the equation of the axis of symmetry.

[b] **Calculate the arithmetic mean and the standard deviation of the set of values :**

8 , 9 , 7 , 6 and 5

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The point (3 , 4) lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

2 .... is one of the measures of the dispersion.

(a) The median (b) The arithmetic mean

(c) The standard deviation (d) The mode

3 The third proportional of the two numbers 3 and 6 is ....

(a) $\frac{1}{2}$ (b) 9 (c) 2 (d) 12

4 If $n(X) = 2$, $n(Y \times X) = 6$, then $n(Y^2) =$ ....

(a) 4 (b) 9 (c) 16 (d) 12

5 The range of the set of the values : 7 , 3 , 6 , 9 and 5 is ....

(a) 3 (b) 4 (c) 6 (d) 12

6 If $X y = 7$, then $y \propto$ ....

(a) $\frac{1}{X}$ (b) $X - 7$ (c) X (d) $X + 7$

---

2 [a] If $X = \{2, 5\}$, $Y = \{1, 2\}$, $Z = \{3\}$

, **find :** 1 $n(X \times Z)$

2 $(Y \cap X) \times Z$

[b] If b is the middle proportional between a and c , **prove that :** $\frac{a-b}{a-c} = \frac{b}{b+c}$

---

3 [a] If $X = \{1, 3, 4, 5\}$, $Y = \{1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y where "a R b" means "$a + b = 7$" for all $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Show that R is a function.

[b] If $5a = 3b$, **find the value of :** $\frac{7a + 9b}{4a + 2b}$

---

4 [a] If $f(X) = 4X + b$ and $f(3) = 15$, **find the value of :** b

[b] If $y \propto X$, $y = 6$ when $X = 3$, **find :**

1 The relation between X and y

2 The value of y when $X = 5$

---

5 [a] Represent graphically the function $f : f(X) = 4 - X^2$, $X \in [-3, 3]$, from the graph deduce the vertex of the curve , the maximum value of the function and the equation of the axis of symmetry.

[b] **The following frequency distribution shows the number of children of some families in a new city :**

Number of children	0	1	2	3	4	Total
Number of families	6	15	40	25	14	100

Calculate the mean and the standard deviation of the number of children.

Model for the merge students

Answer the following questions :

1 Complete :

1. The point (5 , 3) lies in .... quadrant.
2. n : n (X) = $X^3 + 8$ is called a polynomial function of .... degree.
3. The range of the set of the values : 4 , 14 , 25 and 34 is ....
4. If y = 2 X , then y $\propto$ ....
5. If X = $\{2 , 4 , 6\}$, then n (X^2) = ...
6. If (a , 3) = (6 , b) , then a + b = ...

2 Choose the correct answer from those given :

1. If X y = 7 , then y $\propto$ ....

   (a) $\frac{1}{X}$ (b) $X - 7$ (c) X (d) $X + 7$

2. If 2 , 3 , 6 and X are proportional , then X = ...

   (a) 9 (b) 18 (c) 12 (d) 3

3. If 2 a = 5 b , then $\frac{a}{b}$ = ...

   (a) $\frac{-5}{2}$ (b) $\frac{-2}{5}$ (c) $\frac{2}{5}$ (d) $\frac{5}{2}$

4. .... is one of the measures of the dispersion.

   (a) The arithmetic mean (b) The range
   (c) The mode (d) The median

5. If n (X) = 5 , n (X × Y) = 10 , then n (Y) = ...

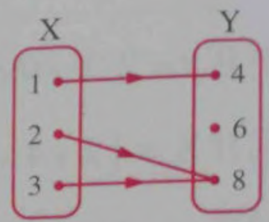
   (a) 4 (b) 3 (c) 2 (d) 1

6. If X = $\{1\}$, then X^2 = ...

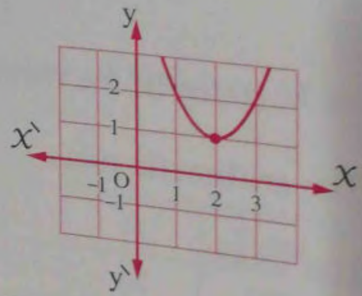
   (a) 1 (b) (1 , 1) (c) $\{(1 , 1)\}$ (d) $\{1\}$

3 **Put (✓) or (✗) :**

1. If the function $f = \{(1, 3), (2, 4), (3, 3)\}$, then the domain of the function is $\{1, 2, 3\}$ ( )
2. If $y \propto x$ and $y = 6$ when $x = 3$, then $y = 2$ when $x = 4$ ( )
3. If $\sum (x - \overline{x})^2 = 36$ for a set of values whose number equals 9, then $\sigma = 4$ ( )
4. The intersection point of the straight line $f(x) = x + 2$ with x-axis is the point $(-2, 0)$ ( )
5. If $f : X \longrightarrow Y$, then X is called the domain of this function. ( )
6. The arrow diagram from X to Y represents a function. ( )



4 **Join from column (A) to column (B) :**

(A)	(B)
1. If $(1, 4) \in \{2, x\} \times \{1, 4\}$, then $x =$	• 6
2. If the function f where $f(x) = x - 4$ is represented graphically by a straight line passing through the point $(a, 2)$, then $a =$	• 1
3. $\frac{1}{2} = \frac{3}{6} = \frac{4}{8} = \frac{\cdots\cdots}{16}$	• 10
4. If $f(x) = 5$, then $f(5) + f(-5) =$	• ± 6
5. The middle proportional of the two numbers 4 and 9 is	• 2
6. **In the opposite figure :** The equation of the line of symmetry is $x =$ .... 	• 8

1 Cairo Governorate

Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer from those given :

1. If $2^{x} = 8$, then $x^2 =$ ....
   (a) 2 (b) 3 (c) 4 (d) 9
2. The degree of the algebraic term $4\,x^2\,y^3$ is ....
   (a) second. (b) third. (c) fourth. (d) fifth.
3. If the point (k – 2 , 4) lies on the y-axis , then k = ...
   (a) 2 (b) 4 (c) 6 (d) 8
4. The middle proportional of the two quantities a , c is ....
   (a) $\pm$ a c (b) $\pm\sqrt{a\,c}$ (c) $\frac{a+c}{2}$ (d) $\frac{1}{2}$ a c
5. The difference between the greatest value and the smallest value for a set of values is called the ....
   (a) range. (b) median.
   (c) arithmetic mean. (d) standard deviation.
6. $\mathbb{R} - \mathbb{Q} =$ ....
   (a) $\mathbb{Z}$ (b) $\grave{\mathbb{Q}}$ (c) $\varnothing$ (d) $\mathbb{R}^+$

2 [a] Find the number which if added to each of the two terms of the ratio 5 : 11 , it becomes 4 : 7

[b] If X = {1 , 2 , 3} and Y = {2 , 3 , 4 , 5} and R is a relation from X to Y where "aRb" means "a + b = 5" for each a ∈ X , b ∈ Y

1. Write R and represent it by an arrow diagram.
2. Show that R is a function.

3 [a] Find the fourth proportional of the quantities : 3 , 5 , 6

[b] If X × Y = {(2 , 1) , (2 , 4) , (2 , 5)} , **find :**

1. Y
2. Y × X
3. $n(Y^2)$

4 [a] If y varies inversely as x and y = 4 when $x = 3$

1. Write the relation between y and x
2. Find the value of y when $x = 6$

[b] If $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$, **prove that :** $\frac{2x+y}{7} = \frac{2y+z}{11}$

5 [a] Graph the curve of the function $f : f(X) = (X-3)^2$, where $X \in [1 , 5]$ **, from the graph find :**

1 The equation of the axis of symmetry of the curve.

2 The minimum value of the function.

[b] **Calculate the standard deviation for the values : 6 , 4 , 5 , 3 , 7**

2 Giza Governorate

Answer the following questions :

1 Choose the correct answer :

1 If $2^X = 1$, then $X =$ ....

(a) zero (b) 1 (c) 2 (d) 3

2 If $\sqrt{X} = 3$, then $X =$ ....

(a) 3 (b) 6 (c) 9 (d) $\sqrt{3}$

3 $\{2\} \times \{5\} =$ ....

(a) $\{10\}$ (b) $\{7\}$ (c) $\{52\}$ (d) $\{(2 , 5)\}$

4 If $X y = 5$, then $y \propto$ ....

(a) $\frac{1}{X}$ (b) X (c) $X + 5$ (d) $\frac{X}{5}$

5 If $\frac{a}{2} = \frac{b}{5} = \frac{2a+b}{k}$, then k = ...

(a) 3 (b) 4 (c) 7 (d) 9

6 The range of the set of the values : 7 , 3 , 6 , 5 and 9 is ....

(a) 3 (b) 9 (c) 6 (d) 12

2 [a] If $\frac{X}{3} = \frac{y}{4} = \frac{c}{5}$ **, then find the value of :** $\frac{2X + 3y}{7c - 2y}$

[b] If $X = \{1 , 2 , 3 , 4\}$, $Y = \{1 , 8 , 9 , 27 , 64\}$ and R is a relation from X to Y where "aRb" means "$a^3 = b$" for each $a \in X$ and $b \in Y$, then :

1 Write R and represent it by an arrow diagram.

2 Is R a function ? and if the relation is a function , then find its range.

3 [a] If $y \propto X$ and y = 6 when $X = 2$ **, then find :**

1 The relation between y and X

2 The value of y when $X = 5$

[b] If b is the middle proportional between a and c **, then prove that :** $\frac{a-b}{a-c} = \frac{b}{b+c}$

4 [a] If $(2x-1, x+y)=(5, 8)$, **then find the value of : y**

[b] If $\frac{x-2y}{x+3y}=\frac{3}{5}$ **, then find the value of :** $x : y$

---

5 [a] **Find the arithmetic mean and the standard deviation of the values :**

$2, 4, 6, 8$

[b] Represent graphically the function $f : f(x)=x^2-4x+3$, taking $x \in [0, 4]$

, and from the graph find :

1 The minimum value of the function.

2 The equation of the axis of symmetry of the function.

3 Alexandria Governorate

Answer the following questions : **(*Calculators are permitted*)**

1 Choose the correct answer from those given :

1 The range of the set of values : 7 , 3 , 6 , 9 and 5 equals ....

(a) 3 (b) 6 (c) 9 (d) 12

2 If $a+3b=7$, $c=3$, then the numerical value of the expression : $a+3(b+c)=$ ....

(a) 10 (b) 16 (c) 21 (d) 30

3 $2^x+2^x=$ ....

(a) 4^x (b) 2^{2x} (c) 2^{2x+1} (d) 2^{x+1}

4 If $(x+5, 8)=(1, 6y+x)$, then $x+y=$ ....

(a) 8 (b) -2 (c) -4 (d) 6

5 If $x-y=5$, $x+y=2$, then $x^2-y^2=$ ....

(a) 10 (b) 3 (c) 2 (d) 5

6 The third proportional of the two numbers 3 , 6 is ....

(a) $\frac{1}{2}$ (b) 9 (c) 2 (d) 12

---

2 [a] If $X=\{0, 1, 2, 3\}$, $Y=\{0, 1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y where "aRb" means "$a=\frac{1}{2}b$" for all $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Show that R is a function , and why ?

[b] If $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$ **, prove that :** $\frac{2y-z}{3x-2y+z}=\frac{1}{2}$

3 [a] If $f(X) = X^2 - 3X$, $g(X) = X - 3$, **find :** $f(\sqrt{2}) + 3\,g(\sqrt{2})$

[b] Find the positive number which if its square is added to each of the two terms of the ratio 5 : 11 , it becomes 3 : 5

4 [a] Represent graphically the function $f : f(X) = -X^2 - 2X$ where $X \in [-4, 2]$

, from the graph deduce :

1 The coordinates of the vertex point of the curve.

2 The equation of the axis of symmetry.

3 The maximum value of the function.

[b] If a , b , c and d are continued proportional quantities , **prove that :** $\frac{a^2 - 3c^2}{b^2 - 3d^2} = \frac{b}{d}$

5 [a] If $y \propto \frac{1}{X}$ and y = 3 when $X = 2$, **find :**

1 The relation between y and X

2 The value of y when $X = 1.5$

[b] **Calculate the standard deviation for the values :** 13 , 14 , 17 , 19 , 22 (rounding the result to three decimal place).

4 El-Kalyoubia Governorate

Answer the following questions :

1 **Choose the correct answer from the given ones :**

1 If $X = \{2\}$, $Y = \{3, 4\}$, then $n(X^2) \times n(Y) =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

2 If $2^{X-4} = \frac{1}{16}$, then $X =$ ....

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

3 The middle proportional between the two numbers 3 and 12 is ....

(a) ± 3 (b) ± 4 (c) ± 6 (d) ± 12

4 The solution set of the equation : $X - 1 = |-1|$ in $\mathbb{N}$ is ....

(a) $\{0\}$ (b) $\{1\}$ (c) $\{2\}$ (d) $\varnothing$

5 If $-1 < X < 3$, $X \in \mathbb{R}$, then $(X + 1) \in$ ....

(a) $\{0, 3\}$ (b) $[-1, 3[$ (c) $\{0, 4\}$ (d) $]0, 4[$

6 The positive square root of the average of squares of deviations of the values from mean is called the ....

(a) range. (b) arithmetic mean.

(c) standard deviation. (d) mode.

---

2 [a] If $X = \{2, -1\}$, $Y = \{-1, 5\}$, $Z = \{2, 3\}$

, **find :** 1 $X \times Y$ 2 $(X - Y) \times Z$

[b] If $y \propto X$ and $y = 5$ when $X = 15$ **, find :**

1 The relation between X and y 2 The value of y when $X = 30$

---

3 [a] If $X = \{-4, -2, 0, 2, 4\}$ and R is a relation on X where "aRb" means "a is the additive inverse of b" where $a \in X$, $b \in X$, write R and represent it by an arrow diagram and show if R is a function or not.

[b] If $\frac{a}{b} = \frac{c}{d}$ **, prove that :** $\frac{a+b}{c+d} = \frac{b}{d}$

---

4 [a] Find the number that if added to each of the two terms of the ratio 7 : 11 , then it becomes 4 : 5

[b] If 2 , a , b , 54 are in continued proportion **, find the value of :** a + b

---

5 [a] Graph the function $f : f(X) = X^2 + 2X - 3$, taking $X \in [-4, 2]$ **, then find :**

1 The minimum value of the function.

2 The equation of the axis of symmetry.

[b] Calculate the arithmetic mean and the standard deviation of the values :

12 , 13 , 16 , 18 , 21

5 El-Sharkia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from those given :

1 If $X y = 3$, then $X \propto$ ....

(a) y (b) $\frac{1}{y}$ (c) y^2 (d) $\frac{1}{y^2}$

2 If the point (k − 2 , 3 k − 2) is at a distance of 4 length units from X-axis , then k = ...

(a) 0 (b) 1 (c) 2 (d) 3

3 If $a : b = 2 : 3$, $b : c = 5 : 6$, then $a : c =$ ....

(a) $1 : 3$ (b) $3 : 5$ (c) $2 : 3$ (d) $5 : 9$

4 If the standard diviation for some values = 3 and the number of these values = 2 , then $\sum (X - \overline{X})^2 =$ ....

(a) 1 (b) 18 (c) 12 (d) 24

5 The result of $\frac{3^{2X} + 3^{2X} + 3^{2X}}{3^{X} \times 3^{X}}$ in the simplest form is ....

(a) 3^{4X} (b) 3^{2X} (c) 3 (d) $\frac{1}{3}$

6 If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 2X + 3 + c$ passes through the origin point , then c = ...

(a) -2 (b) -3 (c) 0 (d) 3

---

2 [a] If $\frac{a}{3} = \frac{b}{2} = \frac{c}{5}$, **prove that :** $\frac{a - 2b + 3c}{2a + b + c} = \frac{14}{13}$

[b] If $(X - Y) \times Y = \{(1 , 2) , (1 , 3)\}$, $n(X \times Y) = 6$
, **find :** 1 X , Y 2 $(X \cap Y) \times Y$

---

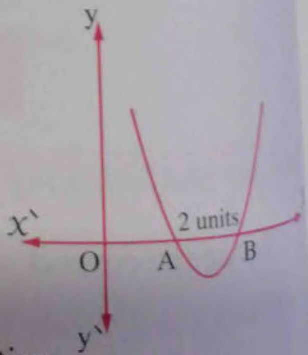
3 [a] If $y = a + 2$, $a \propto X$, write the relation between a and X when $X = 2$ and $a = 4$, then find y at $X = 1$

[b] If $X = \{a : a \in \mathbb{Z} , -2 \le a \le 2\}$ and R is a relation on X where "aRb" means "a is the additive inverse of b" for all $a \in X$, $b \in X$, write R and represent it by an arrow diagram , and show if R is a function or not , give reason.

---

4 [a] If a , b , c and d are in continued proportion
, **prove that :** $\frac{a^2 - 3c^2}{b^2 - 3d^2} = \frac{b}{d}$

[b] **In the opposite figure :**



f is a quadratic function
where $f(X) = X^2 - 6X + m$
, the length of $\overline{AB} = 2$ unit length.
find the value of m , then find the minimum value of the function.

---

5 [a] Find the number which if added to each of the numbers 3 , 5 , 8 , 12 , it will make them proportional.

[b] **Calculate the mean and the standard deviation for the values :** 12 , 13 , 16 , 18 ,

6 El-Gharbia Governorate

Answer the following questions :

1 Choose the correct answer from the given ones :

1. If $\left(2^{X}, \sqrt{y}\right) = (1, 1)$, then $X - y =$ ....
   (a) zero (b) 1 (c) -1 (d) ± 1
2. If $X = \{1, 3\}$, then $n(X^2) =$ ....
   (a) 2 (b) 4 (c) 3 (d) 10
3. If $f(X) = 1$, then $f(1) + f(2) =$ ....
   (a) 1 (b) 2 (c) 3 (d) 4
4. $]-1, 3] \cap \{-3, -1\} =$ ....
   (a) $\varnothing$ (b) $\{-3\}$ (c) $\{-1\}$ (d) $\{3\}$
5. If $X y = 3$, then $y \propto$ ....
   (a) X^{-1} (b) X (c) $3X$ (d) X^2
6. Half the number $4^{20} =$ ....
   (a) 2^{20} (b) 2^{29} (c) 2^{19} (d) 2^{39}

---

2 [a] If $X = \{1, 2, 3\}$, $Y = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{5}\right\}$ and R is a relation from X to Y where "aRb" means "a is the multiplicative inverse of b" for each $a \in X$, $b \in Y$, write R and represent it by an arrow diagram , show if R is a function or not , and why ?

[b] If b is the middle proportional between a and c , **prove that :** $\frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{2a}{c}$

---

3 [a] If y varies inversely as X , and $y = 10$ when $X = 3$, find the relation between y and X , then find also y when $X = 5$

[b] Represent graphically the function $f : f(X) = (X - 2)^2$, $X \in [0, 4]$ **, from the graph deduce :**

1. The coordinates of the vertex point of the curve.
2. The equation of the axis of symmetry.

---

4 [a] Find the number which if we add it to each term of the ratio 3 : 7 , it becomes 1 : 2

[b] If $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$

, find : 1 X , Y 2 $Y \times X$ 3 X^2

[a] If 5 a = 3 b , then find : (7 a + 9 b) : (4 a + 2 b)

[b] Calculate the mean and the standard deviation for the data : 4 , 8 , 12 , 10 , 6 (rounding the result to one decimal place).

7 El-Dakahlia Governorate

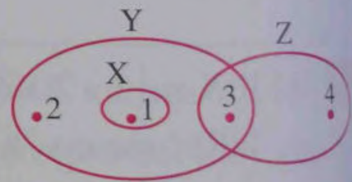
Answer the following questions : *(Calculator is permitted)*

1 [a] Choose the correct answer :

1. The range of the set of values : 23 , 22 , 15 , 18 , 17 is ....
   (a) 8 (b) 18 (c) 19 (d) 23
2. If $f(X) = 2X - 1$, $g(X) = 4$, then $f(g(X)) =$ ....
   (a) 7 (b) 4 (c) – 4 (d) – 7
3. If $X = \{a , a^3\}$, then a may be equal to ....
   (a) – 1 (b) zero (c) 1 (d) 2

[b] In the opposite figure :

By using Venn diagram which represents the sets X , Y and Z , find :
1. $(X \cap Y) \times Z$
2. $(X \cup Y) \times (Z - Y)$



2 [a] Choose the correct answer :

1. If 10 grams of chocolate give 300 calories , then the number of calories which are found in 30 grams of the same chocolate equals ....
   (a) 90 (b) 100 (c) 900 (d) 9000
2. The ratio between the circumference of the circle : the length of its diameter = ...
   (a) $\pi : 1$ (b) $1 : \pi$ (c) $2\pi : 1$ (d) $1 : 2\pi$
3. If $\frac{a}{b} = \frac{3}{5}$, $5a - 2b = 20$, then b = ...
   (a) 3 (b) 5 (c) 15 (d) 20

[b] If b is the middle proportional between a and c , prove that : $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

3 [a] **Find the standard deviation for the values :** 5 , 6 , 7 , 8 , 9

[b] If $X = \{-1 , 0 , 1\}$ and R is a relation on X where "aRb" means "$b = a^{2}$" for each $a \in X$, $b \in X$, write R and show with reason if R is a function or not , and if R is a function , mention its range.

4 [a] If $\frac{X + y}{9} = \frac{y + z}{7}$, **prove that :** $\frac{X - z}{X + 2y + z} = \frac{1}{8}$

[b] A car moves with uniform velocity where the distance varies directly with the time. If the car covered a distance of 150 km. in 6 hours , find the distance covered by that car in 10 hours.

5 [a] If $X^4 y^2 - 14 X^2 y + 49 = 0$, **prove that :** y varies inversely with X^2

[b] **In the opposite figure :**

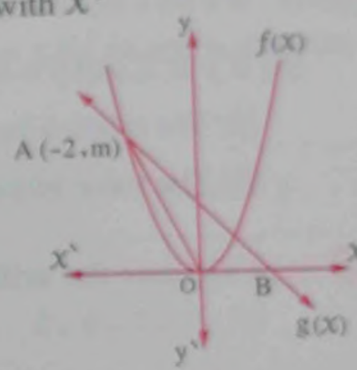
The curve represents the quadratic function $f : f(X) = X^2$

, $\overrightarrow{AB}$ represents the linear function $g : g(X) = k - X$

If $A(-2 , m)$

, find : **1** The values of k , m

2 The area of Δ AOB



8 Kafr El-Sheikh Governorate

Answer the following questions : *(Calculators are permitted)*

1 **Choose the correct answer from the given ones :**

1 The multiplicative inverse of 2 is ....

(a) -2 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 2

2 If $n(X) = 5$, $n(X \times Y) = 10$, then $n(Y) =$ ....

(a) 2 (b) 3 (c) 4 (d) 5

3 The degree of the algebraic term $3 X y^2$ is .... degree.

(a) the second (b) the third (c) the fourth (d) the fifth

4 If $X y = 5$, then $y \propto$ ....

(a) X (b) $\frac{1}{X}$ (c) $5 X$ (d) $\frac{1}{5} X$

5 Half of the number 4^{10} is ....

(a) 2^5 (b) 2^{10} (c) 2^{19} (d) 4^5

6 .... is one of the measures of the dispersions.

(a) The arithmetic mean (b) The median

(c) The mode (d) The range

2 [a] If $X = \{1, 3, 4, 5\}$, $Y = \{1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y where "aRb" means "$a + b = 7$" for all $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Show if R is a function or not, and why ?

[b] If $\frac{a}{b} = \frac{3}{5}$, **find the value of :** $\frac{4a + 2b}{7a + 9b}$

3 [a] If a, b, c and d are proportional, **prove that :** $\frac{a-b}{c-d} = \frac{a}{c}$

[b] If $y \propto X$ and $y = 10$ when $X = 5$, **find :**

1 The relation between X and y

2 The value of y when $X = 3$

4 [a] **Calculate the arithmetic mean and the standard deviation for the values :**
15, 9, 7, 6, 3

[b] If $f(X) = 2X + c$ and $f(1) = 7$

1 Find the value of c

2 Find the value of $f(2)$

5 [a] If b is the middle proportional between a and c, **prove that :** $\frac{b^2 + c^2}{a^2 + b^2} = \frac{c}{a}$

[b] Represent graphically the function $f : f(X) = X^2 - 4$, where $X \in [-3, 3]$, from the graph deduce the vertex of the curve.

9 El-Beheira Governorate

Answer the following questions : *(Calculator is permitted)*

1 **Choose the correct answer from the given ones :**

1 If $n(X) = 3$, $n(X \times Y) = 12$, then $n(Y) =$ ....

(a) 9 (b) 4 (c) 15 (d) 36

2 If $3a - 4b = 0$, then $\frac{a}{b} =$ ....

(a) $\frac{3}{4}$ (b) $\frac{-3}{4}$ (c) $\frac{4}{3}$ (d) $\frac{-4}{3}$

3 The range of the set of the values : 7, 3, 6, 9 and 5 equals ....

(a) 3 (b) 4 (c) 6 (d) 5

4 The solution set of the equation : $(X-1)^2=9$ in $\mathbb{R}$ is ....

(a) $\{4\}$ (b) $\{-2\}$ (c) $\{4,-2\}$ (d) $\{3\}$

5 If $\frac{y}{X}=5$ where $X \neq$ zero , then y $\propto$ ....

(a) X (b) $X-5$ (c) $X+5$ (d) $\frac{1}{X}$

6 If $X^3=27$, $\sqrt{y}=3$, then $X+y=$ ....

(a) 6 (b) 9 (c) 30 (d) 12

---

2 [a] Find the positive number which if we add its square to each of the terms of the ratio 5 : 7 , it becomes 7 : 8

[b] If $X=\{2,3,4\}$, $Y=\{6,9,12,15\}$ and R is a relation from X to Y where "aRb" means "3 a = b" for each a $\in$ X , b $\in$ Y , write R and represent it by an arrow diagram , show that R is a function from X to Y

---

3 [a] If y $\propto$ X and y = 6 when $X=3$, find the relation between y and X , the value of y when $X=5$

[b] If $\frac{X}{2}=\frac{y}{5}=\frac{z}{7}$, **prove that :** $\frac{5y-3z}{2z-3X}=\frac{1}{2}$

---

4 [a] If $X=\{3,4\}$, $Y=\{4,5\}$, $Z=\{5,6,7\}$, **find :**

1 $X \times (Y \cap Z)$ 2 $(X-Y) \times Z$ 3 $n(Z^2)$

[b] If b is the middle proportional between a and c , **prove that :** $\frac{a^2+b^2}{b^2+c^2}=\frac{a}{c}$

---

5 [a] **Calculate the standard deviation for the following values :** 16 , 32 , 5 , 20 , 27

[b] Represent graphically the function $f : f(X)=(X-2)^2$, where $X \in [-1,5]$

, from the graph find :

1 The vertex of the curve.

2 The minimum value of the function and the equation of the axis of symmetry.

10 El-Menia Governorate

Answer the following questions : *(Calculator is allowed)*

1 **Choose the correct answer from the given ones :**

1 If m represents a negative number , which of the following represents a positive number ?

(a) m^3 (b) m^2 (c) 2 m (d) $\frac{m}{2}$

2 .... is one of the measures of the dispersions.

(a) The median
(b) The arithmetic mean
(c) The standard deviation
(d) The mode

3 If the total cost of a trip is (y) , some of it is constant (a) and the other is directly proportional with the number of participants (X) , then .... (m is a constant $\neq 0$)

(a) $y = a X$
(b) $y = \frac{a}{X}$
(c) $y = a + \frac{m}{X}$
(d) $y = a + m X$

4 If $2^X = \frac{1}{8}$, then $X =$ ....

(a) $\frac{1}{2}$
(b) $\frac{1}{3}$
(c) 3
(d) -3

5 If $X - y = 5$, $X + y = \frac{1}{5}$, then $X^2 - y^2 =$ ....

(a) $\frac{1}{25}$
(b) 1
(c) 25
(d) 5

6 If the point $(X - 4 , 2 - X)$ is located in the third quadrant , where $X \in \mathbb{Z}$, then $X =$ ....

(a) 2
(b) 3
(c) 4
(d) 6

---

2 [a] If $X = \{1 , 2 , 3\}$, $Y = \{1 , 3 , 6 , 9 , 12\}$ and R is a relation from X to Y where "aRb" means "$a = \frac{1}{3}$ b" for each $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Is R a function ? And why ?

[b] If $y \propto X$ and $y = 14$ when $X = 42$, **find :**

1 The relation between y and X

2 The value of y when $X = 60$

---

3 [a] If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 4X + a$ intersects the X-axis at the point (2 , b) , **find the value of each of :** a , b

[b] **Calculate the standard deviation of the values :** 8 , 9 , 7 , 6 and 5

---

4 [a] If b is the middle proportional between a and c

, **prove that :** $\left(\frac{b - c}{a - b}\right)^2 = \frac{c}{a}$

[b] If $X^4 y^2 - 14 X^2 y + 49 = 0$, **prove that :** $y \propto \frac{1}{X^2}$

---

5 [a] If a : b = 3 : 5 , **find the ratio :** 20 a − 7 b : 15 a + b

[b] Represent the function $f : f(X) = X^2 - 2$ graphically taking $X \in [-3 , 3]$, and from the graph , deduce the coordinates of the vertex of the curve and the maximum or minimum value of the function.

11 Souhag Governorate

Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer :

1 If $|x| - 4 = 3$, then $x =$ ....

(a) 7 (b) -7 (c) ± 7 (d) 1

2 If $f(x) = 3$, then $f(5) + f(-5) =$ ....

(a) 6 (b) 1 (c) zero (d) -1

3 $\sqrt[3]{125} + \sqrt[3]{\cdots\cdots} = \sqrt{64}$

(a) 8 (b) 3 (c) 9 (d) 27

4 If $x y = 5$, then y changes inversely with ....

(a) $\frac{1}{x}$ (b) x (c) $5x$ (d) $\frac{x}{5}$

5 If $x^2 + y^2 = 25$, $x y = 12$, then $(x - y)^2 =$ ....

(a) 1 (b) 5 (c) 13 (d) 37

6 If all the individuals are equal in value , then ....

(a) $\overline{x} - x > 0$ (b) $\overline{x} - x < 0$ (c) $\sigma = 0$ (d) $\overline{x} = 0$

---

2 [a] If $X = \{3\}$, $Y = \{4, 5\}$, $Z = \{6, 5\}$, **find :**

1 $(X \cap Y) \times Z$ 2 $X \times (Y - Z)$ 3 $n(X^2)$

[b] If $\frac{x - 3y}{x + 2y} = \frac{2}{3}$, **find the value of :** $\frac{x}{y}$

---

3 [a] If a , b , c and d are proportional quantities , **prove that :** $\frac{3a - 2c}{5a + 3c} = \frac{3b - 2d}{5b + 3d}$

[b] If $X = \{0, 1, 2, \frac{1}{2}\}$ and R is a relation on X where "aRb" means "a is the multiplicative inverse of b" for all $a \in X$, $b \in X$

1 Write R as a set of ordered pairs , then represent it by an arrow diagram.

2 Show that R is a function or not ? Why ?

---

4 [a] If a , 2 , 4 , b are in continued proportion , **find :** a + b

[b] Represent the function $f : f(x) = (x + 1)^2$ where $x \in [-4, 2]$

and from the graph deduce :

1 The coordinates of the vertex of the curve.

2 The maximum or the minimum value of the function.

3 The equation of the axis of symmetry.

5 [a] If $y \propto X$ and $y = 20$ when $X = 4$, find :

1 The relation between y and X

2 The value of X when $y = 40$

[b] The following table represents the frequency distribution of the ages of 10 children :

Ages in years	5	8	9	10	12	Total
No. of children	1	2	3	3	1	10

Calculate the standard deviation to ages in years.

12 Qena Governorate

Answer the following questions : (Calculators are permitted)

1 **Choose the correct answer from those given :**

1 If X , Y are two sets non empty and $n(X) = 2$, $n(Y^2) = 9$, then $n(X \times Y) =$ ....

(a) 3 (b) 4 (c) 6 (d) 18

2 $[-2 , 3] - \{-2 , 5\} =$ ....

(a) $[-2 , 3[$ (b) $]-2 , 3[$ (c) $]-2 , 5[$ (d) $]-2 , 3]$

3 If $y \propto X$ and $X = 3$ when $y = 2$, then the constant proportional equals ....

(a) 2 (b) 3 (c) $\frac{2}{3}$ (d) 6

4 $(\sqrt{3} - 1)^2 =$ ....

(a) $4 - 2\sqrt{3}$ (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{3} + 1$

5 If the standard deviation for the values : $X + 1$, y , 4 equals zero , then $X y =$ ....

(a) 4 (b) 12 (c) 16 (d) 20

6 The sum of all real numbers in the interval $]-2 , 2]$ equals ....

(a) 2 (b) -2 (c) zero (d) can not sum

2 [a] If $X = \{-2 , -1 , 0 , 1 , 2 , 3\}$ and R is a relation on X where "aRb" means "a is the additive inverse of b" for each $a \in X$, $b \in X$, write R and show it by an arrow diagram.

Is R a function or not ? And if it is a function , find its range.

[b] If b is the middle proportional between a , c , **then prove that :** $\frac{a^3 + b^3}{b^3 + c^3} = \frac{a^2}{bc}$

3 [a] If the straight line showing the function $f : f(X) = 2X - b$ intersects X-axis at the point $(1 , a - 3)$, **then find the values of :** a , b

[b] If a , b , c and d are proportional quantities , **prove that :** $\frac{a + b}{b} = \frac{c + d}{d}$

4 [a] If $y \propto \frac{1}{X}$ and $y = 3$ when $X = 2$, then find the relation between X and y , then find the value of y when $X = 1.5$

[b] **Find the mean and the standard deviation for the following values :** 3 , 6 , 4 , 7 , 5

5 [a] If $\frac{X}{y} = \frac{2}{3}$ **, then find the value of :** $\frac{3X + 2y}{6y - X}$

[b] Represent graphically the function $f : f(X) = 3 - X^2$. Let $X \in [-2 , 2]$, from the graph find the vertex of the curve , the maximum or minimum value of the function and the equation of the axis of symmetry.

13 Aswan Governorate

Answer the following questions : **(Calculator is allowed)**

1 **Choose the correct answer from those given :**

1 If $2Xy = 5$, then $y \propto$ ....

(a) $\frac{1}{X}$ (b) $X - 5$ (c) X (d) $X + 5$

2 $2^3 \times 2^5 =$ ....

(a) 2^{15} (b) 2^2 (c) 4^8 (d) 2^8

3 The range of the set of values : 7 , 3 , 6 , 5 and 9 equals ....

(a) 3 (b) 5 (c) 6 (d) 7

4 $\frac{1}{2} + \frac{1}{4} =$ .... %

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 75

5 If $(3 , b - 1)$ lies on X-axis , then b = ...

(a) 3 (b) -3 (c) -1 (d) 1

6 $]2 , 5] \cup \{2\} =$ ....

(a) $]2 , 5[$ (b) $[2 , 5]$ (c) $\{2\}$ (d) $]2 , 5]$

2 [a] If $X = \{2 , 3 , 4\}$, $Y = \{2 , 3 , 4 , 5 , 6 , 7 , 8\}$ and R is a relation from X to Y where "aRb" means "$a = \frac{1}{2}$ b" for each $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Show that R is a function from X to Y and find its range.

[b] If $y \propto \frac{1}{X}$ and $y = 6$ when $X = 2$ **, find :**

1 The relation between X and y

2 The value of y when $X = 3$

3 [a] Represent graphically the quadratic function f where $f(X) = X^2 + 2X + 1$, taking $X \in [-4, 2]$ and from the graph deduce the coordinates of the vertex of the curve , the maximum or minimum value of the function and the equation of the symmetry axis.

[b] If b is the middle proportional between a and c

, **prove that :** $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

4 [a] If $f(X) = X^2 - 3X$, $g(X) = X - 3$, **prove that :** $f(3) = g(3)$

[b] If $\frac{a}{b} = \frac{3}{5}$, **then find the value of :** $\frac{7a + 9b}{4a + 2b}$

5 [a] If $X = \{1\}$, $Y = \{2, 3\}$, $Z = \{2, 5, 6\}$

, **find :** 1 $n(Y^2)$ 2 $X \times (Y \cap Z)$

[b] **Calculate the mean and the standard deviation for the following values :**

12 , 13 , 16 , 18 , 21

14 South Sinai Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The solution set of the equation : $X^2 + 9 = 0$ in $\mathbb{R}$ is ....

(a) $\varnothing$ (b) $\{3\}$ (c) $\{-3\}$ (d) $\{-3, 3\}$

2 $\sqrt{4} - \sqrt[3]{64} =$ ....

(a) -4 (b) -2 (c) 2 (d) 4

3 If $(a, b + 1) = (5, -9)$, then $a + b =$ ....

(a) 15 (b) 10 (c) 9 (d) -5

4 If $X = 5y$, then $X \propto$ ....

(a) y (b) $\frac{1}{y}$ (c) $\frac{5}{y}$ (d) $\frac{y}{5}$

5 The range for the values : 7 , 15 , 25 , 19 equals ....

(a) 15 (b) 18 (c) 19 (d) 25

6 If $X = \{1, 3, 4\}$, $Y = \{5, 7\}$, then $n(X \times Y) =$ ....

(a) zero (b) 2 (c) 3 (d) 6

2 [a] If $X = \{-1, 2\}$, $Y = \{3, 2\}$, $Z = \{4, 6, 8\}$, **find :** $(X - Y) \times Z$

[b] If $y^2 - 10Xy + 25X^2 = 0$, **prove that :** $y \propto X$

3 [a] Find the number which if its square is added to each of the two terms of the ratio 7 : 11 , it becomes 4 : 5

[b] If $X = \{1, 2, 3\}$, $Y = \{-1, -2, -3\}$ and R is a relation from X to Y where "aRb" means "a is the additive inverse of b" for all $a \in X$, $b \in Y$, write R as a set of ordered pairs, showing if it is a function or not and represent it by an arrow diagram.

---

4 [a] If y varies inversely as X and $X = 3$ at $y = 2$, find the relation between X and y , then find the value of X when $y = 6$

[b] **The following table shows the marks of 20 students in an algebraic exam :**

The marks	0	1	2	3	4	5	Total
Frequency	1	3	5	6	3	2	20

Calculate the standard deviation for these marks.

---

5 [a] Represent graphically the function $f : f(X) = X^2 - 4$, $X \in [-3, 3]$ and from the graph deduce the vertex of the curve and the equation of the axis of symmetry.

[b] If a , b , c and d are in continued proportion

, **prove that :** $\frac{a^2 - 3c^2}{b^2 - 3d^2} = \frac{b}{d}$

15 Matrouh Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from the given ones :**

1 The range of the set of values : 7 , 3 , 6 , 9 and 5 is ....

(a) 3 (b) 4 (c) 6 (d) 12

2 If $X = \{3\}$, then $X^2 =$ ....

(a) $\{9\}$ (b) 9 (c) $\{(3, 3)\}$ (d) $\{3, 3\}$

3 The algebraic term 4 abc is of the .... degree.

(a) first (b) third (c) fourth (d) seventh

4 If the straight line representing the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 4X - 5$ passes through the point (a , 3) , then a = ...

(a) − 3 (b) − 2 (c) 2 (d) 4

5 The fourth proportional of the quantities 3 , 6 and 6 is ....

(a) 3 (b) 6 (c) 9 (d) 12

6 $\sqrt{25} = \dots\dots\dots\dots$

(a) -5 (b) $|5|$ (c) ± 5 (d) 625

2 [a] If $X = \{2, 4, 6\}$, $Y = \{1, 2, 3, 5\}$ and R is a relation from X to Y where "aRb" means "$a = 2b$" for each $a \in X$, $b \in Y$

1 Write R and represent R by an arrow diagram.

2 Is R a function from X to Y or not ? Why ? And find the range.

[b] If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - 2b + 5c}{3x}$, **then find the value of :** x

3 [a] If $X \times Y = \{(1, 2), (4, 2), (5, 2)\}$

, **find :** 1 X 2 $Y \times X$ 3 $n(X^2)$

[b] If b is the middle proportional between a and c , **prove that :** $\frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{2a}{c}$

4 [a] **Calculate the arithmetic mean and the standard deviation of the following data :**

6 , 8 , 10 , 12 and 14

[b] If y varies inversely as x and $y = 3$ when $x = 2$

, **find :** 1 The relation between y and x 2 The value of y when $x = 6$

5 [a] If $(a - 3, 7) = (2, b^3 - 1)$

, **find :** $\frac{a + 2b}{2a - b}$

[b] Graph the curve of the function $f : f(x) = 1 - x^2$, where $x \in [-2, 2]$ **and from the graph find :**

1 The coordinates of the vertex of the curve.

2 The maximum or minimum value of the function.

3 The equation of the symmetry axis.

Exam 1 Port Said 2023

First Multiple choice questions

Choose the correct answer from those given :

1 $\sqrt{50} - \sqrt{8} =$ ....

(a) $\sqrt{42}$ (b) $\sqrt{58}$ (c) $3\sqrt{2}$ (d) $2\sqrt{5}$

2 If $X = \{2\}$, then $X^2 =$ ....

(a) 4 (b) $\{4\}$ (c) $(2 , 2)$ (d) $\{(2 , 2)\}$

3 $f : f(X) = X^4 - 2X^3 + 7$ is a polynomial function of the .... degree.

(a) first (b) second (c) third (d) fourth

4 If 3 , 6 and X are proportional quantities , then $X =$ ....

(a) 9 (b) 12 (c) 15 (d) 18

5 The range of the set of values 7 , 3 , 6 , 9 , 5 is ....

(a) 3 (b) 4 (c) 6 (d) 12

6 If $\frac{X}{5} = \frac{y}{4} = \frac{X + 2y}{k}$, then k = ...

(a) 8 (b) 9 (c) 13 (d) 14

7 The relation which represents direct variation between y and X is ....

(a) $Xy = 5$ (b) $y = X^2 + 3$ (c) $\frac{X}{3} = \frac{4}{y}$ (d) $\frac{X}{5} = \frac{y}{3}$

8 If $(1 , 2) \in \{(1 , X) , (3 , 4)\}$, then $X =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

9 $f(X) = 5$ is represented by a straight line that is parallel to X-axis and passes through the point ....

(a) $(0 , 5)$ (b) $(5 , 0)$ (c) $(5 , -5)$ (d) $(0 , 0)$

10 If $y \propto X$ and $X = 1$ when $y = 4$, then the variation constant = ...

(a) 4 (b) 3 (c) 2 (d) 1

11 If $\frac{a}{b} = \frac{2}{3}$, then $3a - 2b =$ ....

(a) 3 (b) 2 (c) 1 (d) zero

12 If $x y = 5$, then y varies inversely as ....

(a) x (b) $\frac{1}{x}$ (c) $\frac{5}{x}$ (d) $5 + x$

13 If $\frac{a}{b} = \frac{b}{c} = 2$, then $\frac{a}{c} =$ ....

(a) 2 (b) 4 (c) 6 (d) 8

14 The S.S. of the equation : $x^2 + 9 = 0$ where $x \in \mathbb{R}$ is ....

(a) $\{-3\}$ (b) $\{3\}$ (c) $\{-3 , 3\}$ (d) $\varnothing$

15 If $X \times Y = \{(1 , 2) , (3 , 2)\}$, then $Y =$ ....

(a) $\{1 , 2\}$ (b) $\{3 , 2\}$ (c) $\{2\}$ (d) $\{1 , 3\}$

16 If $f(x) = x + b$, $f(3) = 7$, then $b =$ ....

(a) 10 (b) 7 (c) 4 (d) 3

17 If $y \propto x$, $y \propto \frac{1}{z}$, then $y \propto$ ....

(a) $\frac{x}{z}$ (b) $\frac{z}{x}$ (c) $x z$ (d) $x + z$

18 The point $(-2 , -3)$ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

19 If $n(X) = 3$, $n(X \times Y) = 6$, then $n(Y) =$ ....

(a) 2 (b) 3 (c) 6 (d) 9

20 If $\frac{a}{b} = \frac{c}{d} = \frac{3}{5}$, then $\frac{a + c}{b + d} =$ ....

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) $\frac{6}{5}$ (d) $\frac{5}{6}$

21 If a , b , 2 , 3 are proportional quantities , then $\frac{b}{a} =$ ....

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

Second Essay questions

22 Draw the curve of the function $f : f(X) = X^2 - 1$ where $X \in [-2, 2]$

and from the graph , find : 1 The minimum value of the function.

2 The equation of the symmetry axis of the curve.

---

23 If $y \propto \frac{1}{X}$ and $y = 3$ when $X = 4$, find the value of y when $X = 6$

---

24 **Calculate the standard deviation of the values :** 1 , 3 , 5 , 7 , 9

Exam 2 Port Said 2024

First Multiple choice questions

Choose the correct answer from those given :

1 If $X - y = 3$, $X + y = 7$, then $X^2 - y^2 =$ ....

(a) 4 (b) 10 (c) 14 (d) 21

---

2 If X , Y are two non empty sets and n (X) = n (X × Y) , then n (Y) = ...

(a) 3 (b) 2 (c) 1 (d) zero

---

3 If 3 a = 5 b , then a : b = ...

(a) 3 : 5 (b) 5 : 3 (c) 8 : 5 (d) 5 : 8

---

4 If $\frac{a}{b} = \frac{b}{c} = \frac{c}{3} = 2$, then a = ...

(a) 3 (b) 6 (c) 12 (d) 24

---

5 If $\{2\} \times \{X , y\} = \{(2 , 4) , (2 , 3)\}$, then $X - y =$ ....

(a) 1 (b) − 1 (c) ± 1 (d) zero

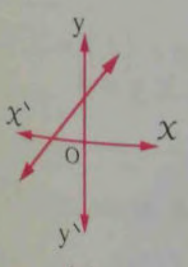
---

6 The line that represents the function $f : f(X) = X + 1$ cuts y-axis at the point ....

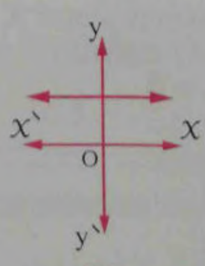
(a) (1 , 0) (b) (0 , 1) (c) (− 1 , 0) (d) (0 , − 1)

---

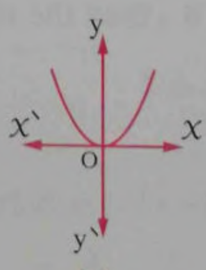
7 Which of the following graphs represents a direct variation between X and y ?



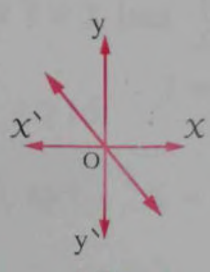
(a)



(b)



(c)



(d)

8 The sum of the two square roots of the number $2\frac{1}{4}$ is ....

(a) $1\frac{1}{2}$ (b) $\frac{1}{2}$ (c) zero (d) 1

9 $f : f(X) = 3$ is a polynomial function of the .... degree.

(a) third (b) second (c) first (d) zero

10 The middle proportional between the two numbers 3 , 27 is ....

(a) 9 (b) -9 (c) ± 9 (d) 81

11 If $X - 2y = 0$, then $X \propto$ ....

(a) y (b) $\frac{1}{y}$ (c) $\frac{2}{y}$ (d) $\frac{y}{2}$

12 The third proportional for the numbers 3 , 5 , ... , 15 is ....

(a) 10 (b) 9 (c) 8 (d) 6

13 If $X = \{3 , 5 , 7\}$ and R is a relation on X , then the relation which represents a function is ....

(a) $R = \{(3 , 5) , (5 , 3) , (3 , 7)\}$ (b) $R = \{(3 , 5) , (5 , 5) , (7 , 5)\}$

(c) $R = \{(3 , 5) , (5 , 7)\}$ (d) $R = \{(3 , 3) , (3 , 5) , (3 , 7)\}$

14 The dispersion for the values : 3 , 3 , 3 , 3 is ....

(a) zero (b) 1 (c) 3 (d) 6

15 If $b < 2$, then the point $(b - 2 , 4)$ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

16 If $\frac{a}{b} = \frac{7}{5}$, then $\frac{a + b}{a - b} =$ ....

(a) 3 (b) 4 (c) 5 (d) 6

17 If $y \propto \frac{1}{X}$ and $X = 1$ when $y = 4$, then the relation between y and X is ....

(a) $X y = 1$ (b) $\frac{X}{y} = 4$ (c) $\frac{y}{X} = 4$ (d) $X y = 4$

18 If $f(X) = X^3$, then $f(2) + f(-2) =$ ....

(a) 8 (b) 4 (c) -8 (d) zero

19 If $\frac{a}{b} = \frac{c}{d} = 5$, then $\frac{2a - 3c}{2b - 3d} =$ ....

(a) 10 (b) 15 (c) 5 (d) 1

20 If $(3, b) \in f(X) = 2X - 1$, then $b =$ ....

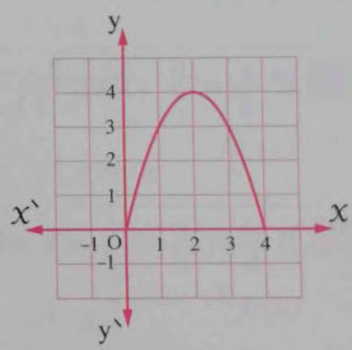
(a) 4 (b) 5 (c) 6 (d) 7

21 If $y^2 X^2 - 4yX + 4 = 0$, then $y \propto$ ....

(a) X (b) X^2 (c) $\frac{1}{X}$ (d) $\frac{1}{X^2}$

Second Essay questions

22 The opposite figure is the graphical representation of $f(X) = 4X - X^2$ where $X \in [0, 4]$, **find from the graph :**



1 The point of the vertex of the curve.

2 The equation of the symmetry axis.

3 The minimum or maximum value of the function.

23 If a , b , c , d are proportional quantities , **show that :** $\frac{a + 2c}{b + 2d} = \frac{c - a}{d - b}$

24 **Calculate the arithmetic mean and the standard deviation for the following values :**

8 , 9 , 7 , 6 , 5

Exam 3

First Multiple choice questions

Choose the correct answer from those given :

1 The simplest and easiest method of measuring dispersion is the ....

(a) mean. (b) median.

(c) range. (d) standard deviation.

2 If $X = \{3\}$, $n(Y) = 5$, then $n(X \times Y) =$ ....

(a) 1 (b) 5 (c) 8 (d) 15

3 The relation which represents an inverse variation between X and y is ....

(a) $Xy = 5$ (b) $y = X + 3$ (c) $\frac{X}{5} = \frac{y}{3}$ (d) $y = 2X$

4 $-2x^2 \times 3x =$ ....

(a) $6x^3$ (b) $6x^2$ (c) $-6x^3$ (d) $-5x^3$

5 If $f(x) = 3$, then $f(1) + f(-1) =$ ....

(a) 0 (b) 6 (c) 1 (d) 3

6 If $(x + 5 , 8) = (1 , y + x)$, then $y =$ ....

(a) 12 (b) 8 (c) -8 (d) -12

7 The middle proportional between 2 and 8 is ....

(a) 16 (b) ± 16 (c) 4 (d) ± 4

8 If $\frac{a}{3} = \frac{b}{5}$, then $\frac{2a + 2b}{3b - a} =$ ....

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{8}{5}$ (d) $\frac{5}{8}$

9 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$ where $k \in \mathbb{R}$, then $\frac{ace}{bdf} =$ ....

(a) k^3 (b) k^2 (c) k (d) 3

10 The function $f : f(x) = x - 2$ is represented by a straight line cutting the y-axis at the point ....

(a) $(2 , 0)$ (b) $(0 , 2)$ (c) $(-2 , 0)$ (d) $(0 , -2)$

11 The third proportional of 4 , 12 , ... , 48 is ....

(a) 7 (b) 32 (c) 16 (d) 36

12 Twice the number 2^5 is ....

(a) 4^5 (b) 2^{10} (c) 2^6 (d) 4^{10}

13 If $y \propto x$ and $y = 20$ when $x = 4$, then $y =$ .... when $x = 6$

(a) 30 (b) 15 (c) 60 (d) 24

14 If $(3 , 5) \in \{1 , 3\} \times \{x , 7\}$, then $x =$ ....

(a) 7 (b) 5 (c) 1 (d) 3

15 If $X = \{1 , 2 , 5\}$, R represents a function on X where $R = \{(1 , 2) , (a , 5) , (b , 5)\}$, then $a + b =$ ....

(a) 10 (b) 4 (c) 8 (d) 7

16 If $f(x) = x^2 - 1$, $g(x) = x + 1$, then $f(-1) + g(-1) =$ ....

(a) 0 (b) -2 (c) 2 (d) 4

17 If $3a = 4b$, then $a : b =$ ....

(a) $3 : 4$ (b) $4 : 3$ (c) $3 : 7$ (d) $4 : 7$

18 If $y \propto \frac{1}{x^2}$, $y = 6$ when $x = 2$, then the variation constant equals ....

(a) 3 (b) 1.5 (c) 12 (d) 24

19 If $\frac{a}{4} = \frac{4}{8}$, then $a =$ ....

(a) 2 (b) 16 (c) 8 (d) 4

20 The function $f : f(x) = 2(x^2 - 1)$ is of the .... degree.

(a) first (b) second (c) third (d) fourth

21 If $4x^2 - 4xy + y^2 = 0$, then $y \propto$ ....

(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) x (d) x^2

Second Essay questions

22 Represent graphically the function $f : f(x) = x^2 + 2x + 1$ where $x \in [-4 , 2]$ and from the graph deduce the coordinates of the vertex of the curve and the minimum or the maximum value of the function.

23 If $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, **find the value of :** $\frac{2y - z}{3x - 2y + z}$

24 **Calculate the mean and the standard deviation for the values :** 3 , 6 , 7 , 9 , 15

Exam 4

First Multiple choice questions

Choose the correct answer from those given :

1 The point $(-3 , 4)$ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

2 The range of the values : 7 , 3 , 6 , 9 , 5 is ....

(a) 3 (b) 4 (c) 5 (d) 6

3 If $y \propto X$ and $y = 2$ when $X = 8$, then $y = 3$ when $X =$ ....

(a) 16 (b) 12 (c) 24 (d) 6

4 If $f(X) = kX + 8$, $f(2) = 0$, then $k =$ ....

(a) 8 (b) -6 (c) 4 (d) -4

5 If X , 3 , 4 , 6 are proportional quantities , then $X =$ ....

(a) 0 (b) 1 (c) 2 (d) 3

6 If $X^2 = 25$ where $X \in \mathbb{Z}$, then $X =$ ....

(a) 5 (b) -5 (c) ± 5 (d) -25

7 If $n(X) = 2$, $n(X \times Y) = 6$, then $n(Y^2) =$ ....

(a) 4 (b) 9 (c) 16 (d) 12

8 If 3 , 6 , X are in continued proportion , then $X =$ ....

(a) 12 (b) 18 (c) 24 (d) 36

9 If $(-1 , 2) \in$ the function $f : f(X) = 2X + c$, then $c =$ ....

(a) 2 (b) -2 (c) 4 (d) -4

10 If $\frac{a}{3} = \frac{b}{5}$, then $\frac{b}{a} =$ ....

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{5}{8}$ (d) $\frac{3}{8}$

11 $\sqrt{(10)^2 - (6)^2} = 10 -$ ....

(a) 6 (b) 8 (c) 2 (d) 4

12 If $2X = 5y$, then $y \propto$ ....

(a) X (b) $\frac{1}{X}$ (c) X^2 (d) $\frac{1}{X^2}$

13 If $\frac{a}{2} = \frac{b}{5} = \frac{c}{7} = \frac{a+b+c}{2X}$, then $X =$ ....

(a) 14 (b) 7 (c) 28 (d) 21

14 If $X = \{2\}$, then $X^2 =$ ....

(a) $\{4\}$ (b) $(2 , 2)$ (c) $\{(4 , 4)\}$ (d) $\{(2 , 2)\}$

15 If the relation $R = \{(1, 2), (2, 3), (3, 4)\}$, then R represents a function where its range is ....

(a) $\{1, 2, 3\}$ (b) $\{2, 3, 4\}$ (c) $\{1, 2, 3, 4\}$ (d) $\{1, 4\}$

16 All the following functions are polynomial except $f : f(X) =$ ....

(a) $\frac{3}{4} X + 1$ (b) $\sqrt{2} X - 2$ (c) $X\left(\frac{1}{X} + 3\right)$ (d) $X(X - 5)$

17 If y varies inversely as X, then ....

(a) $y = X$ (b) $y = m X$ (c) $X = m y$ (d) $y = \frac{m}{X}$

18 If b is the middle proportional between a and c, then ....

(a) $b = \pm a c$ (b) $b^2 = a^2 c^2$ (c) $b^2 = 2 a c$ (d) $b = \pm\sqrt{a c}$

19 If a, 4, b, 8 are proportional quantities, then $\frac{a}{b} =$ ....

(a) $\frac{1}{2}$ (b) 2 (c) 16 (d) 32

20 The function $f : f(X) = 5$ is represented by a straight line passing through the point ....

(a) $(5, -5)$ (b) $(5, 0)$ (c) $(0, 5)$ (d) $(0, -5)$

21 If $X^2 y^2 + 16 = 8 X y$, then $y \propto$ ....

(a) X^2 (b) X (c) $4 X$ (d) $\frac{1}{X}$

Second Essay questions

22 If b is the middle proportional between a and c, **prove that :** $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

23 Represent graphically the function $f : f(X) = X^2 - 2$, $X \in [-3, 3]$ **and deduce :**

1 The coordinates of the vertex of the curve.

2 The equation of the axis of symmetry.

24 **Calculate the mean and the standard deviation for the values :** 72, 53, 61, 70, 59

Exam 5

First Multiple choice questions

Choose the correct answer from those given :

1 If 2, 5, X, 15 are proportional, then $X =$ ....

(a) 4 (b) 10 (c) 6 (d) 30

2 The positive square root of the average of squares of deviations of the values from their mean is called the ....

(a) range. (b) mean.

(c) standard deviation. (d) mode.

3 The multiplicative inverse of 2 is ....

(a) 2 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) -2

4 If $X = \{2 , 1\}$, $Y = \{0 , 2\}$, then $n (X \times Y) =$ ....

(a) 0 (b) 2 (c) 4 (d) 5

5 If $f(x) = x + 1$, then which of the following points belongs to the function f ?

(a) (2 , 1) (b) (− 1 , 1) (c) (− 2 , 1) (d) (1 , 2)

6 If $y \propto x$, $y = 15$ when $x = 3$, then $y =$ .... when $x = 5$

(a) 25 (b) 45 (c) 20 (d) 30

7 If $3a = 4b$, then $b : a =$ ....

(a) 3 : 7 (b) 4 : 3 (c) 3 : 4 (d) 4 : 7

8 The third proportional of 5 , 25 is ....

(a) 5 (b) 125 (c) ± 125 (d) ± 25

9 If $\frac{a}{3} = \frac{b}{4} = \frac{c}{5} = \frac{3a - b + c}{2x}$, then $x =$ ....

(a) 5 (b) 12 (c) 4 (d) 8

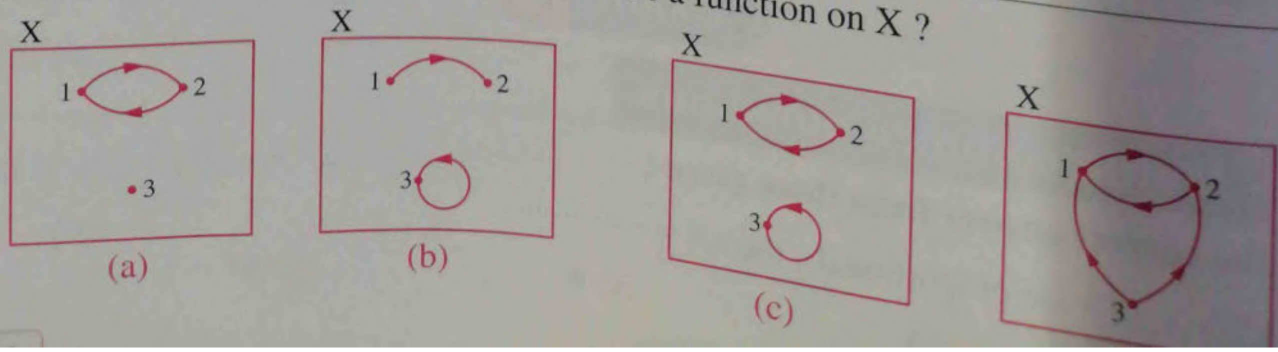
10 If $x^2 = 4$, then $|x| =$ ....

(a) ± 2 (b) 2 (c) − 2 (d) ± 4

11 If $Y \times X = \{(1 , 2) , (1 , 3)\}$, then $X =$ ....

(a) {1} (b) {1 , 2 , 3} (c) (2 , 3) (d) {2 , 3}

12 Which of the following diagrams represents a function on X ?



13 The function $f : f(X) = 2X^2 + 3(X + 1)$ is a polynomial of the .... degree.

(a) first (b) second (c) third (d) fourth

14 If $X^2 + 9y^2 = 6Xy$, then $y \propto$ ....

(a) X^2 (b) $\frac{1}{X^2}$ (c) $\frac{1}{X}$ (d) X

15 If $(2 + a , 1) = (3 , 3 - b)$, then $a + b =$ ....

(a) 5 (b) 3 (c) 4 (d) 2

16 If $f(X) = X - 3$, then $f(3) + f(2) =$ ....

(a) -1 (b) 5 (c) -3 (d) 3

17 If $Xy = 5$, then $y \propto$ ....

(a) $\frac{1}{X}$ (b) $X - 5$ (c) X (d) $X + 5$

18 If $\frac{a}{b} = \frac{b}{3} = 5$, then $a =$ ....

(a) 15 (b) 45 (c) 75 (d) 125

19 If $f(X) = X^2 - 4$, then the minimum value of the function f is ....

(a) -5 (b) -4 (c) -3 (d) zero

20 If $\frac{X}{y} = \frac{3}{4}$, then $4X - 3y =$ ....

(a) 1 (b) -1 (c) 4 (d) zero

21 If $y \propto \frac{1}{X^2}$, $y = 2$ when $X = 2$, then X could be .... when $y = \frac{1}{2}$

(a) $\frac{1}{2}$ (b) 4 (c) 8 (d) 16

Second Essay questions

22 If a , b , c , d are proportional quantities , **prove that :** $\frac{a-b}{a} = \frac{c-d}{c}$

23 **Calculate the standard deviation of :** 8 , 9 , 6 , 7 , 5

24 Represent graphically the function $f : f(X) = (X - 2)^2$ where $X \in [0 , 4]$, **then deduce :**

1. The vertex of the curve.
2. The equation of the axis of symmetry.
3. The minimum or maximum value.

Second

Trigonometry and Geometry

Accumulative Tests

on Trigonometry and Geometry

Accumulative Tests

on Trigonometry and Geometry

Accumulative test 1 on lesson 1 – unit 4

1 **Choose the correct answer from those given :**

1 If x , y are the measures of two complementary angles and $\sin x = \frac{3}{5}$, then cos y = ... « El-Beheira 18 »

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{5}{3}$

2 If sin X = cos X , where X is an acute angle , then m (∠ X) = ... « Cairo 24 »

(a) 30° (b) 45° (c) 60° (d) 90°

3 For any angle A , $\frac{\sin A}{\cos A}$ = ... « New Valley 19 »

(a) sin A (b) cos A (c) tan A (d) 1

4 ABC is a right-angled triangle at B , and $2\,AB = \sqrt{3}\,AC$, then cos C = ... « New Valley 17 »

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) 1

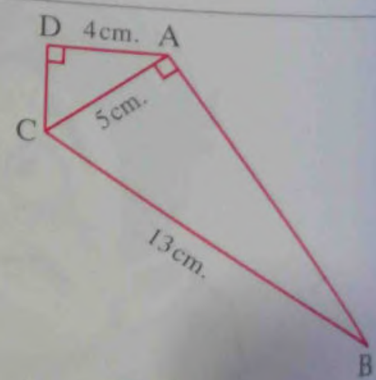
5 The surface area of a square is 25 cm^2, then the length of its diagonal is .... cm. « El-Monofia 20 »

(a) 5 (b) 10 (c) $5\sqrt{2}$ (d) $10\sqrt{2}$

6 Δ ABC is a right-angled triangle at A , then cosine angle B : sine angle C equals .... « El-Sharkia 18 »

(a) $\frac{3}{5}$ (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) 1

2 **In the opposite figure :**



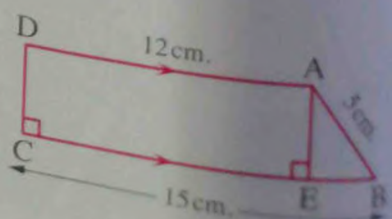
m (∠ ADC) = 90° , m (∠ BAC) = 90°
, AD = 4 cm. , AC = 5 cm. , BC = 13 cm.

Find the value of each of :

1 tan (∠ ACB) + tan (∠ ACD)

2 sin (∠ B) cos (∠ CAD) + cos (∠ B) sin (∠ CAD)

3 **In the opposite figure :** « El-Gharbia 17 »



ABCD is a trapezium right-angled at C
, $\overline{AD} // \overline{BC}$, $\overline{AE} \perp \overline{BC}$
, AD = 12 cm. , AB = 5 cm. , BC = 15 cm.

Find : 1 The length of $\overline{AE}$

2 The value of : tan (∠ BAE) × tan (∠ ACB)

Accumulative test 2 till lesson 2 – unit 4

1 Choose the correct answer from those given :

1 If $\cos 3X = \frac{1}{2}$ where $(3X)$ is the measure of an acute angle , then $X =$ ....

« El-Sharkia 17 »

(a) 15° (b) 20° (c) 30° (d) 45°

2 If $\tan \frac{3X}{2} = 1$ where X is the measure of an acute angle , then $X =$ ....

« Qena 16 »

(a) 15° (b) 30° (c) 45° (d) 60°

3 If $m(\angle A) = 75°$, $\sin B = \cos A$, $\angle B$ is acute , then $m(\angle B) =$ ....

« El-Dakahlia 20 »

(a) 45° (b) 75° (c) 15° (d) 105°

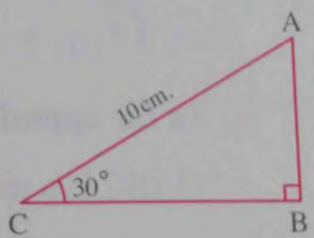
4 If $\sin X = \frac{1}{2}$, X is the measure of an acute angle , then $\sin 2X =$ .... « New Valley 24 »

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

5 If ABCD is a square , then $m(\angle CAB) =$ .... « Kafr El-Sheikh 19 »

(a) 90° (b) 45° (c) 60° (d) 30°

6 **In the opposite figure :**



AB = ... cm.

(a) 5 (b) 15

(c) 20 (d) 40

« Assiut 20 »

---

2 ABC is a right-angled triangle at B

1 **Prove that :** $\sin^2 A + \cos^2 A = 1$

2 If AB = 5 cm. , AC = 13 cm.

, **find :** $m(\angle C)$ to the nearest minute.

« El-Dakahlia 19 »

---

3 Find the value of X if : $4X = (\cos 30° \tan 30° \tan 45°)^2$

« El-Gharbia 24 »

Accumulative test 3 till lesson 1 – unit 5

1 **Choose the correct answer from those given :**

1. The distance between the point $(-6, 8)$ and y-axis is .... length units. « Damietta »
   (a) 6 (b) -6 (c) 8 (d) -8

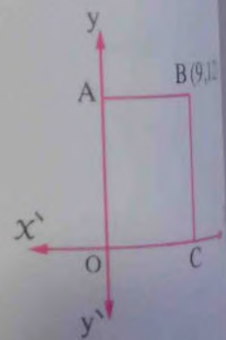
2. The distance between the point $A(\sqrt{2}, 4)$ and the origin point is .... length units. « El-Sharkia »
   (a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $3\sqrt{2}$ (d) $4\sqrt{2}$

3. The number of axes of symmetry of any isosceles triangle is .... « Beni Suef »
   (a) 0 (b) 1 (c) 2 (d) 3

4. A circle its centre is the origin point and its radius length equals 5 cm. , then the point $(3, 4)$ lies .... the circle. « Port Said »
   (a) inside (b) outside
   (c) on (d) on the centre of

5. If $\cos \frac{X}{2} = \frac{\sqrt{3}}{2}$ where $\frac{X}{2}$ is the measure of an acute angle , then $\tan(X - 15^\circ) =$ .... « El-Monofia »
   (a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$

6. **In the opposite figure :**
   OABC is a rectangle in the Cartesian coordinates plane , then AC = ... length units.
   (a) 12 (b) 9
   (c) 15 (d) 25



« El-Monofia »

2 ABCD is a quadrilateral where :
A (2 , 4) , B (–3 , 0) , C (–7 , 5) and D (–2 , 9)
Prove that : ABCD is a square. « North Sinai »

3 **Find m (∠ X) where X is an acute angle if :** $3 \tan^2 X = 4 \sin^2 30^\circ + 8 \cos^2 60^\circ$ « El-Fayoum »

Accumulative test 4 till lesson 2 – unit 5

1 Choose the correct answer from those given :

1 If the origin point is the midpoint of $\overline{AB}$, where A (5 , – 2) , then the point B is .... « Port Said 19 »

(a) (2 , 5) (b) (5 , – 2) (c) (– 2 , – 5) (d) (– 5 , 2)

2 If (3 , – 1) is the midpoint of $\overline{AB}$ where A (X , 2) , B (– 1 , – 4) , then X = ... « El-Kalyoubia 16 »

(a) 17 (b) 6 (c) 13 (d) 7

3 ABC is a right-angled triangle at B , then sin A + 2 cos C = ... « El-Gharbia 20 »

(a) 2 sin C (b) 3 sin A (c) 2 sin A (d) 3 cos A

4 If the side lengths of a triangle are 5 cm. , 12 cm. and 13 cm. , then its area equals .... cm^2. « Matrouh 18 »

(a) 30 (b) 32.5 (c) 78 (d) 144

5 If sin X = cos 30° , then tan X = ... (where X is the measure of an acute angle) « Assiut 24 »

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

6 If $\overline{AB}$ is a diameter in a circle of centre M , where A (2 , 4) and B (– 2 , 0) , then M = ... « Beni Suef 20 »

(a) (0 , 2) (b) (2 , 0) (c) (0 , 0) (d) (2 , 2)

---

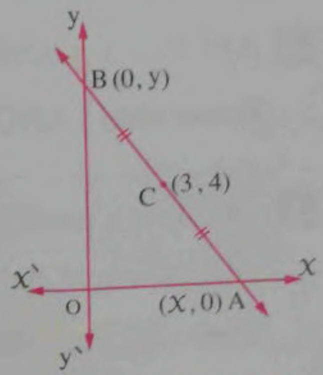
2 ABCD is a trapezium , $\overline{AD}$ // $\overline{BC}$, m (∠ B) = 90° , AB = 3 cm. , BC = 6 cm. , AD = 2 cm. Find the length of $\overline{DC}$ and the value of cos (∠ BCD) « El-Beheira 19 »

---

3 In the opposite figure :

The point C is the midpoint of $\overline{AB}$ where C (3 , 4)

Find the perimeter of the triangle AOB



« El-Kalyoubia 20 »

Accumulative test 5 till lesson 3 – unit 5

1 Choose the correct answer from those given :

1 The slope of the straight line which makes with the positive direction of X-axis an angle whose positive measure is X equals …………… « Giza 20 »

(a) $\sin X$ (b) $\cos X$ (c) $\frac{\sin X}{\cos X}$ (d) $\sin X + \cos X$

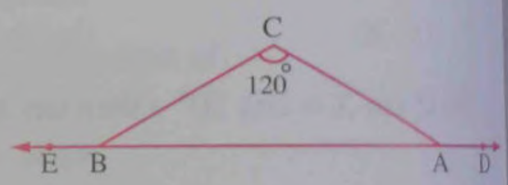
2 Two perpendicular straight lines , if the slope of one of them is $-\frac{1}{4}$ and the slope of the other is 4 k , then k = …………… « Ismailia 24 »

(a) -4 (b) 4 (c) 1 (d) $\frac{1}{4}$

3 If the two straight lines whose slopes are $-\frac{2}{3}$ and $\frac{k}{2}$ are parallel , then k = …………… « Alexandria 17 »

(a) $\frac{-3}{4}$ (b) $\frac{1}{3}$ (c) 3 (d) $\frac{-4}{3}$

4 **In the opposite figure :**



If m (∠ C) = 120° , A ∈ $\overleftrightarrow{DE}$, B ∈ $\overleftrightarrow{DE}$, then m (∠ DAC) + m (∠ EBC) = ……………

(a) 60° (b) 180°

(c) 240° (d) 300° « El-Dakahlia 24 »

5 The slope of the perpendicular straight line to the straight line which passes through the two points (2 , 3) and (5 , 1) equals …………… « Giza 17 »

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{-3}{2}$ (d) $\frac{-2}{3}$

6 If A (5 , 7) and B (1 , – 1) , then the midpoint of $\overline{AB}$ is …………… « El-Beheira 20 »

(a) (2 , 3) (b) (3 , 3) (c) (3 , 2) (d) (3 , 4)

2 ABCD is a quadrilateral , where A (2 , 3) , B (6 , 2) , C (– 2 , – 2) and D (– 2 , 1)

Prove that : ABCD is a trapezoid. « Damietta 18 »

3 ABCD is a parallelogram where A (3 , 4) , B (2 , – 1) and C (– 5 , 2) , M is the point of intersection of its diagonals.

Find : 1 The coordinates of the point M

2 The coordinates of the point D « Cairo 24 »

Accumulative test 6 till lesson 4 – unit 5

1 **Choose the correct answer from those given :**

1 The perpendicular length between $X = 5$ and $X + 3 = 0$ equals …………… length units. « El-Kalyoubia 20 »

(a) 2 (b) 8 (c) -8 (d) 5

2 In the square XYZL , if the slope of $\overleftrightarrow{XZ} = 1$, then the slope of $\overleftrightarrow{YL} =$ …………… « El-Sharkia 20 »

(a) 1 (b) -1 (c) ± 1 (d) $45°$

3 The equation of the straight line which passes through the point $(-5 , 3)$ and is parallel to X-axis is …………… « Port Said 24 »

(a) $X = -5$ (b) $y = -5$ (c) $y = 3$ (d) $X = 3$

4 If the lengths $3 , 7 , \ell$ are lengths of sides of a triangle , then ℓ can be equal to …………… « El-Gharbia 19 »

(a) 3 (b) 7 (c) 4 (d) 10

5 If $X + y = 5 , k X + 2 y = 0$ are two perpendicular straight lines , then $k =$ …………… « Giza 23 »

(a) -2 (b) -1 (c) 1 (d) 2

6 If $\tan (X + 20°) = \sqrt{3}$ where X is the measure of an acute angle , then $X =$ …………… « El-Sharkia 18 »

(a) $20°$ (b) $30°$ (c) $40°$ (d) $50°$

---

2 Find the equation of the straight line which passes through the point $(1 , 6)$ and the midpoint of $\overline{AB}$ where $A (1 , -2) , B (3 , -4)$ « Souhag 23 »

---

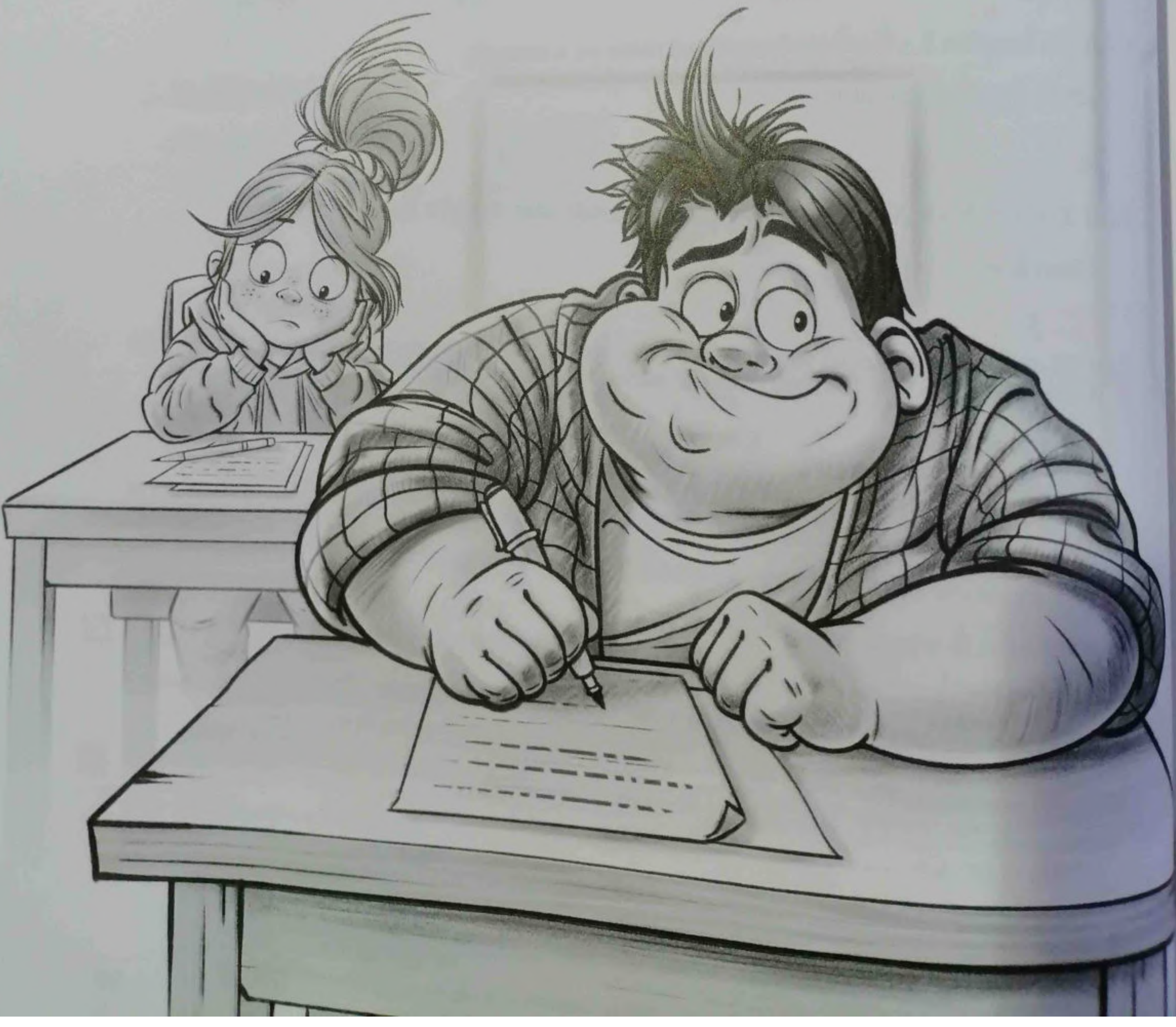
3 Δ ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm.

Find : 1 $\cos A \cos C - \sin A \sin C$

2 $m (\angle C)$ « Beni Suef 24 »

Important Questions

on Trigonometry and Geometry



First Multiple choice questions

1 $2\cos^2 30° - 1 =$ .... *(Cairo 18)*

(a) $\cos 60°$ (b) $\sin 60°$ (c) $2\sin 30°$ (d) $\tan 60°$

2 If $\angle X$, $\angle Y$ are two complementary angles and $\sin X = \frac{3}{5}$, then $\cos Y =$ .... *(Giza 20)*

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{5}{3}$

3 If $\sin 70° = \cos 2X$ where $2X$ is the measure of an acute angle , then $X =$ .... *(El-Monofia 23)*

(a) $10°$ (b) $20°$ (c) $45°$ (d) $60°$

4 In ΔABC , if $m(\angle A) = 85°$, $\sin B = \cos B$, then $m(\angle C) =$ .... *(El-Beheira 17)*

(a) $30°$ (b) $45°$ (c) $50°$ (d) $60°$

5 In ΔABC , if $m(\angle A) : m(\angle B) : m(\angle C) = 3 : 4 : 5$, then $\cos B =$ .... *(El-Gharbia 16)*

(a) zero (b) $\frac{1}{2}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$

6 If $\sin H = \frac{1}{2}$ where H is an acute angle , then $m(\angle H) =$ .... *(Cairo 23)*

(a) $30°$ (b) $45°$ (c) $60°$ (d) $90°$

7 If $\cos X = \frac{\sqrt{3}}{2}$ where X is an acute angle , then $\sin 2X =$ .... *(Aswan 23)*

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$

8 If $X = \cos 60° \tan 45°$, then $X^2 =$ .... *(Cairo 18)*

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

9 In ΔABC , if $m(\angle B) = 90°$, then $\sin A + \cos C =$ .... *(El-Sharkia 20)*

(a) $2\sin C$ (b) $2\cos A$ (c) $2\cos C$ (d) $\tan A$

10 If $\sin 2X = 0.5$ where X is the measure of an acute angle , then $X =$ .... *(El-Kalyoubia 17)*

(a) $70°$ (b) $60°$ (c) $15°$ (d) $30°$

11 If $\sin \frac{X}{2} = \frac{1}{2}$ where X is the measure of an acute angle , then $X =$ .... (Red Sea 17)

(a) $30°$ (b) $60°$ (c) $15°$ (d) $45°$

12 If $\tan (X + 15°) = \sqrt{3}$ where X is the measure of an acute angle , then $\tan X =$ .... (El-Monofia 16)

(a) 1 (b) $\sqrt{3}$ (c) $45°$ (d) $\frac{1}{\sqrt{2}}$

13 If $\tan \frac{a}{b} = 1$, then $\tan \frac{2a}{3b} =$ .... (El-Kalyoubia 24)

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{2}{3}$ (d) $\sqrt{3}$

14 ABC is a right-angled triangle at B where $3\,AC = 5\,BC$, then $\tan A =$ .... (El-Sharkia 20)

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

15 ABC is a triangle in which $m(\angle B) = 90°$, $3 \tan C - 4 = 0$, then $25 \sin C \cos C =$ .... (El-Dakahlia 18)

(a) 3 (b) 4 (c) 25 (d) 12

16 ABC is a right-angled triangle at A , $\tan B = 1$, then $\tan C - \sin C \cos C =$ .... (Red Sea 16)

(a) zero (b) 1 (c) 2 (d) $\frac{1}{2}$

17 If the triangle ABC is a right-angled triangle at A , then $\sin B : \cos C =$ .... (Alex. 24)

(a) $\frac{3}{5}$ (b) $\frac{3}{4}$ (c) 1 (d) $\frac{4}{3}$

18 If $\tan (2X - 5)° = 1$ where X is the measure of an acute angle , then $X =$ .... (El-Gharbia 16)

(a) $45°$ (b) $35°$ (c) $25°$ (d) $15°$

19 In ΔABC , if $\sin A = \cos C$, then ΔABC is .... (El-Sharkia 23)

(a) an acute-angled triangle. (b) a right-angled triangle.

(c) an obtuse-angled triangle. (d) an isosceles triangle.

Second Essay questions

1 If the ratio between the measures of two supplementary angles is 3 : 5 , find the degree measure of each one. *(Aswan 15)*

2 If the ratio among the measures of the interior angles of a triangle is 3 : 4 : 7 , find the degree measure of each angle. *(El-Beheira 13)*

3 If Δ ABC is a right-angled triangle at C , AB = 13 cm. , BC = 12 cm.
, prove that : $\sin A \cos B + \cos A \sin B = 1$ *(New Valley 24)*

4 **Without using calculator , find the value of :**
$\sin 45^\circ \cos 45^\circ + \sin 30^\circ \cos 60^\circ - \cos^2 30^\circ$ *(El-Monofia 24)*

5 **Without using calculator , prove that :** $\tan^2 60^\circ - \tan^2 45^\circ = 4 \sin 30^\circ$ *(Giza 17)*

6 **Without using calculator , prove that :** $\sin^3 30^\circ = 9 \cos^3 60^\circ - \tan^2 45^\circ$ *(El-Monofia 16)*

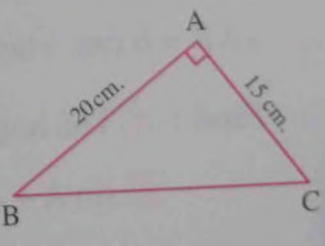
7 **Find the value of X which satisfies that :**
$X \sin 30^\circ = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$ *(Cairo 20)*

8 **Find the value of X which satisfies that :** $X \sin 30^\circ \cos^2 45^\circ = \sin^2 60^\circ$ *(South Sinai 16)*

9 **Without using calculator , find the value of X which satisfies the equation :**
$\tan X = 4 \sin 30^\circ \cos 60^\circ$ where X is the measure of a positive acute angle. *(Giza 20)*

10 **Find m ($\angle$ E) where E is an acute angle , if :** $\sin E = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$ *(Aswan 23)*

11 **In the opposite figure :**

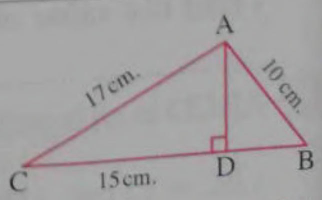


ABC is a triangle in which : m ($\angle$ A) = 90°
, AC = 15 cm. , AB = 20 cm.

Prove that :

$\cos C \cos B - \sin C \sin B = \text{zero}$ *(El-Beheira 17)*

12 **In the opposite figure :**

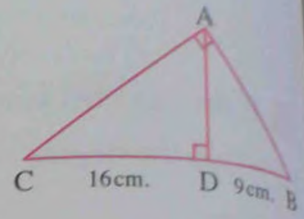


$\overline{AD} \perp \overline{BC}$, AB = 10 cm.
, AC = 17 cm. and DC = 15 cm.

Find the value of : $3 \tan C + \sin B$ *(El-Beheira 24)*

13 **In the opposite figure :**

Find the value of : tan B tan C

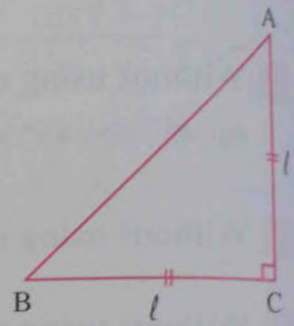


(El-Fayoum 24)

14 ABC is a right-angled triangle at B , if $2\ AB = \sqrt{3}\ AC$, find the main trigonometrical ratios of the angle C *(Alex. 15)*

15 **In the opposite figure :**

ABC is an isosceles triangle and right-angled at C , and the length of each of its legs is ℓ



Find :

1. The ratio among the lengths of the triangle sides AC : BC : AB
2. tan B , sin A

(Alex. 20)

16 If ABC is a right-angled triangle at B

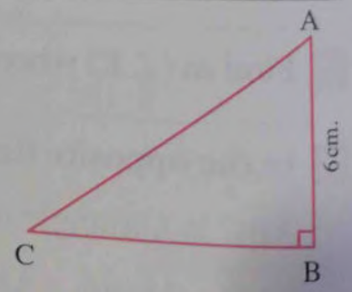
, **find the value of :** $\frac{\sin A}{\cos C}$ and if $\tan E = \frac{\sin A}{\cos C}$

, **find :** m ($\angle$ E) where $\angle$ E is an acute angle. *(Ismailia 19)*

17 **In the opposite figure :**

ABC is a right-angled triangle at B

, AB = 6 cm. , $\tan C = \frac{3}{4}$



Find : 1. The length of each of $\overline{BC}$ and $\overline{AC}$

2. sin A + cos A

(Matrouh 24)

18 If $2 \cos X - \sqrt{3} = 0$ where X is the measure of an acute angle

, **find the value of :** $\tan 2X$

(Red Sea 24)

19 ABCD is an isosceles trapezoid in which : $\overline{AD} \,//\, \overline{BC}$, AD = 4 cm. , AB = 5 cm.

, BC = 12 cm. , **then calculate :** $\frac{\tan B \cos C}{\cos^2 C + \sin^2 C}$

(Kafr El-Sheikh 20)

20 ABCD is a trapezoid in which : $\overline{AD} \parallel \overline{BC}$, $m(\angle B) = 90°$, $AB = 3$ cm. , $AD = 6$ cm. and $BC = 10$ cm.

Prove that : $\cos(\angle DCB) - \tan(\angle ACB) = \frac{1}{2}$ *(Matrouh 18)*

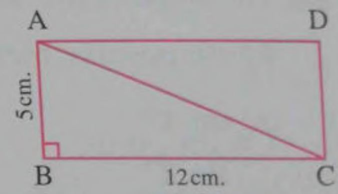
---

21 ABC is an isosceles triangle in which : AB = AC = 10 cm. and BC = 12 cm.

Find : 1 m (∠ B) 2 The area of Δ ABC *(Beni Suef 16)*

---

22 **In the opposite figure :**



If ABCD is a rectangle in which :

AB = 5 cm. , BC = 12 cm.

, **find :**

1 The length of $\overline{AC}$

2 The value of : $5\tan(\angle ACD) - 13\sin(\angle DAC)$ *(El-Sharkia 20)*

First Multiple choice questions

1 If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$, the slope of $\overleftrightarrow{AB} = \frac{1}{2}$, then the slope of $\overleftrightarrow{CD} =$ .... *(Cairo 19)*

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{-1}{2}$ (d) -2

2 The slope of the straight line that makes with the positive direction of X-axis an angle whose positive measure is $X°$ equals .... *(Giza 20)*

(a) $\sin X°$ (b) $\cos X°$

(c) $\frac{\sin X°}{\cos X°}$ (d) $\sin X° + \cos X°$

3 If ABCD is a rectangle , A (1 , 0) , C (4 , 4) , then BD = ... length units. *(New Valley 19)*

(a) 5 (b) 8 (c) 9 (d) 10

4 ABCD is a square , A (1 , 1) , C (4 , 4) , then its area = ... square units. *(Luxor 24)*

(a) 3 (b) 6 (c) 9 (d) 18

5 The straight line whose equation is : $2y = 3X + 6$ cuts from the positive part of y-axis a part of length .... length units. *(Ismailia 18)*

(a) $\frac{3}{2}$ (b) 2 (c) 3 (d) 6

6 The radius length of the circle whose centre is (– 2 , 3) and passes through the point (2 , – 1) equals .... length units. *(Alex. 24)*

(a) 5 (b) $4\sqrt{2}$ (c) 2 (d) 3

7 The slope of the straight line whose equation is : $X - y + 3 = 0$ is .... *(Luxor 16)*

(a) – 3 (b) – 1 (c) 1 (d) 3

8 In the Cartesian coordinates plane , the point that is at the distance 2 length units from the origin may be .... *(Cairo 09)*

(a) (1 , 2) (b) (2 , 1) (c) (0 , 2)

9 A circle its centre is the origin and its radius length is 2 length unit , which of the following points belongs to the circle ? *(Beni Suef 16)*

(a) $(1, 2)$ (b) $(-2, 1)$ (c) $(\sqrt{3}, 1)$ (d) $(\sqrt{2}, 1)$

---

10 If $\overline{AB}$ is a diameter in a circle where $A(3, -5)$, $B(5, 1)$, then the centre of the circle is .... *(El-Monofia 24)*

(a) $(2, 2)$ (b) $(4, -2)$ (c) $(4, 2)$ (d) $(8, -2)$

---

11 If m_1 , m_2 are the slopes of two parallel straight lines , then .... *(Cairo 23)*

(a) $m_1 = m_2$ (b) $m_1 m_2 = -1$ (c) $m_1 - m_2 = -1$ (d) $m_1 m_2 = 1$

---

12 If m_1 , m_2 are the slopes of two perpendicular straight lines , $m_1 = \frac{1}{3}$, then $m_2 =$ .... *(Kafr El-Sheiekh 24)*

(a) -3 (b) $\frac{1}{3}$ (c) $-\frac{1}{3}$ (d) -1

---

13 The straight line whose equation is : $3X - 3y + 5 = 0$ makes a positive angle with the positive direction of X-axis , its measure = ... *(El-Monofia 11)*

(a) $30°$ (b) $45°$ (c) $60°$ (d) $90°$

---

14 The straight line whose equation is : $2X + 5y - 10 = 0$ cuts from the positive part of X-axis a part of length .... length units. *(El-Dakahlia 11)*

(a) $\frac{2}{5}$ (b) 2 (c) $\frac{5}{2}$ (d) 5

---

15 If the two straight lines : $3X - 4y - 3 = 0$, $ky + 4X - 8 = 0$ are perpendicular , then $k =$ .... *(El-Beheira 15)*

(a) -4 (b) -3 (c) 3 (d) 4

---

16 If the two straight lines : $X + y = 5$, $kX + 2y = 0$ are parallel , then $k =$ .... *(Souhag 16)*

(a) -2 (b) -1 (c) 1 (d) 2

---

17 The straight line whose equation is : $2X - 3y - 6 = 0$ cuts from the negative part of y-axis a part of length .... length units. *(Cairo 14)*

(a) -6 (b) -2 (c) $\frac{2}{3}$ (d) 2

18 If $\frac{-2}{3}$, $\frac{6}{k}$ are the slopes of two perpendicular straight lines , then k = ... (El-Menia 24)

(a) – 9 (b) – 4 (c) 4 (d) 9

19 If the slope of the straight line : a X – y + 3 = 0 is 2 , then a = ... (El-Kalyoubia 18)

(a) $-\frac{1}{3}$ (b) – 2 (c) $\frac{1}{3}$ (d) 2

20 The perpendicular distance between the two straight lines : $X + 2 = 0$, $X - 4 = 0$ equals .... length units. (El-Gharbia 18)

(a) 2 (b) 4 (c) 5 (d) 6

21 If the distance between the two points (a , 7) , (– 2 , 3) is 5 length units , then a = ... (Alex. 23)

(a) 5 or – 1 (b) 10 (c) – 5 or 1 (d) 7

22 The equation of a straight line is : $\frac{X}{2} - \frac{y}{3} = 6$, then it intercepts from the positive part of X-axis a part of length .... length units. (El-Monofia 20)

(a) 3 (b) 12 (c) 6 (d) 18

23 The slope of the straight line perpendicular to y-axis is .... (El-Dakahlia 24)

(a) undefined. (b) zero (c) – 1 (d) 1

24 The equation of the straight line whose slope is 1 and passes through the origin point is .... (Souhag 24)

(a) $y = X$ (b) $X = 1$ (c) $y = 1$ (d) $y = -X$

25 The equation of the straight line which passes through (3 , – 4) and parallel to y-axis is .... (El-Menia 18)

(a) $y = -4$ (b) $X = 3$ (c) $y = 3$ (d) $X = -4$

26 If the two straight lines : $3X - 4y - 3 = 0$, $ky = 1 - 8X$ are perpendicular , then k = ... (El-Monofia 18)

(a) – 6 (b) – 3 (c) 3 (d) 6

27 If the straight line passing through the two points $(\sqrt{3} , 1)$, $(2\sqrt{3} , y)$ its slope equals tan 60° , then y = ... (Kafr El-Sheikh 20)

(a) 2 (b) 3 (c) 4 (d) 5

28 If the straight line : $y = x \sin 30° + c$ passes through the point $(4, 6)$, then $c = \dots\dots$ *(El-Monofia 16)*

(a) 4 (b) 6 (c) 8 (d) 2

---

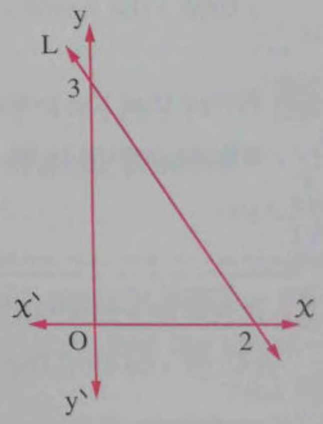
29 The distance between the point $(\ell, -4)$ and y-axis is $\dots\dots$ length units where $\ell \in \mathbb{R}$ *(Damietta 18)*

(a) 4 (b) ℓ (c) -4 (d) $|\ell|$

---

30 **In the opposite figure :**

The equation of the straight line L is $\dots\dots$ *(Suez 17)*



(a) $y = 2x + 3$

(b) $2x + 3y = 0$

(c) $\frac{x}{2} + \frac{y}{3} = 1$

(d) $\frac{x}{2} + \frac{y}{3} = 5$

---

31 If the straight line : $ax + (2 - a)y = 5$ is parallel to the straight line passing through the two points $(1, 4)$, $(3, 5)$, then $a = \dots\dots$ *(Kafr El-Sheikh 20)*

(a) 3 (b) -2 (c) 1 (d) zero

---

32 The distance between the two straight lines : $y + 1 = 0$, $y + 3 = 0$ is $\dots\dots$ length units. *(El-Beheira 17)*

(a) 4 (b) 2 (c) 1 (d) 5

---

33 If the two straight lines $y = \ell x + e$, $y = n x + o$ are parallel, (where ℓ, e, n, o are real numbers), then $\ell - n = \dots\dots$ *(Assiut 24)*

(a) -2 (b) -1 (c) 1 (d) zero

---

34 The area of the triangle which is bounded by the straight lines : $3x - 4y = 12$, $x = 0$, $y = 0$ equals $\dots\dots$ square units. *(El-Kalyoubia 15)*

(a) 6 (b) 7 (c) 12 (d) -6

---

35 If the straight line passing through the two points $(k, 0)$, $(0, 4)$ is perpendicular to the straight line which makes with the positive direction of x-axis a positive angle of measure $45°$, then $k = \dots\dots$ *(Aswan 13)*

(a) 4 (b) -4 (c) 1 (d) -1

Second Essay questions

1. Prove that the points A $(-3, -1)$, B $(6, 5)$ and C $(3, 3)$ are collinear. *(Port Said 24)*

2. Show the type of the triangle whose vertices are A $(-2, 4)$, B $(3, -1)$ and C $(4, 5)$ according to its side lengths. *(Beni Suef 24)*

3. If the distance between the two points $(x, 5)$ and $(6, 1)$ is $2\sqrt{5}$ length units, **find :** the value of x *(Giza 24)*

4. Prove that the points A $(3, -1)$, B $(-4, 6)$ and C $(2, -2)$ are located on a circle whose centre is M $(-1, 2)$, then find the circumference of the circle in terms of π *(Luxor 23)*

5. If $\overline{AD}$ is a median in Δ ABC, M is the midpoint of $\overline{AD}$ where M $(-3, -2)$, B $(-2, 4)$, C $(0, 6)$ find the point A *(El-Dakahlia 23)*

6. Prove that the triangle whose vertices are A $(1, 4)$, B $(-1, -2)$ and C $(2, -3)$ is right-angled at B, then find its surface area. *(El-Monofia 24)*

7. ABCD is a quadrilateral where A $(5, 3)$, B $(6, -2)$, C $(1, -1)$ and D $(0, 4)$, prove by using the slope that ABCD is a parallelogram, then show that the parallelogram ABCD is a rhombus. *(El-Dakahlia 17)*

8. If C $(6, -4)$ is the midpoint of $\overline{AB}$, where A $(5, -3)$, find the point B *(Giza 23)*

9. If C $(3, 1)$ is the midpoint of $\overline{AB}$, where A $(1, y)$, B $(x, 3)$, **find :** (x, y) *(El-Kalyoubia 24)*

10. Prove that the points A $(3, 3)$, B $(0, 3)$, C $(0, 0)$ and D $(3, 0)$ in the Cartesian coordinates plane are the vertices of a square and calculate the length of its diagonal and its area. *(Luxor 09)*

11. ABCD is a rhombus in which A $(5, 3)$, B $(6, -2)$, C $(1, m)$ Find the value of m *(El-Dakahlia 24)*

12. If the points A $(3, y)$, B $(x, 3)$ and C $(5, 2)$ are collinear, B is the midpoint of $\overline{AC}$, **find the value of :** $x + y$ *(El-Dakahlia 17)*

13 $\overline{AB}$ is a diameter in a circle M , where B (8 , 11) and M (5 , 7)

Find : 1 The coordinates of A

2 The circumference of the circle where ($\pi = 3.14$) *(Kafr El-Sheikh 18)*

14 Prove that the points A (– 3 , 0) , B (3 , 4) and C (1 , – 6) are the vertices of an isosceles triangle and find its surface area. *(Alex. 24)*

15 ABCD is a parallelogram in which A (3 , 3) , B (2 , – 2) and C (5 , – 1) , **find :**

1 The point of intersection of the two diagonals

2 The point D *(Kafr El-Sheikh 24)*

16 Prove that the straight line which passes through the two points (– 3 , – 2) , (4 , 5) is parallel to the straight line which makes an angle of measure 45° with the positive direction of X-axis. *(Alex. 23)*

17 If the straight line L_1 passes through the points (3 , 1) , (2 , k) and the straight line L_2 makes with the positive direction of X-axis an angle of measure 45° , then find the value of k which makes the two straight lines L_1 , L_2 perpendicular. *(Giza 23)*

18 Find the equation of the straight line passing through the point (3 , 2) and makes with the positive direction of X-axis a positive angle of measure 45° *(El-Sharkia 17)*

19 Find the equation of the straight line which passes through the point (3 , – 5) and is parallel to the straight line $X + 2y - 7 = 0$ *(El-Gharbia 23)*

20 Find the equation of the straight line which makes with the positive direction of X-axis a positive angle whose tan = 2 and intercepts from the positive part of y-axis 7 length units. *(New Valley 24)*

21 Find the equation of the straight line passing through the point (1 , 2) and perpendicular to the straight line passing through the two points A (2 , – 3) , B (5 , – 4) *(Aswan 24)*

22 Find the equation of the straight line which intercepts from the positive parts of the coordinate axes «X-axis and y-axis» two parts of lengths 4 and 9 length units respectively. *(El-Kalyoubia 19)*

23 Find the equation of the straight line passing through the two points (4 , 2) , (– 2 , – 1) , then prove that it passes through the origin point. *(Alex. 18)*

24 If the two points A (3 , – 1) , B (5 , 3) , find the equation of the axis of symmetry of $\overline{AB}$ *(El-Sharkia 20)*

25 Find the equation of the straight line whose slope equals the slope of the straight line :
$\frac{y-1}{x} = \frac{1}{3}$ and intercepts a negative part of y-axis of 4 length units. *(Damietta 24)*

26 If A (5 , 1) , B (3 , – 7) and C (1 , 3) are three non-collinear points , find the equation of the straight line which passes through the point A and is parallel to $\overleftrightarrow{BC}$ *(El-Sharkia 19)*

27 Find the slope and the intercepted part of y-axis by the straight line whose equation is :
$\frac{x}{3} + \frac{y}{2} = 1$ *(Beni Suef 16)*

28 Find the equation of the straight line which is perpendicular to the straight line :
$3x - 4y + 7 = 0$ and intercepts from the positive part of y-axis a part of length 4 units.
(El-Monofia 20)

29 ABC is a triangle in which A (1 , 2) , B (5 , – 2) and C (3 , 4) , D is the midpoint of $\overline{AB}$ and $\overline{DE}$ // $\overleftrightarrow{BC}$ and intersects $\overline{AC}$ at E

Find : 1 The length of $\overline{DE}$ 2 The equation of $\overleftrightarrow{DE}$ *(Matrouh 18)*

30 Find the equation of the straight line which intercepts 3 units from the positive part of y-axis and perpendicular to the straight line whose equation is : $\frac{x}{2} + \frac{y}{3} = 1$ *(El-Sharkia 19)*

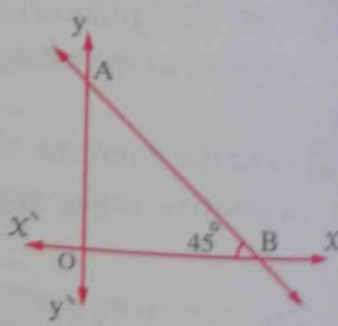
31 In the opposite figure :

$\overleftrightarrow{AB}$ intercepts from the positive part of x-axis a part of length 3 units

, m (∠ ABO) = 45°

Find : 1 The coordinates of the point A

2 The equation of $\overleftrightarrow{AB}$



(Red Sea 24)

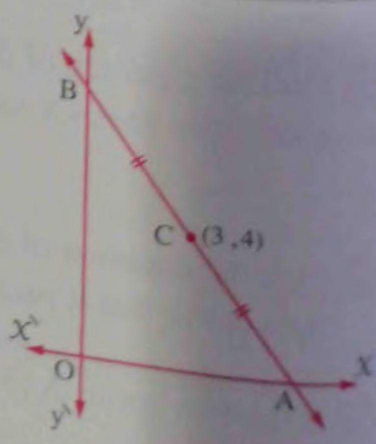
32 In the opposite figure :

The point C is the midpoint of $\overline{AB}$ where C (3 , 4) , O is the origin point in the perpendicular coordinates system.

Find :

1 The coordinates of the two points A and B

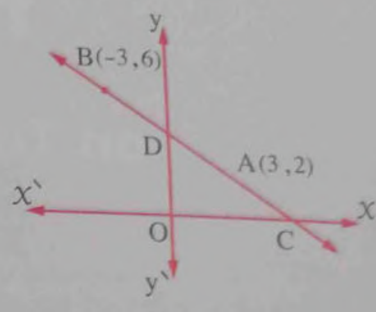
2 The equation of $\overleftrightarrow{AB}$



(El-Gharbia 19)

33 **In the opposite figure :**

$\overleftrightarrow{CD}$ passes through the two points A $(3, 2)$, B $(-3, 6)$ and cuts the two axes at C and D respectively.



Find with the proof :

1. The equation of $\overleftrightarrow{CD}$
2. The area of the triangle DOC where O is the origin point.

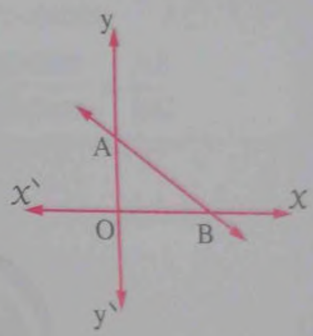
(El-Monofia 19)

---

34 **In the opposite figure :**

$\overleftrightarrow{AB}$ cuts from the positive part of y-axis 3 length units , $AB = 5$ length units.

Find : the equation of $\overleftrightarrow{AB}$



(El-Sharkia 17)

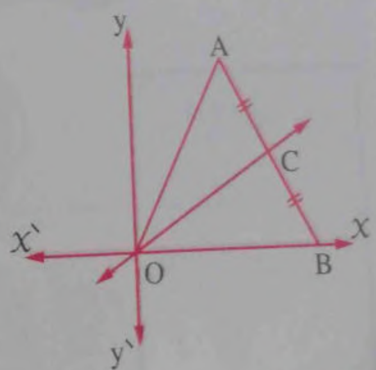
---

35 **In the opposite figure :**

ABO is an equilateral triangle

, C is the midpoint of $\overline{AB}$

Find : the equation of $\overleftrightarrow{OC}$ where O is the origin point.



(Giza 20)

Final Revision

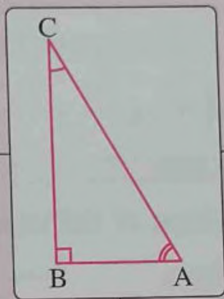
on Trigonometry and Geometry

First Trigonometry

Remember: The main trigonometrical ratios of the acute angle and the important relations between them

The trigonometrical ratios of the angle A

- $\sin A = \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{BC}{AC}$
- $\cos A = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{AB}{AC}$
- $\tan A = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{BC}{AB}$



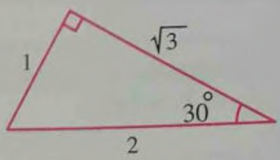
The trigonometrical ratios of the angle C

- $\sin C = \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{AB}{AC}$
- $\cos C = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{BC}{AC}$
- $\tan C = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{AB}{BC}$

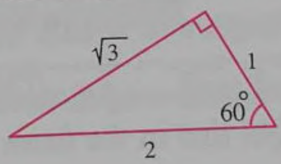
Some important relations

- $\tan A = \dfrac{\sin A}{\cos A}$
- If $m(\angle A) + m(\angle C) = 90^\circ$, then $\sin A = \cos C$, $\cos A = \sin C$
- If $\sin A = \cos C$ or $\cos A = \sin C$, then $m(\angle A) + m(\angle C) = 90^\circ$

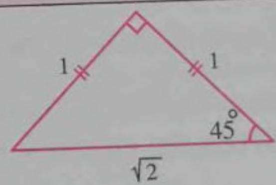
Remember: The trigonometrical ratios of some angles



- $\sin 30^\circ = \dfrac{1}{2}$
- $\cos 30^\circ = \dfrac{\sqrt{3}}{2}$
- $\tan 30^\circ = \dfrac{1}{\sqrt{3}}$



- $\sin 60^\circ = \dfrac{\sqrt{3}}{2}$
- $\cos 60^\circ = \dfrac{1}{2}$
- $\tan 60^\circ = \sqrt{3}$



- $\sin 45^\circ = \dfrac{1}{\sqrt{2}}$
- $\cos 45^\circ = \dfrac{1}{\sqrt{2}}$
- $\tan 45^\circ = 1$

Notice that :

If $\cos \theta = 0.7152$, then we use the calculator to find θ by using the keys as the following sequence from left : SHIFT cos · 7 1 5 2 = °,,,

, then $\theta \simeq 44^\circ\, 20'\, 25''$

Second Analytical geometry

Remember — The important laws

If $A(x_1, y_1)$, $B(x_2, y_2)$

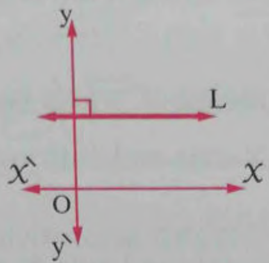
- The law of the distance between the two points A , B (the length of $\overline{AB}$):
  $AB = \sqrt{(\text{difference between } x\text{-coordinates})^2 + (\text{difference between y-coordinates})^2}$
- The law of finding the coordinates of the midpoint of $\overline{AB}$:
  The midpoint of $\overline{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- The law of finding the slope of the straight line $\overleftrightarrow{AB}$:
  $m = \frac{y_2 - y_1}{x_2 - x_1}$

Remember — How to find the slope of the straight line

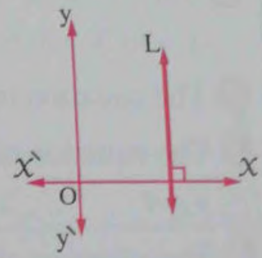
1. Given two points on the line as : $A(x_1, y_1)$, $B(x_2, y_2)$ → $m = \frac{y_2 - y_1}{x_2 - x_1}$
2. Given the measure of the positive angle which the straight line makes with the positive direction of x-axis , say θ → $m = \tan\theta$
3. Given the equation of the straight line in the form : $y = b x + c$ → $m = b$ where b is the coefficient of x
4. Given the equation of the straight line in the form : $a x + b y + c = 0$ → $m = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b}$
5. Given the slope of the parallel straight line to it , say m_1 → $m = m_1$ because the two slopes are equal.
6. Given the slope of the perpendicular straight line to it , say m_2 → $m = \frac{-1}{m_2}$ because : $m \times m_2 = -1$

! Important remarks on the slope of the straight line

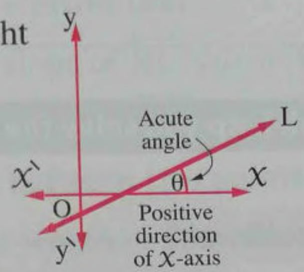
- The slope of X-axis equals 0
- The slope of the straight line parallel to X-axis equals 0



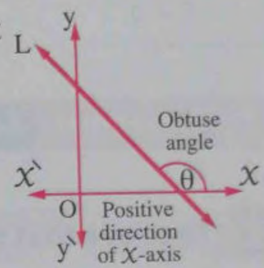
- The slope of y-axis is undefined.
- The slope of the straight line parallel to y-axis is undefined.



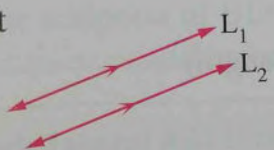
- The slope of the straight line which makes an acute angle with the positive direction of X-axis is positive.



- The slope of the straight line which makes an obtuse angle with the positive direction of X-axis is negative.

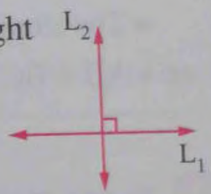


- The two parallel straight lines their slopes are equal.



i.e. If $L_1 // L_2$, then $m_1 = m_2$

- The two perpendicular straight lines the product of their slopes equals – 1

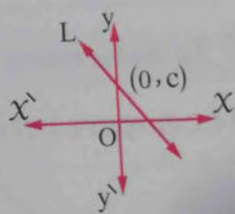


i.e. If $L_1 \perp L_2$, then $m_1 \times m_2 = -1$

Remember | The equation of the straight line

- The equation of the straight line whose slope = m and cuts y-axis at the point (0 , c) is :

$y = m X + c$



For example :

- The equation of the straight line whose slope is – 2 and cuts from the positive part of y-axis 7 units is : $y = -2X + 7$
- To find the equation of the straight line whose slope is 3 and passes through the point (1 , – 2) :

$\because$ The slope = 3 $\quad\therefore$ The equation of the straight line is : $y = 3X + c$

, then substitute by the point (1 , – 2) to find the value of c as the following :

$-2 = 3 \times 1 + c$, then : $c = -5$

$\therefore$ The equation of the straight line is : $y = 3X - 5$

Important remarks on the equation of the straight line

1. The equation of the straight line which passes through the origin point O $(0, 0)$ is : $y = m\,X$ where m is the slope.
2. The equation of X-axis is : $y = 0$ and the equation of y-axis is : $X = 0$
3. The equation of the straight line parallel to X-axis and cuts y-axis at the point $(0, c)$ is : $y = c$
4. The equation of the straight line parallel to y-axis and cuts X-axis at the point $(a, 0)$ is : $X = a$

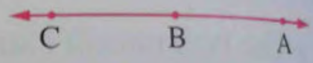
Remember **Some rules and remarks which help you solve the exercises**

1 To prove that the points A , B and C are collinear

We will prove that :

- The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$

or • $AB + BC = AC$ (where AC is the greatest length)

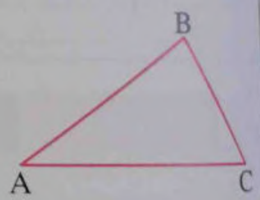


2 To prove that the points A , B and C are vertices of a triangle

We prove that :

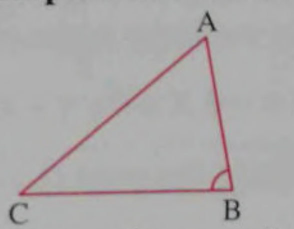
- The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

or • $AB + BC > AC$ (where AC is the greatest length)



3 To determine the type of the triangle ABC according to its angle measures

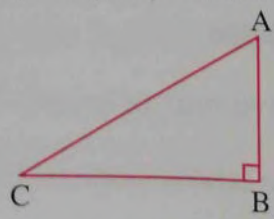
We compare between : $(AC)^2$, $(AB)^2 + (BC)^2$ where $\overline{AC}$ is the longest side , if



$(AC)^2 < (AB)^2 + (BC)^2$

, then :

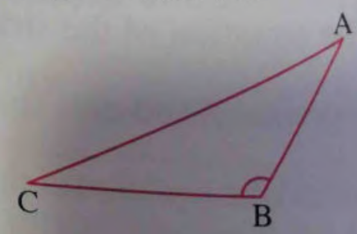
Δ ABC is acute-angled.



$(AC)^2 = (AB)^2 + (BC)^2$

, then :

Δ ABC is right-angled at B



$(AC)^2 > (AB)^2 + (BC)^2$

, then :

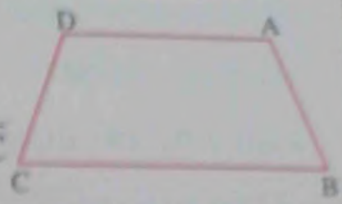
Δ ABC is obtuse-angled at B

4 To prove that the quadrilateral ABCD is a trapezium

We prove that :

The slope of $\overleftrightarrow{AD}$ = the slope of $\overleftrightarrow{BC}$, then $\overline{AD} \parallel \overline{BC}$

, the slope of $\overleftrightarrow{AB}$ ≠ the slope of $\overleftrightarrow{DC}$, then $\overline{AB}$ is not parallel to $\overline{DC}$

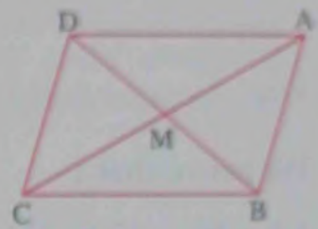


5 To prove that the quadrilateral ABCD is a parallelogram

- **By using the slope , we prove that :**

  The slope of $\overleftrightarrow{AD}$ = the slope of $\overleftrightarrow{BC}$, then $\overline{AD} \parallel \overline{BC}$

  , the slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{DC}$, then $\overline{AB} \parallel \overline{DC}$



- **By using the distance between two points , we prove that :**

  The length of $\overline{AD}$ = the length of $\overline{BC}$, the length of $\overline{AB}$ = the length of $\overline{DC}$

- **By using the midpoint of a line segment , we prove that :**

  The midpoint of $\overline{AC}$ is the midpoint of $\overline{BD}$, then : $\overline{AC}$, $\overline{BD}$ bisect each other.

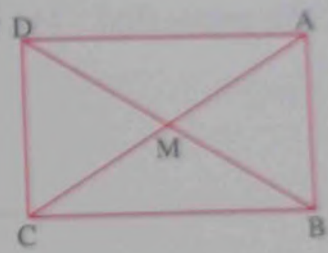
6 To prove that the quadrilateral ABCD is a rectangle

* **First we prove that :** The quadrilateral ABCD is a parallelogram by one of the previous methods

, then prove that :

- AC = BD (By using the distance between two points)

or • The slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{BC}$ = − 1 , then $\overline{AB} \perp \overline{BC}$



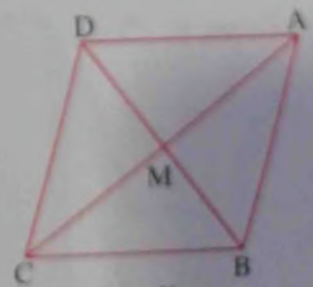
7 To prove that the quadrilateral ABCD is a rhombus

* **First we prove that :** The quadrilateral ABCD is a parallelogram

, then prove that :

- AB = BC (By using the distance between two points)

or • The slope of $\overleftrightarrow{AC}$ × the slope of $\overleftrightarrow{BD}$ = − 1 , then $\overline{AC} \perp \overline{BD}$



* We can prove that the quadrilateral ABCD is a rhombus directly by using the distance between two points

  We prove that :

  AB = BC = CD = DA

8 To prove that the quadrilateral ABCD is a square

* **First we prove that :** The quadrilateral ABCD is a parallelogram

, then prove that :

• AB = BC (By using the distance between two points)

and the slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{BC} = -1$, then $\overline{AB} \perp \overline{BC}$

or • AC = BD (By using the distance between two points)

and the slope of $\overleftrightarrow{AC}$ × the slope of $\overleftrightarrow{BD} = -1$, then $\overline{AC} \perp \overline{BD}$

* We can prove that the quadrilateral ABCD is a square by using the distance between two points

We prove that :

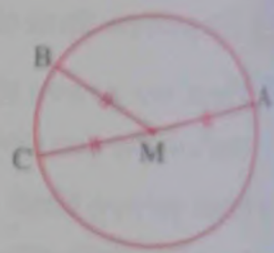
AB = BC = CD = DA , then the quadrilateral is a rhombus

, then prove that : AC = BD

9 To prove that the points A , B and C lie on one circle of centre M

By using the distance between two points

We prove that : MA = MB = MC



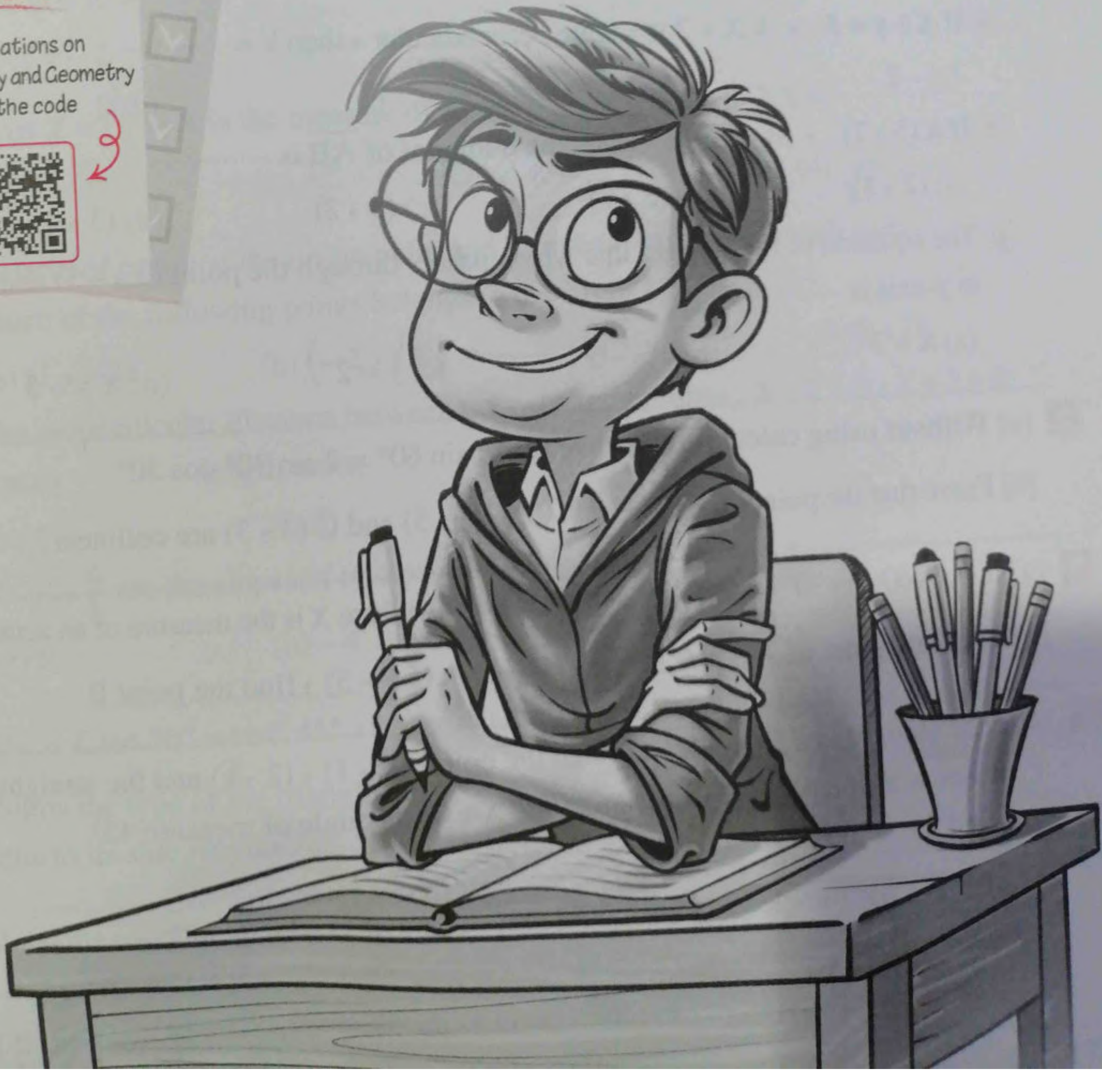
Final Examinations

on Trigonometry and Geometry

- School book examinations
- Governorates' examinations
- Examinations on Port Said specifications

For more

examinations on Trigonometry and Geometry scan the code



Model 1

Answer the following questions :

1 **Choose the correct answer from those given :**

1. $\tan 45° =$ ....
   (a) 1 (b) $2\sqrt{2}$ (c) $\frac{1}{2}$ (d) $\sqrt{2}$
2. If $\sin X = \frac{1}{2}$, X is an acute angle , then m ($\angle$ X) = ...
   (a) 45° (b) 60° (c) 30° (d) 90°
3. The distance between the two points (3 , 0) and (0 , – 4) equals .... length units.
   (a) 4 (b) 5 (c) 6 (d) 7
4. If $x + y = 5$, $k x + 2 y = 0$ are perpendicular , then k = ...
   (a) – 2 (b) – 1 (c) 1 (d) 2
5. If A (5 , 7) , B (1 , – 1) , then the midpoint of $\overline{AB}$ is ....
   (a) (2 , 3) (b) (3 , 3) (c) (3 , 2) (d) (3 , 4)
6. The equation of the straight line which passes through the point (3 , – 5) and parallel to y-axis is ....
   (a) $x = 3$ (b) $y = -5$ (c) $y = 2$ (d) $x = -5$

2 [a] **Without using calculator , prove that :** $\sin 60° = 2 \sin 30° \cos 30°$

[b] Prove that the points A (– 3 , – 1) , B (6 , 5) and C (3 , 3) are collinear.

3 [a] If $4 \cos 60° \sin 30° = \tan x$, find the value of x , where x is the measure of an acute angle.

[b] If the midpoint of $\overline{AB}$ is C (6 , – 4) where A (5 , – 3) , find the point B

4 [a] If the straight line L_1 passes through the points (3 , 1) , (2 , k) and the straight line L_2 makes with the positive direction of the x-axis an angle of measure 45° , find the value of k if $L_1 // L_2$

[b] ABC is a right-angled triangle at C , AC = 6 cm. , BC = 8 cm.

Find : 1. $\cos A \cos B - \sin A \sin B$

2. m ($\angle$ B)

5 [a] Find the equation of the straight line whose slope is 2 and passes through the point (1 , 0)

[b] Prove that the points A (3 , – 1) , B (– 4 , 6) and C (2 , – 2) which belongs to an orthogonal Cartesian coordinates plane lie on the circle whose centre is M (– 1 , 2) Find the circumference of the circle.

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 $2 \sin 30° \tan 60° =$ ....

(a) $\sqrt{3}$ (b) 3 (c) $\frac{\sqrt{3}}{3}$ (d) $\frac{1}{2}$

2 The equation of the straight line which passes through the point (– 2 , – 3) and parallel to X-axis is ....

(a) $X = -2$ (b) $X = -3$ (c) $y = -2$ (d) $y = -3$

3 If $\cos X = \frac{\sqrt{3}}{2}$, X is the measure of an acute angle , then $\sin 2X =$ ....

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) – 2 (d) $\frac{1}{\sqrt{3}}$

4 A circle of centre at the origin point and its radius length is 2 length units , which of the following points belongs to the circle ?

(a) (1 , – 2) (b) $(-2 , \sqrt{5})$ (c) $(\sqrt{3} , 1)$ (d) (0 , 1)

5 The perpendicular distance between the two straight lines : $X - 2 = 0$, $X + 3 = 0$ equals .... length units.

(a) 1 (b) 5 (c) 2 (d) 3

6 If $\frac{-3}{2}$, $\frac{6}{k}$ are the slopes of two parallel straight lines , then k = ...

(a) 6 (b) – 4 (c) $\frac{3}{2}$ (d) 2

2 [a] If $\cos E \tan 30° = \cos^2 45°$, **find :** m (∠ E) , E is an acute angle.

[b] Show the type of the triangle whose vertices are A (3 , 3) , B (1 , 5) and C (1 , 3) due to its side lengths.

3 [a] Find the equation of the straight line which passes through the points (1 , 3) and (– 1 , – 3) and prove that it is passing through the origin point.

[b] If the point (3 , 1) is the midpoint of (1 , y) , (X , 3) , find the point (X , y)

4 [a] Find the equation of the straight line which intercepts from the two axes two positive parts of lengths 1 and 4 for X and y axes respectively and find its slope.

[b] ABC is a right-angled triangle at B , AC = 10 cm. and BC = 8 cm.

Prove that : $\sin^2 A + 1 = 2\cos^2 C + \cos^2 A$

---

5 [a] Prove that the straight line which passes through the points $(-1 , 3)$ and $(2 , 4)$ is parallel to the straight line : $3y - X - 1 = 0$

[b] ABCD is a trapezium , $\overline{AD} \parallel \overline{BC}$, $m(\angle B) = 90°$, AB = 3 cm. , BC = 6 cm. and AD = 2 cm.

Find : the length of $\overline{DC}$ and the value of $\cos(\angle BCD)$

Model for the merge students

Answer the following questions :

1 **Put (✓) or (✗) :**

1 The distance between the points (9 , 0) , (4 , 0) equals 5 length units. ( )

2 If $\tan E = 1$, then $m(\angle E) = 45°$ ( )

3 The straight line $y = 2x + 1$ intercepts a part of length – 1 from y-axis ( )

4 If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$, then the slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{CD} = 1$
(both of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ aren't parallel to any axis) ( )

5 $\tan 60° = \frac{1}{\sqrt{3}}$ ( )

6 If A (1 , 2) , B (3 , 4) , then the midpoint of $\overline{AB}$ is (2 , 3) ( )

---

2 **Choose the correct answer from those given :**

1 The distance between the point (4 , 3) and x-axis is .... length units.

(a) – 3 (b) 3 (c) 4 (d) – 4

2 $4 \cos 30° \tan 60° =$ ....

(a) 3 (b) $2\sqrt{3}$ (c) 6 (d) 12

3 If $x + y = 5$, $k x + 2y = 0$ are parallel , then k = ...

(a) – 2 (b) – 1 (c) 1 (d) 2

4 The points (0 , 1) , (3 , 0) and (0 , 4) ....

(a) form a right-angled triangle. (b) form an acute-angled triangle.

(c) form an obtuse-angled triangle. (d) are collinear.

5 If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$, then the slope of $\overleftrightarrow{CD} =$ ....

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{-2}{3}$ (d) $\frac{-3}{2}$

6 If $\sin x = \frac{1}{2}$, x is the measure of an acute angle , then $\sin 2x =$ ....

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

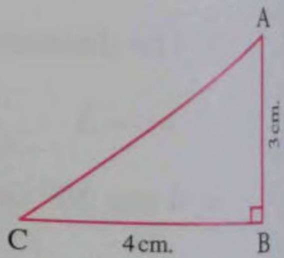
3 **Join from column (A) to column (B) :**

(A)	(B)
1 The slope of the straight line which is parallel to X-axis is	• 10
2 $\sin^2 30° + \cos^2 30° =$	• 0
3 If ABCD is a rectangle where A $(-1, -4)$, C $(5, 4)$, then the length of $\overline{BD}$ = ... length units.	• 1
4 The equation of the straight line which passes through the origin point and its slope is 2 is y = ... X	• -3
5 The equation of the straight line which passes through the point $(2, -3)$ and parallel to X-axis is y = ...	• 2
6 The value of : $\frac{2 \tan 30°}{1 + \tan^2 30°}$ = ...	• $\frac{\sqrt{3}}{2}$

4 **Complete the following :**

1 If $\overline{AB} \parallel \overline{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{1}{2}$, then the slope of $\overleftrightarrow{CD}$ = ...

2 **In the opposite figure :**



ABC is a right-angled triangle at B

, AB = 3 cm. and BC = 4 cm.

, then sin C = ...

3 If the point $(0, a)$ belongs to the straight line : $3X - 4y = -12$, then a = ...

4 If $X \cos 60° = \tan 45°$, then X = ...

5 The distance between the point $(4, 3)$ and the origin point in the coordinates plane is ....

6 If the origin point is the midpoint of $\overline{AB}$ where A $(5, -2)$

, then B (............... , ...)

1 Cairo Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from those given :

1 The sum of measures of the interior angles of the parallelogram equals ....

(a) 90° (b) 180° (c) 270° (d) 360°

2 The length of the perpendicular distance between the two straight lines $X + 2 = 0$ and $X = 3$ equals .... length units.

(a) 1 (b) 2 (c) 3 (d) 5

3 The number of axes of symmetry of the rectangle is ....

(a) zero (b) 2 (c) 4 (d) an infinite number.

4 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) quarter (b) third (c) half (d) twice

5 If $\sin X = \cos X$ where X is an acute angle , then $m(\angle X) =$ ....

(a) 30° (b) 45° (c) 60° (d) 90°

6 The slope of the straight line whose equation is $a X + by + c = 0$ equals .... (where $b \neq 0$)

(a) $\frac{-a}{b}$ (b) $\frac{a}{b}$ (c) $\frac{-b}{a}$ (d) $\frac{b}{a}$

---

2 [a] **Without using calculator , find the value of X which satisfies :**

$2X \tan 45^\circ = \tan 60^\circ \cos 30^\circ$ (Show steps of solution)

[b] Find the equation of the straight line which passes through the point (1 , 5) and its slope equals 3

---

3 [a] **Without using calculator , prove that :**

$\cos^2 60^\circ = \tan 45^\circ - \sin^2 60^\circ$ (Show steps of solution)

[b] ABCD is a parallelogram where A (3 , 4) , B (2 , – 1) and C (– 5 , 2) , M is the point of intersection of its diagonals.

Find : 1 The coordinates of the point M

2 The coordinates of the point D

4 [a] ABC is a right-angled triangle at B where AB = 5 cm. , AC = 13 cm.

Prove that : $\sin^2 C + \cos^2 C = 1$

[b] Prove that the straight line passing through the two points (3 , 2) , (1 , 3) is perpendicular to the straight line $y = 2X + 5$

---

5 [a] Find the diameter length of the circle whose centre is M (2 , 7) and passes through the point A (– 1 , 3)

[b] A straight line its slope equals 3 and intercepted 6 units from the positive part of the y-axis.

Find : 1 The equation of this straight line.

2 Its point of intersection with the X-axis.

2 Giza Governorate

Answer the following questions :

1 **Choose the correct answer :**

1 If $\sin 30° = \cos X$, where X is the measure of an acute angle , then the value of X = ...

(a) 15° (b) 30° (c) 45° (d) 60°

2 The straight line whose equation is $y = 2X - 8$ intercepts from the positive part of the X-axis a part of length .... length units.

(a) 1 (b) 3 (c) 4 (d) 7

3 The distance between the point (3 , – 4) and the X-axis equals .... length units.

(a) 3 (b) 4 (c) 7 (d) 12

4 If Δ ABC is an isosceles triangle in which AB = 3 cm. , BC = 7 cm. , then AC = ... cm.

(a) 3 (b) 4 (c) 7 (d) 10

5 If the area of a square is 100 cm.², then its perimeter is .... cm.

(a) 40 (b) 50 (c) 60 (d) 100

6 The slope of the straight line which is parallel to the X-axis is ....

(a) undefined (b) zero (c) 1 (d) – 1

---

2 [a] If $2 \sin X = \sin 30° \cos 60° + \cos 30° \sin 60°$, then without using calculator find the value of X where X is the measure of an acute angle.

[b] Find the equation of the straight line which passes through the point (2 , – 5) and it is parallel to the straight line whose equation is $2X + y - 7 = 0$

3 [a] If ABC is a right-angled triangle at B , where AC = 5 cm. , BC = 4 cm.
, **then find the value of :** sin A cos C + cos A sin C

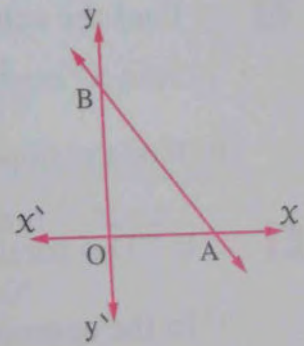
[b] If the point C (3 , 4) is the midpoint of $\overline{AB}$, where A (1 , 2) , then find the coordinates of the point B

4 [a] Find the slope of the straight line and the length of the intercepted part of y-axis where its equation is $2x - 3y + 6 = 0$

[b] If the distance between the two points $(x , 5)$ and $(6 , 1)$ is $2\sqrt{5}$ length units , then find the value of x

5 [a] State the type of the triangle ABC , where its vertices are A (– 2 , 4) , B (3 , – 1) , C (4 , 5) with respect to its sides.

[b] **In the opposite figure :**



If OA = 3 length units , OB = 4 length units where O is the origin point , **then find :**

1 The coordinates of the midpoint of $\overline{AB}$

2 The equation of $\overleftrightarrow{AB}$

3 Alexandria Governorate

Answer the following questions : *(Calculators are permitted)*

1 Choose the correct answer from those given :

1 The length of the radius of the circle whose center is (– 2 , 3) and passes through the point (2 , – 1) equals .... length units.

(a) 5 (b) $4\sqrt{2}$ (c) 2 (d) 3

2 The quadrilateral whose diagonals are equal in length and perpendicular is the ....

(a) rhombus. (b) rectangle. (c) square. (d) parallelogram.

3 ABCD is a parallelogram , m (∠ A) + m (∠ C) = 200° , then m (∠ B) = ...

(a) 80° (b) 50° (c) 100° (d) 110°

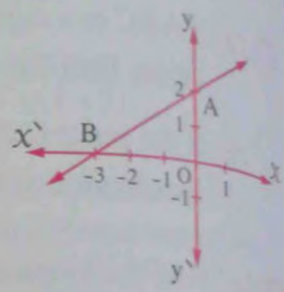
4 The volume of the cuboid whose dimensions are $\sqrt{2}$ cm. , $\sqrt{3}$ cm. , $\sqrt{6}$ cm. equals .... cm^3.

(a) $2\sqrt{6}$ (b) $3\sqrt{6}$ (c) $3\sqrt{2}$ (d) 6

5 If the triangle ABC is a right-angled triangle at A , then sin B : cos C = ...

(a) $\frac{3}{5}$ (b) $\frac{3}{4}$ (c) 1 (d) $\frac{4}{3}$

6 **In the opposite figure :**



The slope of $\overleftrightarrow{AB} =$ ....

(a) $\frac{-3}{2}$ (b) $\frac{-2}{3}$

(c) $\frac{3}{2}$ (d) $\frac{2}{3}$

2 [a] **Without using the calculator , prove that :** $\sin^3 30° = 9 \cos^3 60° - \tan^2 45°$

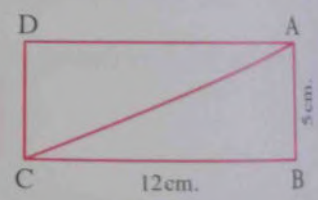
[b] Prove that the points A (– 3 , 0) , B (3 , 4) , C (1 , – 6) are the vertices of an isosceles triangle and find its surface area.

3 [a] **Find the value of X , where X is the measure of an acute angle , if :**
$3 \sin X° \cos^2 45° = \sin^2 60°$

[b] Find the slope of the straight line $\frac{X}{2} + \frac{y}{3} = 1$, then find the length of y intercept.

4 [a] If $\overleftrightarrow{CD}$ is parallel to the X-axis , where C (4 , 2) , D (– 5 , y) , find the value of y

[b] **In the opposite figure :**



ABCD is a rectangle , AB = 5 cm. , BC = 12 cm.

Find : 1 The length of $\overline{AC}$

2 The value of : 5 tan (∠ ACD) – 13 sin (∠ DAC)

5 [a] If the straight line whose equation is : $y - aX + 3 = 0$ is perpendicular to the straight line which passes through the points (5 , 2) , (6 , – 3) , find the value of a

[b] ABC is a triangle , where A (1 , 2) , B (– 2 , 3) , C (– 4 , – 3) , $\overline{AD}$ is a median of the triangle ABC , find the equation of the straight line which passes through the points A , D

4 El-Kalyoubia Governorate

Answer the following questions :

1 **Choose the correct answer from the given ones :**

1 If ABCD is a parallelogram , then AD + BC = ...

(a) 2 AC (b) 2 BD (c) 2 AB (d) 2 BC

2 The length of the radius of the circle whose center is $(7 , 4)$ and passes through the point $(3 , 1)$ equals .... length units.

(a) 8 (b) 6 (c) 5 (d) 4

3 If 4 , 9 , L are the side lengths of a triangle , then L may equal ....

(a) 3 (b) 4 (c) 5 (d) 6

4 If the slopes of two parallel straight lines are $\frac{-3}{2}$, $\frac{6}{k}$, then k = ...

(a) -4 (b) $\frac{3}{2}$ (c) 2 (d) 9

5 If ABC is a right-angled triangle at B , $m(\angle C) = 30°$, AB = 6 cm. , then AC = ... cm.

(a) 3 (b) 6 (c) 10 (d) 12

6 If $\tan \frac{a}{b} = 1$, then $\tan \frac{2a}{3b} =$ ....

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{2}{3}$ (d) $\sqrt{3}$

---

2 [a] If C (3 , 1) is the midpoint of $\overline{AB}$, where A (1 , y) , B (x , 3) , **find :** (x , y)

[b] **Find the value of x which satisfies :** $x \sin 30° \cos^2 45° = \sin^2 60°$

---

3 [a] If ABCD is a quadrilateral where A (2 , 4) , B (– 3 , 0) , C (– 7 , 5) and D (– 2 , 9) , prove that the figure ABCD is a square.

[b] If ABC is a right-angled triangle at C , AB = 13 cm. , BC = 12 cm.

, **find :** 1 The length of $\overline{AC}$ 2 $1 + \tan^2 A$

---

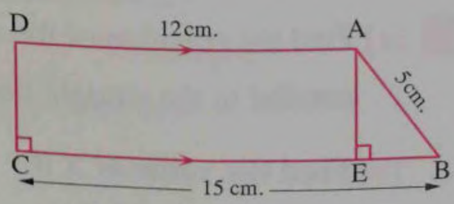
4 [a] If (0 , 1) , (a , 3) , (2 , 5) are collinear , find the value of a

[b] Prove that the straight line which passes through the two points $(4 , 3\sqrt{3})$ and $(5 , 2\sqrt{3})$ is perpendicular to the straight line which makes with the positive direction of the x-axis an angle of measure 30°

---

5 [a] Find the equation of the straight line whose slope is 3 and passes through the point (1 , 0)

[b] **In the opposite figure :**



ABCD is a trapezium right-angled at C , $\overline{AD} \parallel \overline{BC}$, $\overline{AE} \perp \overline{BC}$, AD = 12 cm. , AB = 5 cm. , BC = 15 cm.

Find : 1 The length of $\overline{AE}$

2 The value of : $\tan(\angle BAE) \times \tan(\angle ACB)$

5 El-Sharkia Governorate

Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer from those given :

1 If the straight line which passes through the two points (2 , 4) , (3 , k) makes an angle of measure 45° with the positive direction of X-axis , then k = ...

(a) 3 (b) 1 (c) 5 (d) 6

2 If $\sin (X + 20)^\circ = \frac{1}{2}$ where $(X + 20)^\circ$ is the measure of an acute angle , then $\tan (55 - X)^\circ =$ ....

(a) $\frac{\sqrt{3}}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{\sqrt{2}}{2}$

3 The point (4 , 6) is the image of the point (– 2 , 2) by reflection in the point ....

(a) origin point. (b) (– 1 , – 4) (c) (1 , 4) (d) (4 , 1)

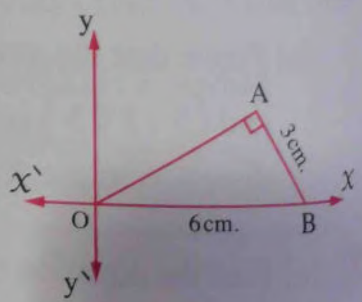
4 If $\overline{AB}$ is a diameter of a circle where A (– 1 , 4) , B (– 3 , – 2) , then the area of the circle equals .... π square units.

(a) 10 (b) $2\sqrt{10}$ (c) 20 (d) 80

5 If the ratio between the measures of two supplementary angles is 4 : 5 , then the measure of the greater angle equals ....

(a) 40° (b) 50° (c) 80° (d) 100°

6 **In the opposite figure :**



The equation of $\overleftrightarrow{OA}$ is y = ...

(a) $\sqrt{3}\, X$ (b) $\frac{1}{2}\, X$

(c) $\frac{1}{\sqrt{3}}\, X$ (d) $\frac{1}{3}\, X$

2 [a] Find the equation of the straight line which passes through the point (– 6 , – 1) and is parallel to the straight line whose equation is $\frac{1}{2} X + 3\,y = 1$

[b] **Find the value of X if :**

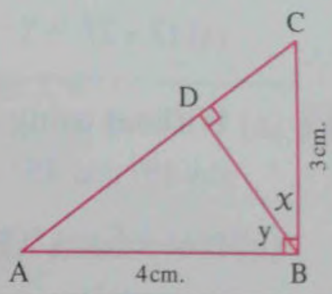
$\cos X \tan X + \sin 30^\circ = 1$ where X is the measure of an acute angle.

3 [a] ABCD is a rectangle in which A $(1, 1)$, B $(3, 3)$, C $(0, -3x)$, D (x, y), find the value of each of x, y

[b] **In the opposite figure :**

Δ ABC is right-angled at B where $\overline{BD} \perp \overline{AC}$, AB = 4 cm., BC = 3 cm.

Find the value of : $\tan x \tan y + \sin A$



4 [a] Find the equation of the straight line which passes through the point $(5, -2)$ and is perpendicular to the straight line which passes through the two points $(3, 2)$, $(-1, 0)$

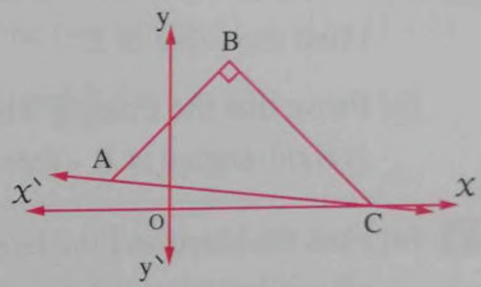
[b] Prove that the points A $(1, 4)$, B $(-1, -2)$, C $(2, -3)$ are the vertices of a right-angled triangle at B, then find its area.

5 [a] **Without using the calculator, prove that :** $\cos 60° = 2 \cos^2 30° - \tan 45°$

[b] **In the opposite figure :**

A $(-2, 1)$, B $(2, 5)$

Find the equation of $\overleftrightarrow{AC}$



6 El-Monofia Governorate

Answer the following questions : *(Calculator is allowed)*

1 **Choose the correct answer from those given :**

1. The triangle whose side lengths are 5 cm., 5 cm., ... cm. is an isosceles triangle.
   (a) 12 (b) 11 (c) 10 (d) 9
2. The number of the axes of symmetry of an equilateral triangle equals ....
   (a) zero (b) 1 (c) 2 (d) 3
3. If XYZ is a triangle, $(XY)^2 > (YZ)^2 + (XZ)^2$, then $\angle$ Z is ....
   (a) acute. (b) right. (c) obtuse. (d) straight.
4. If $\cos 2x = \frac{1}{2}$, where x is the measure of an acute angle, then $x =$ ....
   (a) 30° (b) 45° (c) 60° (d) 90°
5. If $\frac{2}{3}$, $\frac{k}{2}$ are the slopes of two parallel straight lines, then k = ...
   (a) $\frac{-4}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{4}{3}$ (d) 3

6 If $\overline{AB}$ is a diameter in a circle of center M , where A $(3 , -5)$, B $(5 , 1)$, then the center of the circle M = ...

(a) $(2 , 2)$ (b) $(4 , -2)$ (c) $(4 , 2)$ (d) $(8 , -2)$

---

2 [a] **Without using a calculator , find the value of :**

$\sin 45° \cos 45° + \sin 30° \cos 60° - \cos^2 30°$

[b] Prove that Δ ABC whose vertices are A $(1 , -2)$, B $(-4 , 2)$, C $(1 , 6)$ is isosceles.

---

3 [a] If Δ ABC is a right-angled triangle at C , AB = 10 cm. , BC = 8 cm.

, **find the value of :** $\sin A \cos B + \cos A \sin B$

[b] Find the equation of the straight line which passes through the point $(3 , 4)$ and is perpendicular to the straight line $3 x - 2 y + 7 = 0$

---

4 [a] If $2 \sin E = \tan^2 60° - 2 \tan 45°$, where E is the measure of an acute angle , find the value of E

[b] Prove that the triangle whose vertices are A $(1 , 4)$, B $(-1 , -2)$, C $(2 , -3)$ is right-angled at B , then find its surface area.

---

5 [a] Find the slope and the length of the intercepted part from y-axis of the straight line whose equation is $3 x + 2 y = 6$

[b] If the points A $(0 , 1)$, B $(k , 3)$, C $(2 , 5)$ are collinear , find the value of k

7 El-Gharbia Governorate

Answer the following questions :

1 **Choose the correct answer from the given answers :**

1 The number of axes of symmetry of half a circle equals ....

(a) 0 (b) 1 (c) 2 (d) an infinite number.

2 The straight line whose equation is $y = 3 x + 4$ cuts from the positive part of y-axis a part of length .... length units.

(a) 2 (b) 3 (c) 4 (d) 7

3 The image of the point $(3 , -2)$ by the reflection in the origin point is ....

(a) $(3 , 2)$ (b) $(2 , 3)$ (c) $(-3 , 2)$ (d) $(-3 , -2)$

4 ABCD is a parallelogram , $m (\angle A) + m (\angle C) = 200°$, then $m (\angle B) =$ ....

(a) 50° (b) 80° (c) 100° (d) 120°

5 The equation of the straight line passing through the point (2 , 3) and parallel to y-axis is ....

(a) $X = 2$ (b) $X = 3$ (c) $y = 2$ (d) $y = 3$

6 If $2 \sin X = \tan X$ where X is the measure of an acute angle , then $X =$ ....

(a) 30° (b) 45° (c) 60° (d) 150°

---

2 [a] **Without using calculator , find the value of X if :** $4\,X = (\cos 30° \tan 30° \tan 45°)^2$

[b] If A (3 , 2) , B (4 , – 3) , C (– 1 , – 2) and D (– 2 , 3) are the vertices of a rhombus , find : 1 The coordinates of the point of intersection of the two diagonals.

2 The area of the rhombus.

---

3 [a] If A (5 , 1) , B (3 , – 7) and C (1 , 3) , prove that the points A , B , C are not collinear.

[b] **Without using the calculator , find the value of :** $3 - \tan 45° \div 4 \sin 30°$

---

4 [a] Find the equation of the straight line passing through the two points (2 , – 1) , (1 , 1)

[b] XYZ is a right-angled triangle at Y where XY = 5 cm. and XZ = 13 cm.

Find the value of : tan X + tan Z

---

5 [a] If the straight line L_1 passes through the two points (3 , 1) , (2 , k) and the straight line L_2 makes with the positive direction of the X-axis an angle of measure 45° , then find the value of k if the two straight lines are perpendicular.

[b] Find the equation of the straight line which passes through (0 , 3) and is parallel to the straight line whose equation is $X + 2\,y - 1 =$ zero

8 El-Dakahlia Governorate

Answer the following questions : *(Calculator is permitted)*

1 [a] **Choose the correct answer :**

1 The slope of the straight line which is perpendicular to y-axis equals ....

(a) undefined. (b) zero. (c) – 1 (d) 1

2 If the ratio between the measures of two complementary angles is 4 : 5 , then the measure of the smaller angle equals ....

(a) 40° (b) 50° (c) 80° (d) 100°

3 If $\tan (X + 10)° = \sqrt{3}$, where $(X + 10)°$ is the measure of an acute angle , then the value of $X =$ ....

(a) 20° (b) 40° (c) 50° (d) 70°

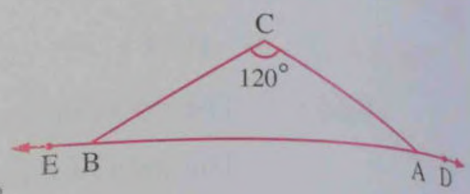
[b] If $\overline{AB}$ is a diameter in the circle M where A (8 , y) , B (X , 3) , M (5 , 7) , find the value of $X + y$

2 [a] **Choose the correct answer :**

1 If the point C is the midpoint of $\overline{AB}$, then $(AB)^2 = \dots\dots\dots\dots (AC)^2$

(a) 4 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

2 **In the opposite figure :**



If m (∠ C) = 120° , A ∈ $\overleftrightarrow{DE}$, B ∈ $\overleftrightarrow{DE}$, then m (∠ DAC) + m (∠ EBC) = ...

(a) 60° (b) 180°

(c) 240° (d) 300°

3 The area of the triangle which is bounded by the straight lines $X = 0$, $y = 0$, $\frac{X}{3} - \frac{y}{4} = 1$ equals .... square units.

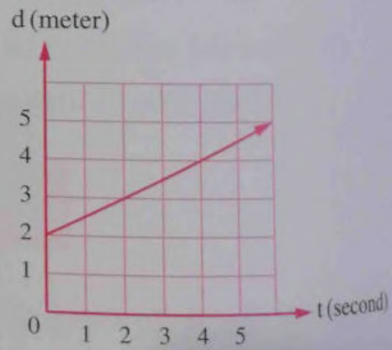
(a) – 6 (b) 6 (c) 7 (d) 12

[b] ABCD is a rhombus in which A (5 , 3) , B (6 , – 2) , C (1 , m) , find the value of m

3 [a] **Find the value of X which satisfies that :**

$3 \tan X - 4 \cos^2 60° = 8 \sin^2 30°$, where X is the measure of an acute angle.

[b] The opposite graph represents the motion of a particle moving with a uniform velocity (v) where the distance (d) is measured in meters and the time (t) in seconds.



Find : 1 The distance at the beginning of the motion.

2 The velocity of the particle.

3 The equation of the straight line representing the motion of the particle.

4 [a] If the straight line which passes through the two points A (4 , 3) , B (– 2 , – 3) is parallel to the straight line whose equation is : $(2k + 1) X - k y + 7 = 0$, find the value of k

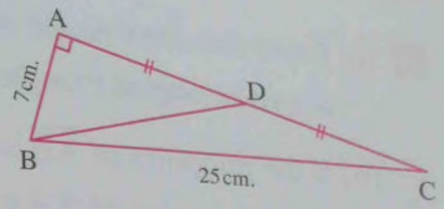
[b] A ladder $\overline{AB}$ is of length 6 meters , its upper edge A lies on a vertical wall and its other edge B on a horizontal floor. If C is the projection of the point A on the surface of the floor and its angle of slope on the surface of the floor was of measure 60° , then find the length of $\overline{AC}$

5 **[a] In the opposite figure :**

$\overline{AB} \perp \overline{AC}$, AB = 7 cm.

, BC = 25 cm. , AD = CD

Find : $\tan C + \frac{1}{\tan(\angle ABD)}$



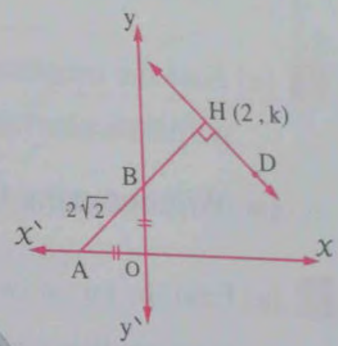
[b] In the opposite figure :

O is the origin point , OA = OB , AB = $2\sqrt{2}$ length units.

If the point H (2 , k) , $\overleftrightarrow{AB} \perp \overleftrightarrow{HD}$

, find : 1 The value of k

2 The equation of $\overleftrightarrow{HD}$



9 Ismailia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from those given :

1 The triangle has two .... angles at least.

(a) acute (b) right (c) obtuse (d) straight

2 Two perpendicular straight lines , if the slope of one is $-\frac{1}{4}$ and the slope of the other is 4 k , then k = ...

(a) – 4 (b) 4 (c) 1 (d) $\frac{1}{4}$

3 .... = 7 cm.

(a) $\overleftrightarrow{AB}$ (b) $\overrightarrow{AB}$ (c) $\overline{AB}$ (d) AB

4 If $\cos(x + 15)° = \frac{1}{2}$, then $\tan x°$ = ...

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

5 The distance between the two points (6 , 0) , (0 , 8) equals .... length units.

(a) 6 (b) 8 (c) 10 (d) 14

6 If 3 cm. , 7 cm. , L cm. are the lengths of sides of a triangle , then one of the values of L = ...

(a) 3 (b) 4 (c) 7 (d) 10

2 **[a]** If $2 \sin x° = \tan^2 60° - 2 \tan^2 45°$, find the value of x

(where x is the measure of an acute angle)

[b] Prove that the straight line whose equation is : $4x - 2y = 7$ is parallel to the straight line which passes through the two points (1 , 3) and (2 , 5)

3 **[a]** Prove that the triangle whose vertices are : A $(-1, -1)$, B $(2, 3)$ and C $(6, 0)$ is a right-angled triangle at B

[b] If the midpoint of $\overline{AB}$ is C $(4, 2)$ where A $(X, 4)$ and B $(6, y)$, find the value of $X + y$

4 **[a]** Find the equation of the straight line which passes through the point $(2, -5)$ and is perpendicular to the straight line $2X - y + 3 = 0$

[b] Without using the calculator , prove that : $\tan 60° = \frac{2 \tan 30°}{1 - \tan^2 30°}$

5 **[a]** Find the equation of the straight line which makes an angle of measure 45° with the positive direction of the X-axis and the length of the intercepted part of the y-axis is 3 units from the positive part.

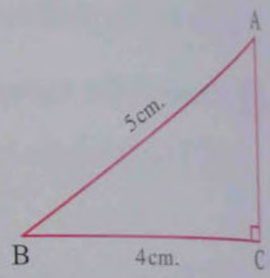
[b] In the opposite figure :

ABC is a right-angled triangle at C

, AB = 5 cm. , BC = 4 cm.

Prove that :

$\sin A \cos B + \cos A \sin B = 1$



10 Suez Governorate

Answer the following questions : ***(Calculator is permitted)***

1 **Choose the correct answer from those given :**

1. If $\tan (X + 30°) = \sqrt{3}$, X is the measure of an acute angle , then $X =$ ....
   (a) 60° (b) 30° (c) 45° (d) 90°

2. The number of axes of symmetry of the equilateral triangle is ....
   (a) 1 (b) 2 (c) 3 (d) 4

3. If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$, and the slope of $\overleftrightarrow{AB} = \frac{1}{3}$, then the slope of $\overleftrightarrow{CD} =$ ....
   (a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

4. The distance between the point $(-3, 4)$ and y-axis is .... length units.
   (a) 4 (b) -4 (c) 3 (d) -3

5. The area of the rhombus whose diagonals lengths are 6 cm. , 8 cm. is .... cm^2
   (a) 48 (b) 24 (c) 14 (d) 7

6 The volume of the cube whose edge length is 2 cm. is .... cm^3.

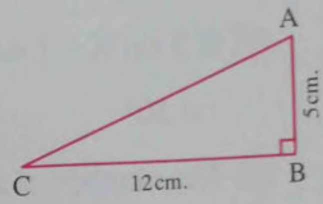
(a) 8 (b) 4 (c) 16 (d) 64

2 [a] In the opposite figure :

ABC is a right-angled triangle at B

, AB = 5 cm. , BC = 12 cm.

Prove that : cos A cos C = sin A sin C



[b] Find the equation of the straight line which passes through the point (0 , 3) and makes a positive angle of measure 45° with the positive direction of X-axis.

3 [a] If A = (– 1 , 1) , B = (0 , 5) , C = (5 , 6) and D = (4 , 2)

, **prove that :** ABCD is a parallelogram.

[b] **Without using calculator , prove that :** $2 \sin 30° = \tan^2 60° - 2 \tan 45°$

4 [a] If the point C = (5 , 4) is the midpoint of $\overline{AB}$, A = (3 , – 1) , find the coordinates of the point B

[b] Prove that the straight line passing through the points (– 1 , 4) and (2 , 5) is parallel to the straight line whose equation is $3y = X + 4$

5 [a] If the distance between the two points (X , 3) and (0 , 2) is $5\sqrt{2}$ length units , find X

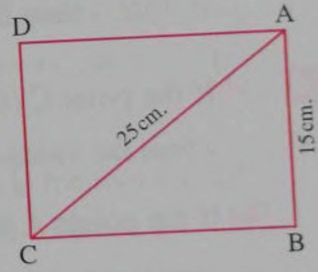
[b] **In the opposite figure :**

ABCD is a rectangle

, AB = 15 cm. , AC = 25 cm.

Find : 1 m (∠ ACB)

2 The area of the rectangle ABCD



11 Damietta Governorate

Answer the following questions : *(Calculator is allowed)*

1 **Choose the correct answer from those given :**

1 The equation of the y-axis is ....

(a) $X = 0$ (b) $y = X$ (c) $y = 0$ (d) $y = -X$

2 The sum of the measures of the accumulative angles at a point equals ....

(a) 90° (b) 180° (c) 270° (d) 360°

3 The perpendicular distance between the two straight lines :
$X = 2$ and $X + 3 = 0$ equals .... length units.

(a) 6 (b) 5 (c) 3 (d) 2

4 If $2 \sin X - 1 = 0$ (where X is an acute angle) , then m (∠ X) = ...

(a) 30° (b) 45° (c) 60° (d) 90°

5 The number of axes of symmetry of the isosceles triangle equals ....

(a) 3 (b) 2 (c) 1 (d) zero

6 ABC is a triangle , if m (∠ B) > m (∠ C) , then ....

(a) $AC - AB < 0$ (b) $AC - AB > 0$ (c) $BC \leq AB$ (d) $AC - AB \leq 0$

---

2 [a] **Without using calculator , prove that :** $\tan^2 60° - 2 \sin 45° \cos 45° = 2$

[b] Find the equation of the straight line whose slope equals the slope of the straight line $\frac{y-1}{X} = \frac{1}{3}$ and intercepts a negative part of y-axis of 4 length units.

---

3 [a] If $3 \tan X = 4 \sin^2 30° + 8 \cos^2 60°$, find the value of X
(where X is the measure of an acute angle)

[b] If the straight line L_1 passes through the two points (3 , 1) and (2 , k) and the straight line L_2 makes with the positive direction of the X-axis a positive angle whose measure is 135° , then find k if the two straight lines L_1 and L_2 are parallel.

---

4 [a] If the point C (4 , y) is the midpoint of $\overline{AB}$ where A (X , 3) and B (6 , 5) , find the value of $X + y$

[b] If the points A (6 , 0) , B (2 , 0) and C $\left(4 , 2\sqrt{3}\right)$ are three points in a cartesian coordinates plane , **prove that :** Δ ABC is equilateral.

---

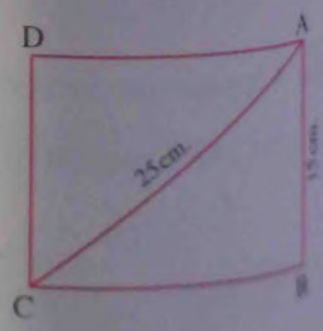
5 [a] Find the equation of the straight line which passes through the point (– 2 , 3) and is perpendicular to the straight line whose equation is $2y + X + 1 = 0$

[b] **In the opposite figure :**

ABCD is a rectangle in which
AB = 15 cm. and AC = 25 cm.

Find : 1 cos (∠ ACB)

2 The surface area of the rectangle ABCD



12 Beni Suef Governorate

Answer the following questions : **(Calculator is allowed)**

1 Choose the correct answer from those given :

1 If $\cos X = \frac{\sqrt{3}}{2}$ where X is the measure of an acute angle , then $X =$ ....

(a) 30° (b) 45° (c) 60° (d) 90°

2 If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$, then the slope of $\overleftrightarrow{CD} =$ ....

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

3 The distance between the point (– 5 , 3) and the y-axis is .... length units.

(a) – 5 (b) – 3 (c) 3 (d) 5

4 In the triangle ABC , if $(AC)^2 < (AB)^2 + (BC)^2$, then ∠ B is ....

(a) an acute angle. (b) an obtuse angle. (c) a right angle. (d) a reflex angle.

5 ABCD is a parallelogram , if m (∠ A) = 80° , then m (∠ C) = ...

(a) 40° (b) 80° (c) 100° (d) 160°

6 If the lengths of two sides in a triangle are 5 cm. and 9 cm. , then the length of the third side can be equal to .... cm.

(a) 3 (b) 4 (c) 14 (d) 8

---

2 [a] State the kind of the triangle whose vertices are the points A (– 2 , 4) , B (3 , – 1) , C (4 , 5) with respect to its sides.

[b] If $\tan X - 4 \cos 60° \sin 30° =$ zero , find the value of X where X is the measure of an acute angle.

---

3 [a] Δ ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm.

1 **Find the value of :** cos A cos C – sin A sin C

2 **Calculate :** m (∠ C)

[b] Find the slope of the straight line whose equation is $\frac{y-2}{X} = \frac{1}{2}$, then find the length of the intercepted part of y-axis.

---

4 [a] **Prove that :** $\sin^2 45° = 2 \cos^2 30° - 1$

[b] Find the equation of the straight line which passes through the point (3 , – 5) and is parallel to the straight line which makes with the positive direction of the X-axis an angle of measure 45°

5 [a] If the point C (4 , y) is the midpoint of $\overline{AB}$ where A = (6 , 5) and B = (X , 3) , find the value of $X + y$

[b] Prove that the straight line passing through the two points (– 2 , 5) and (– 2 , 4) is perpendicular to the straight line passing through the two points (2 , 3) and (5 , 3)

13 Assiut Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from those given :**

1. The distance between the point (– 4 , – 3) and the X-axis equals .... length units.
   (a) – 3 (b) 3 (c) 4 (d) – 4
2. If Δ ABC $\equiv \Delta$ XYZ , m ($\angle$ A) = 50° , m ($\angle$ B) = 60° , then m ($\angle$ X) + m ($\angle$ Y) = ...
   (a) 110° (b) 120° (c) 140° (d) 70°
3. If sin X° = cos 30° , then tan X° = ... (where X is the measure of an acute angle)
   (a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$
4. If two vertically opposite angles are supplementary , then the measure of each angle of them equals ....
   (a) 45° (b) 60° (c) 90° (d) 180°
5. If the two straight lines $y = \ell X + e$, $y = n X + o$ are parallel , (where ℓ , e , n , o are real numbers) , then $\ell - n =$ ....
   (a) – 2 (b) – 1 (c) 1 (d) zero
6. A triangle has only one axis of symmetry and the lengths of two sides are 4 cm. and 8 cm. , so the length of the third side is .... cm.
   (a) 12 (b) 8 (c) 4 (d) 2

2 [a] **Without using the calculator , find the value of :** $\sin^2 60° + \cos^2 60° + \tan^2 45°$

[b] Find the equation of the straight line which passes through the two points (2 , – 1) , (1 , 1)

3 [a] If ABC is a right-angled triangle at B , AB = 12 cm. , AC = 13 cm. , find m ($\angle$ C) to the nearest degree.

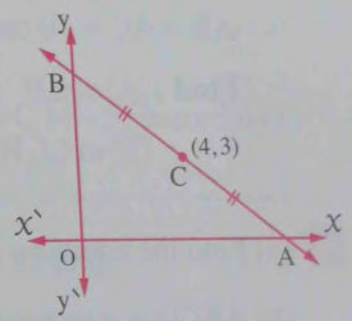
[b] If the straight line L_1 passes through the two points (X , – 1) , (6 , 3) and the straight line L_2 makes with the positive direction of the X-axis an angle of measure 45° , find the value of X if L_1 is perpendicular to L_2

4 [a] **Without using the calculator , prove that :** $\cos 30° = \frac{\sin 30° \sin 60°}{\sin 45° \cos 45°}$

[b] If the points A (1 , 0) , B (– 1 , 4) , C (7 , 8) and D (9 , 4) are in perpendicular coordinates plane , prove that the figure ABCD is a parallelogram.

5 [a] Find the slope of the straight line and the length of the y-intercept by the straight line whose equation is $\frac{x}{2} + \frac{y}{3} = 1$

[b] **In the opposite figure :**



The point C is the midpoint of $\overline{AB}$

where C (4 , 3)

Find (show the steps) :

1 The coordinates of the points A and B

2 The equation of $\overleftrightarrow{AB}$

14 Luxor Governorate

Answer the following questions :

1 **Choose the correct answer :**

1 If C is an acute angle where sin C = cos C , then tan C = ...

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{3}$

2 The straight line whose equation is $2x + 3y = 6$ intersects the x-axis at the point ....

(a) (2 , 0) (b) (3 , 0) (c) (0 , 2) (d) (0 , 3)

3 ABCD is a square , A (1 , 1) , C (4 , 4) , then its surface area = ... square units.

(a) 3 (b) 6 (c) 9 (d) 18

4 ABC is a triangle , m (∠ A) : m (∠ B) : m (∠ C) = 3 : 4 : 5 , then m (∠ B) = ...

(a) 30° (b) 45° (c) 60° (d) 90°

5 ABCD is a parallelogram , then $\overline{AB}$ // ....

(a) CD (b) $\overline{AD}$ (c) AD (d) $\overline{CD}$

6 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) half (b) double (c) third (d) quarter

2 [a] If X is an acute angle , **find the value of m (∠ X) when**

sin X = sin 60° cos 30° – cos 60° sin 30°

[b] Prove that the points A (0 , 1) , B (1 , 2) , C (2 , 3) are collinear.

3 [a] **Without using calculator , prove that :** $\tan 30° \tan 60° = \sin^2 45° + \cos^2 45°$

[b] If the straight line $k X - 2 y - 5 = 0$ makes a positive angle with the positive direction of the X-axis of measure 45° , find the value of k

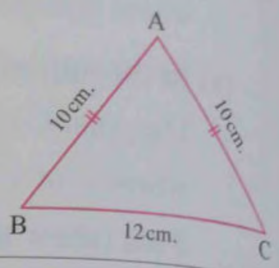
4 [a] If AB = 5 units of length , A (6 , X) , B (2 , 0) , find the value of X

[b] **In the opposite figure :**

AB = AC = 10 cm. , BC = 12 cm.

Find : 1 cos B

2 m (∠ B)



5 [a] Find the equation of the axis of symmetry of $\overline{AB}$ where A (– 1 , 4) , B (1 , 2)

[b] ABCD is a rectangle , A (1 , 1) , B (3 , 3) , C (0 , – 3 X) , D (X , y)

Find the value of each of X and y

15 New Valley Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 Δ ABC is a right-angled triangle at B , m (∠ C) = 30° , AB = 6 cm. , then AC = ... cm.

(a) 3 (b) 6 (c) 12 (d) 9

2 The distance between the two points (3 , 0) and (0 , – 4) equals .... length units.

(a) 4 (b) 3 (c) 7 (d) 5

3 If $\sin X = \frac{1}{2}$ where X is the measure of an acute angle , then $\sin 2X =$ ....

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

4 If the two straight lines whose slopes are $\frac{-2}{3}$ and $\frac{k}{2}$ are parallel , then k = ...

(a) $\frac{-4}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{1}{3}$ (d) 3

5 The measure of each interior angle of the regular pentagon equals ....

(a) 60° (b) 108° (c) 120° (d) 135°

6 The two diagonals are equal in length and not perpendicular in the ....

(a) square. (b) rhombus. (c) rectangle. (d) parallelogram.

2 [a] **Find the value of X, where $0° < X < 90°$, if :**

$\sin X = \sin 60° \cos 30° - \cos 60° \sin 30°$

[b] Prove that the points A (1 , 1) , B (2 , 2) and C (3 , 3) are collinear.

3 [a] Find the equation of the straight line which makes with the positive direction of X-axis a positive angle whose tan = 2 and intercepts from the positive part of y-axis 7 length units.

[b] Show the type of Δ ABC such that A (– 2 , 4) , B (3 , – 1) and C (4 , 5) according to its side lengths.

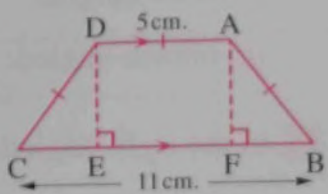
4 [a] If Δ ABC is a right-angled triangle at C , AB = 13 cm. , BC = 12 cm.

, **prove that** : $\sin A \cos B + \cos A \sin B = 1$

[b] Find the equation of the straight line which passes through the point (1 , 6) and the midpoint of $\overline{AB}$ where A (1 , – 2) , B (3 , – 4)

5 [a] Prove that the straight line passing through the two points (3 , – 4) and (1 , – 2) is perpendicular to the straight line that makes a positive angle of measure 45° with the positive direction of X-axis.

[b] **In the opposite figure :**



ABCD is an isosceles trapezium in which

$\overline{AD} \parallel \overline{BC}$, AB = AD = DC = 5 cm.

, BC = 11 cm.

Find : m ($\angle$ B) and the area of the trapezium ABCD

Exam 1 Port Said 2023

First Multiple choice questions

Choose the correct answer from those given :

1 If $\sin X = \frac{1}{2}$ where X is the measure of an acute angle , then $\cos 2X =$ ....

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{\sqrt{3}}{3}$

2 If $\overleftrightarrow{AB} // \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$, then the slope of $\overleftrightarrow{CD} =$ ....

(a) $\frac{3}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

3 The radius length of the circle whose centre is (0 , 0) and passes through the point (3 , 4) equals .... length units.

(a) 3 (b) 4 (c) 5 (d) 7

4 The triangle whose side lengths are 3 cm. , 4 cm. , 5 cm is ....

(a) acute-angled. (b) right-angled.

(c) obtuse-angled. (d) with congruent angles.

5 In the right-angled triangle ABC , if m (∠ B) = 90° , then sin A – cos C = ...

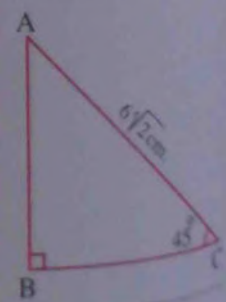
(a) 2 sin A (b) 2 cos C (c) zero (d) 1

6 If $\overline{AB}$ is a diameter in a circle where A (3 , – 5) and B (5 , 1) , then the centre of this circle is the point ....

(a) (4 , – 2) (b) (4 , 2) (c) (2 , – 2) (d) (8 , – 4)

7 **In the opposite figure :**

AB = ... cm.



(a) 3 (b) 4

(c) 5 (d) 6

8 A square of perimeter = 16 cm. , its area = ...

(a) 4 cm.² (b) 8 cm.² (c) 16 cm.² (d) 24 cm.²

9 The equation of the line which passes through the point $(2 , -3)$ and is parallel to y-axis is ....

(a) $y = -3$ (b) $X = 2$ (c) $y = 3$ (d) $X = -2$

10 The slope of the line that makes an angle of measure 45° with the positive direction of X-axis is ....

(a) $\frac{1}{2}$ (b) 1 (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

11 The complementary angle for the angle of measure 60° is an angle of measure ....

(a) 120° (b) 0° (c) 30° (d) 90°

12 $4 \cos 60° \sin 30° =$ ....

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 4 (d) 1

13 The line whose equation is $y = 3X + 4$ cuts from the positive part of the y-axis .... length units.

(a) 2 (b) 3 (c) 4 (d) 7

14 The slope of the line that is parallel to the X-axis is ....

(a) -1 (b) zero (c) 1 (d) undefined.

15 The sum of the measures of all interior angles of any quadrilateral is ....

(a) 90° (b) 180° (c) 360° (d) 540°

16 For any angle of measure a , then $\frac{\sin a}{\cos a} =$ ....

(a) sin a cos a (b) 1 (c) tan a (d) -1

17 The slope of the line whose equation is : $2X - 2y = 3$ is ....

(a) 3 (b) 2 (c) -2 (d) 1

18 If $\sin H = 0.6214$, then $m(\angle H) \simeq$ ....

(a) 55° 38̀ (b) 38° 25̀ (c) 83° 52̀ (d) 48° 52̀

19 The length of the perpendicular from $(3 , -4)$ to the X-axis is .... length units.

(a) 3 (b) -4 (c) 4 (d) 5

20 $\sin 70° = \cos$ ....

(a) 110° (b) 20° (c) 290° (d) 360°

21 The line whose equation is : $2X + 3y = 0$ passes through the point ....

(a) $(3 , 2)$ (b) $(2 , 3)$ (c) $(0 , 0)$ (d) $(1 , -1)$

Second Essay questions

22 Find the equation of $\overleftrightarrow{AB}$ which passes through A (0 , 4) and B (4 , 0)

23 ABC is a triangle in which $\angle$ B is a right angle , AB = 5 cm. and BC = 12 cm.
Find : $\sin^2 A + \cos^2 A$

24 Show the type of Δ ABC with respect to its sides where : A (3 , 3) , B (1 , 5) and C (1 , 3)

Exam 2 Port Said 2024

First Multiple choice questions

Choose the correct answer from those given :

1 If the origin point is the midpoint of $\overline{AB}$ and A (5 , − 2) , then B = ...
(a) (2 , 5) (b) (5 , − 2) (c) (− 2 , − 5) (d) (− 5 , 2)

2 $2 \sin 30° \tan 60° =$ ....
(a) $\sqrt{3}$ (b) 3 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

3 The distance between the two points (3 , a) and (− 1 , a) equals .... length units.
(a) 3 (b) 4 (c) 9 (d) 16

4 If X , y are the measures of two complementary angles and $\sin X = \frac{3}{5}$, then cos y = ...
(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{5}{4}$ (d) $\frac{5}{3}$

5 44.125° = ... in degrees , minutes and seconds.
(a) 44° 7′ 30″ (b) 44° 30′ 7″ (c) 44° 17′ 30″ (d) 44° 30′ 17″

6 The sum of measures of all interior angles of a triangle equals ....
(a) 120° (b) 150° (c) 180° (d) 360°

7 For any acute angle of measure a , then sin a − cos a tan a = ...
(a) − 1 (b) 0 (c) 1 (d) 2

8 If m_1 , m_2 are the slopes of two parallel lines , then ....
(a) $m_1 - m_2 = 0$ (b) $m_1 + m_2 = 0$ (c) $m_1 m_2 = 0$ (d) $m_1 - m_2 \neq 0$

9 A circle its centre is the origin point and its radius length equals 5 cm. , then the point (3 , 4) lies .... the circle.
(a) inside (b) outside (c) on (d) on the centre of

10 If $X \cos 60° = \tan 45°$, then $X =$ ……………

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\sqrt{2}$

11 If ABCD is a square , then m ($\angle$ ABD) = ……………

(a) 30° (b) 45° (c) 60° (d) 90°

12 The product of the slopes of two perpendicular lines equals ……………

(a) zero (b) 1 (c) – 1 (d) $\frac{1}{2}$

13 The line whose equation is : $y - 3X + 1 = 0$ passes through the point ……………

(a) (1 , 2) (b) (2 , 1) (c) (0 , 3) (d) (3 , 0)

14 If $\sin (X + 7)° = \frac{1}{2}$ where X is the measure of an acute angle , then $X =$ ……………

(a) 60° (b) 30° (c) 23° (d) 13°

15 The number of symmetry axes of an isosceles triangle equals ……………

(a) zero (b) 1 (c) 2 (d) 3

16 If A = (5 , 7) and B = (1 , – 1) , then the midpoint of $\overline{AB}$ is ……………

(a) (2 , 3) (b) (3 , 3) (c) (3 , 2) (d) (3 , 4)

17 ABC is a triangle in which m ($\angle$ A) = 85° , sin B = cos B , then m ($\angle$ C) = ……………

(a) 30° (b) 45° (c) 50° (d) 60°

18 The equation of the line that passes through the origin point and has slope = 1 is ……………

(a) $y = X$ (b) $y = -X$ (c) $y = 2X$ (d) $y = 0$

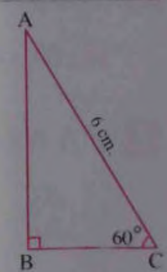
19 The equation of the line which passes through the point (– 5 , 3) and is parallel to X-axis is ……………

(a) $X = -5$ (b) $y = -5$ (c) $y = 3$ (d) $X = 3$

20 The line whose equation is : $3y = 2X - 6$ cuts from the y-axis a part of length …………… units.

(a) 6 (b) – 6 (c) 2 (d) – 2

21 **In the opposite figure :**



AB = …………… cm.

(a) 3 (b) $2\sqrt{3}$

(c) $3\sqrt{3}$ (d) $\sqrt{6}$

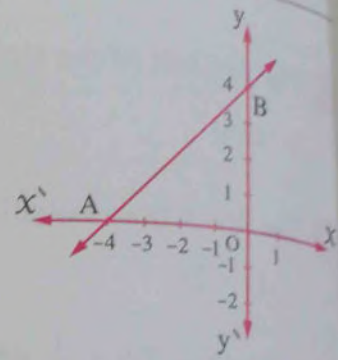
Second Essay questions

22 If $\cos H = \sin^2 45° \tan 60°$, find the measure of the acute angle H

23 Show that A (– 3 , – 1) , B (6 , 5) and C (3 , 3) are three collinear points.

24 **In the opposite figure :**

Find the equation of $\overleftrightarrow{AB}$ which cuts from the negative the X-axis and positive the y-axis two equal parts of length 4 length units.



Exam 3

First Multiple choice questions

Choose the correct answer from those given :

1 If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{1}{2}$, then the slope of $\overleftrightarrow{CD}$ = ...

(a) 2 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) – 2

2 The perpendicular distance between the two straight lines : y + 1 , y + 3 = 0 equals .... length units.

(a) 4 (b) 2 (c) 1 (d) 5

3 The equation of the straight line which is passing through (2 , 3) and parallel to the X-axis is ....

(a) $X = 2$ (b) $X = 3$ (c) $y = 2$ (d) $y = 3$

4 The number of the axes of symmetry of the isosceles triangle is ....

(a) 1 (b) 2 (c) 3 (d) 4

5 The distance between (4 , 3) and the y-axis is .... length units.

(a) – 3 (b) – 4 (c) 3 (d) 4

6 The point (– 1 , 3) is the image of the point (5 , 3) by reflection in the point ....

(a) (0 , 0) (b) (4 , 6) (c) (2 , 3) (d) (– 2 , – 3)

7 If Δ ABC is right-angled at A , then sin B = ...

(a) $\frac{AC}{BC}$ (b) $\frac{AB}{AC}$ (c) $\frac{BC}{AC}$ (d) $\frac{AC}{AB}$

8 The slope of the straight line : $3X + 2y - 5 = 0$ is ....

(a) $\frac{3}{2}$ (b) $-\frac{3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{5}{2}$

9 If $\sin X = \frac{1}{2}$, where X is the measure of an acute angle , then $\sin 2X =$ ....

(a) $\frac{1}{4}$ (b) 1 (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

10 The parallelogram whose diagonals are perpendicular and equal in length is ....

(a) a square. (b) a rhombus. (c) a rectangle. (d) a trapezium.

11 If ΔABC is right-angled at B and $\sin A = \frac{3}{5}$, then $\cos C =$ ....

(a) $\frac{5}{3}$ (b) $\frac{4}{5}$ (c) $\frac{5}{4}$ (d) $\frac{3}{5}$

12 The distance between $(3 , -4)$ and the origin point is .... length units.

(a) 5 (b) 1 (c) -1 (d) -5

13 The straight line : $X + 2y = 6$ cuts from the positive part of the y-axis a part of length .... units.

(a) 6 (b) 3 (c) 2 (d) -3

14 If $\tan (X + 10°) = 1$ where X is the measure of an acute angle , then $X =$ ....

(a) 45° (b) 35° (c) 55° (d) 50°

15 If ΔXYZ is right-angled at Y , XY = 12 cm. , YZ = 5 cm. , then $\sin^2 X + \sin^2 Z =$ ....

(a) 1 (b) $\frac{25}{144}$ (c) $\frac{144}{169}$ (d) $\frac{25}{169}$

16 The angle of measure 40° complements an angle of measure ....

(a) 50° (b) 80° (c) 90° (d) 140°

17 If $a \sin 30° = 4 \sin 45° \cos 45°$, then $a =$ ....

(a) 2 (b) 4 (c) 8 (d) 16

18 The slope of the straight line passing through the two points $(3 , -1)$, $(1 , -2)$ is ....

(a) 2 (b) $\frac{1}{2}$ (c) -2 (d) $-\frac{1}{2}$

19 If C $(2 , 1)$ is the midpoint of $\overline{AB}$ where A $(4 , -1)$, then B = ...

(a) $(6 , 0)$ (b) $(2 , 2)$ (c) $(0 , 3)$ (d) $(2 , 0)$

20 If the straight line whose equation is : $a X + y = 5$ is parallel to the straight line passing through $(1 , 4)$, $(3 , 5)$, then $a =$ ....

(a) $-\frac{1}{4}$ (b) $\frac{3}{5}$ (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

21 If $\sin 2X = 2 \sin 30° \cos 60°$, where X is the measure of an acute angle , then $X =$ ....

(a) $15°$ (b) $30°$ (c) $45°$ (d) $60°$

Second Essay questions

22 Determine the type of Δ ABC where A $(1 , 1)$, B $(5 , 1)$, C $(3 , 4)$ according to the lengths of its sides.

23 Find the equation of the straight line which passes through $(3 , -5)$ and parallel to the straight line : $X + 3y = 7$

24 **Find the value of :** $\cos 60° \sin 30° - \sin 60° \tan 60° + \cos^2 30°$

Exam 4

First Multiple choice questions

Choose the correct answer from those given :

1 The equation of the straight line which is perpendicular to the y-axis is ....

(a) $X = 0$ (b) $y = X$ (c) $y = -X$ (d) $y = 0$

2 If $X \cos 60° = \tan 45°$, then $X =$ ....

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

3 The measure of the exterior angle of the equilateral triangle is ....

(a) $120°$ (b) $90°$ (c) $60°$ (d) $30°$

4 The slope of the straight line that makes with the positive direction of the X-axis a positive angle of measure θ equals ....

(a) $\sin \theta$ (b) $\cos \theta$ (c) $\frac{\sin \theta}{\cos \theta}$ (d) $\sin \theta + \cos \theta$

5 The slope of the straight line which makes an angle of measure $60°$ with the positive direction of the X-axis is ....

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

6 If the y-axis bisects $\overline{AB}$ such that A (3 , 2) , B (x , y) , then x = ...

(a) – 3 (b) – 2 (c) 2 (d) 3

7 If A (2 , – 1) , B (– 4 , 3) , then the midpoint of $\overline{AB}$ is ....

(a) (2 , 2) (b) (– 2 , – 4) (c) (– 1 , 1) (d) (3 , – 2)

8 If $\cos x = \frac{\sqrt{3}}{2}$, where x is the measure of an acute angle , then $\sin 2x$ = ...

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) – 2 (d) $\frac{1}{\sqrt{3}}$

9 If Δ XYZ is right-angled at Y , XY = 16 cm. , m (∠ X) = 54° , then YZ ≃ .... cm.

(a) 22 (b) 14 (c) 12 (d) 15

10 The distance between the two points (– 2 , 5) , (– 2 , – 4) is .... length units.

(a) – 2 (b) 1 (c) 0 (d) 9

11 If A lies on the axis of symmetry of $\overline{XY}$, then $\overline{AX}$ .... $\overline{AY}$

(a) // (b) = (c) ≡ (d) ⊥

12 If Δ ABC is right-angled at B , AB = 8 cm. , BC = 6 cm. , then sin C = ...

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

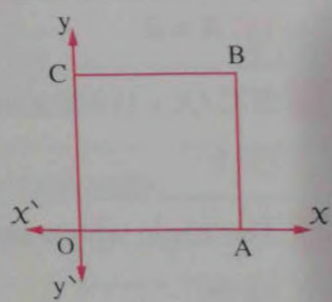
13 The straight line passing through (2 , 1) , (4 , 0) is parallel to the straight line whose equation is ....

(a) $2x + y = 1$ (b) $y = \frac{1}{2}x + 3$ (c) $x + 2y = 5$ (d) $2x + 3y = 3$

14 If AB = 5 length units , A (4 , – 1) , then B could be ....

(a) (– 1 , 4) (b) (2 , 1) (c) (1 , 3) (d) (5 , 0)

15 **In the opposite figure :**



OABC is a square of side length 4 cm.

, then the equation of $\overleftrightarrow{AC}$ is ....

(a) $y = x + 4$ (b) $y = x - 4$

(c) $y = -x + 4$ (d) $x = 4y + 4$

16 If Δ ABC is right-angled at B , then sin C + cos C 1

(a) = (b) > (c) < (d) ≤

17 The sum of measures of the accumulative angles at a point equals ....

(a) $90°$ (b) $180°$ (c) $270°$ (d) $360°$

18 $\overline{AB}$ is a diameter in a circle whose centre is M (2 , – 1) , if A (– 2 , 3) , then B = ...

(a) (0 , 1) (b) (0 , 2) (c) (2 , – 2) (d) (6 , – 5)

19 If Δ ABC is right-angled at B , 2 AB = $\sqrt{3}$ AC , then m ($\angle$ C) = ...

(a) $30°$ (b) $45°$ (c) $60°$ (d) $75°$

20 The straight line whose equation is : $2 x - 3 y = 6$ cuts from the negative part of the y-axis a part of length .... units.

(a) 6 (b) 2 (c) – 3 (d) 3

21 If $\cos 70° = \sin x$ where x is the measure of an acute angle , then x = ...

(a) $60°$ (b) $45°$ (c) $30°$ (d) $20°$

Second Essay questions

22 **Prove that :** $\sin 60° = 2 \sin 30° \cos 30°$

23 Prove that the points A (– 3 , – 1) , B (6 , 5) , C (2 , 4) , D (– 7 , – 2) are the vertices of a parallelogram.

24 Find the equation of the straight line whose slope = 2 and passes through the point (1 , 3)

Exam 5

First Multiple choice questions

Choose the correct answer from those given :

1 The equation of the y-axis is ....

(a) $x = 0$ (b) $y = 0$ (c) $x = y$ (d) $y = 1$

2 If C (x , 1) is the midpoint of $\overline{AB}$ where A (5 , y) , B (3 , 3) , then $x + y$ = ...

(a) 5 (b) 3 (c) – 1 (d) 4

3 The angle whose measure is $30°$ supplements an angle of measure ....

(a) $60°$ (b) $120°$ (c) $150°$ (d) $180°$

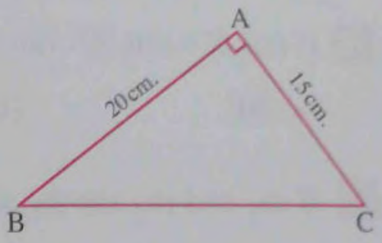
4 If $\sin x = \frac{1}{2}$ where x is the measure of an acute angle , then $\tan (x + 15°)$ = ...

(a) 1 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) – 1

5 ABCD is a parallelogram , its diagonals interesect at M where A (3 , – 1) , C (1 , 7) , then the point M is ....

(a) (3 , 1) (b) (2 , 3) (c) (3 , 2) (d) (1 , 3)

6 **In the opposite figure :**



cos C cos B – sin C sin B = ...

(a) 0 (b) 1
(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

7 A circle its centre is the origin point and its radius length is 2 length units. Which of the following points lies on the circle ?

(a) (1 , 2) (b) (– 2 , 1) (c) $(\sqrt{3} , 1)$ (d) $(\sqrt{2} , 1)$

8 The slope of the straight line which makes an angle of measure 45° with the positive direction of the X-axis is ....

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 1 (d) – 1

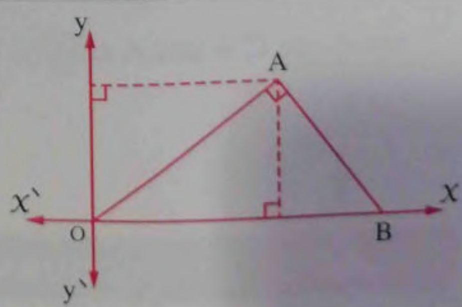
9 The equation of the straight line which passes through (2 , – 1) and is parallel to the X-axis is ....

(a) $X = 2$ (b) y = 2 (c) $X = -1$ (d) y = – 1

10 The image of the point (3 , 2) by reflection in the origin point is ....

(a) (– 3 , – 2) (b) (– 3 , 2) (c) (3 , – 2) (d) (2 , 3)

11 **In the opposite figure :**



Δ ABO is right-angled at A , A (6 , 3)

, then tan (∠ AOB) = ...

(a) 2 (b) $\frac{1}{2}$
(c) $\frac{\sqrt{3}}{2}$ (d) $-\frac{1}{2}$

12 The distance between the two points (3 , 2) , (– 1 , 5) is .... length units.

(a) 4 (b) 5 (c) 6 (d) $5\sqrt{2}$

13 sin 30° cos 60° = ...

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $2\sqrt{3}$

14 The straight line whose equation is : $y - X = 3$ makes an angle with the positive direction of the X-axis of measure ....

(a) $45°$ (b) $30°$ (c) $60°$ (d) $135°$

15 If $\cos X = \sin 30° \tan 45°$ where X is the measure of an acute angle , then $X =$ ....

(a) $30°$ (b) $60°$ (c) $90°$ (d) $180°$

16 If m_1 and m_2 are the slopes of two parallel straight lines , then ....

(a) $m_1\ m_2 = 2$ (b) $m_1\ m_2 = 1$ (c) $m_1 - m_2 = 0$ (d) $m_1\ m_2 = -1$

17 If Δ ABC is right-angled at B , AB = 3 BC , then tan C = ...

(a) 3 (b) $\frac{1}{3}$ (c) $\frac{3}{\sqrt{10}}$ (d) $\frac{1}{\sqrt{10}}$

18 The number of the axes of symmetry of the equilateral triangle is ....

(a) 0 (b) 1 (c) 2 (d) 3

19 The straight line whose equation is : $\frac{X}{2} - \frac{y}{3} = 6$ cuts from the positive part of the X-axis a part of length .... units.

(a) 3 (b) 12 (c) 6 (d) 18

20 The straight line whose equation is : $2X + y - 2 = 0$ is perpendicular to the straight line whose equation is ....

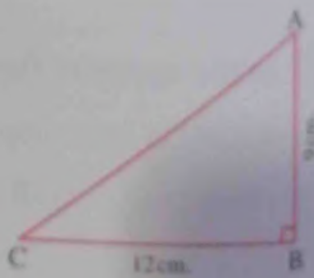
(a) $y = 2X + 2$ (b) $2y - X = 3$ (c) $y = 2X$ (d) $2X + 3y = 0$

21 **In the opposite figure :**

$\sin A \cos C + \cos A \sin C =$ ....

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{3}{5}$ (d) $\frac{4}{5}$



Second Essay questions

22 **Find the value of X where :** $\sin X = \sin 60° \cos 30° - \cos 60° \sin 30°$, $0° < X < 90°$

23 Prove that the points A (3 , – 1) , B (– 4 , 6) , C (2 , – 2) lie on one circle whose centre is M (– 1 , 2)

24 Find the equation of the straight line which passes through (1 , 3) and is perpendicular to the straight line : $X + 3y = 4$

EL-MOASSER

By a group of supervisors

GUIDE ANSWERS

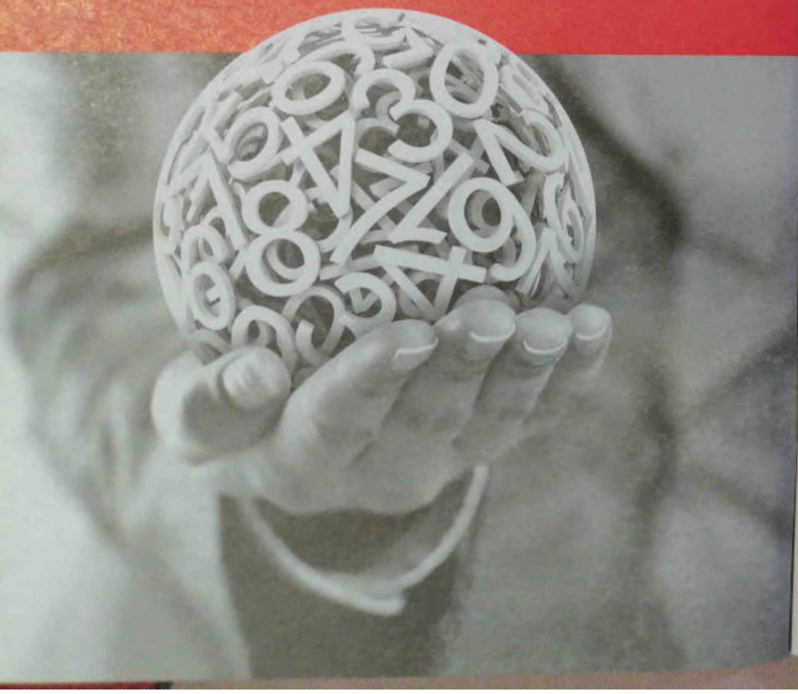
3rd PREP. 2025

FIRST TERM

Maths

Guide Answers

Of Algebra and Statistics Exercises



Answers of unit one

Answers of Exercise 1

1

1 $a = -5$, $b = 9$ 2 $a = \sqrt{25} = 5$, $b = \sqrt[3]{27} = 3$

3 $a - 2 = 2$ $\therefore a = 4$, $b + 1 = -3$ $\therefore b = -4$

4 $2 - a = 6$ $\therefore a = -4$, $b - 3 = -1$ $\therefore b = 2$

5 $a - 7 = -2$ $\therefore a = 5$, $b^3 - 1 = 26$

$\therefore b^3 = 27$ $\therefore b = \sqrt[3]{27} = 3$

6 $\because a = 2 - a$ $\therefore 2a = 2$ $\therefore a = 1$

, $b = 2b - 3$ $\therefore b = 3$

7 $a^5 = 32$ $\therefore a^5 = 2^5$ $\therefore a = 2$

, $b^2 - 1 = \sqrt[3]{27}$ $\therefore b^2 - 1 = 3$

$\therefore b^2 = 4$ $\therefore b = \pm 2$

8 $b = 7$, $a = b^2 = 49$

9 $a = 7$, $2a = 2b + 1$ $\therefore 2b + 1 = 14$

$\therefore 2b = 13$ $\therefore b = 6.5$

10 $5a - 1 = 3$ $\therefore 5a = 4$ $\therefore a = \frac{4}{5}$

, $b = 4a$ $\therefore b = 4 \times \frac{4}{5}$ $\therefore b = \frac{16}{5}$

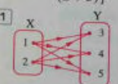
2

1 a   2 d   3 c   4 c   5 d

3

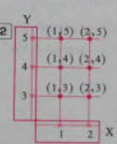
$X \times Y = \{(1 , 3) , (1 , 4) , (1 , 5) , (2 , 3) , (2 , 4) , (2 , 5)\}$

1



The arrow diagram

2

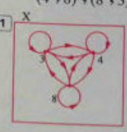


The Cartesian diagram

4

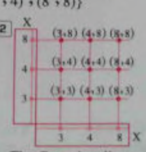
$X^2 = \{(3 , 3) , (3 , 4) , (3 , 8) , (4 , 3) , (4 , 4) , (4 , 8) , (8 , 3) , (8 , 4) , (8 , 8)\}$

1



The arrow diagram

2



The Cartesian diagram

5

1 $X \times Y = \{(1 , 4) , (2 , 4) , (3 , 4)\}$

2 $Y \times X = \{(4 , 1) , (4 , 2) , (4 , 3)\}$

3 $Y^2 = \{(4 , 4)\}$

4 $n(X^2) = 3^2 = 9$

6

1 $X \times Y = \{(2 , 4) , (2 , 0) , (-1 , 4) , (-1 , 0)\}$

2 $Y \times Z = \{(4 , 4) , (4 , 5) , (4 , -2) , (0 , 4) , (0 , 5) , (0 , -2)\}$

3 $X^2 = \{(2 , 2) , (2 , -1) , (-1 , 2) , (-1 , -1)\}$

4 $n(X \times Z) = 2 \times 3 = 6$

5 $n(Y^2) = 2 \times 2 = 4$

6 $n(Z^2) = 3 \times 3 = 9$

7

1 b	2 b	3 d	4 d	5 a
6 d	7 a	8 a	9 b	10 b
11 a	12 d	13 c	14 c	15 c

8

$X = \{2 , 3 , 5\}$, $Y = \{6 , 9\}$

9

1 $X = \{1\}$, $Y = \{1 , 3 , 5\}$

2 $Y \times X = \{(1 , 1) , (3 , 1) , (5 , 1)\}$

3 $Y^2 = \{(1 , 1) , (1 , 3) , (1 , 5) , (3 , 1) , (3 , 3) , (3 , 5) , (5 , 1) , (5 , 3) , (5 , 5)\}$

10

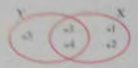
$X = \{1 , 2\}$

11

$X = \{3\}$

$\therefore X^2 = \{3\} \times \{3\} = \{(3 , 3)\}$

12



1 $(X \cap Y) \times Y = \{3 , 4\} \times \{3 , 4 , 5\}$
$= \{(3 , 3) , (3 , 4) , (3 , 5) , (4 , 3) , (4 , 4) , (4 , 5)\}$

2 $(X - Y) \times Y = \{1 , 2\} \times \{3 , 4 , 5\}$
$= \{(1 , 3) , (1 , 4) , (1 , 5) , (2 , 3) , (2 , 4) , (2 , 5)\}$

3 $(Y - X) \times X = \{5\} \times \{1 , 2 , 3 , 4\}$
$= \{(5 , 1) , (5 , 2) , (5 , 3) , (5 , 4)\}$

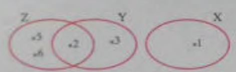
13

1 $X \times (Y \cap Z) = \{3 , 4\} \times \{5\} = \{(3 , 5) , (4 , 5)\}$

2 $(X - Y) \times Z = \{3\} \times \{6 , 5\} = \{(3 , 6) , (3 , 5)\}$

3 $(X - Y) \times (Y - Z) = \{3\} \times \{4\} = \{(3 , 4)\}$

14



First :

1 $X \times Y = \{(1 , 2) , (1 , 3)\}$

2 $Y \times Z = \{(2 , 2) , (2 , 5) , (2 , 6) , (3 , 2) , (3 , 5) , (3 , 6)\}$

3 $X \times Z = \{(1 , 2) , (1 , 5) , (1 , 6)\}$

4 $Y^2 = \{(2 , 2) , (2 , 3) , (3 , 2) , (3 , 3)\}$

Second :

$(X \times Y) \cup (Y \times Z) = \{(1 , 2) , (1 , 3) , (2 , 2) , (2 , 5) , (2 , 6) , (3 , 2) , (3 , 5) , (3 , 6)\}$

Third : $X \times (Y \cap Z) = \{1\} \times \{2\} = \{(1 , 2)\}$

Fourth : $(X \times Y) \cap (X \times Z) = \{(1 , 2)\}$

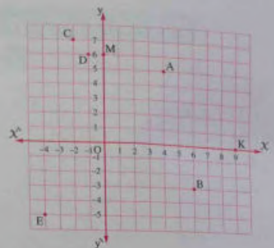
Fifth : $(Z - Y) \times (X \cup Y) = \{5 , 6\} \times \{1 , 2 , 3\}$
$= \{(5 , 1) , (5 , 2) , (5 , 3) , (6 , 1) , (6 , 2) , (6 , 3)\}$

15

1 $\because Y = \{2 , 3\}$ $\therefore n(Y) = 2$
$, \because n(X \times Y) = 6$ $\therefore n(X) = 3$
$\therefore X = \{1 , 2 , 3\}$

2 $(X \cap Y) \times Y = \{2 , 3\} \times \{2 , 3\}$
$= \{(2 , 2) , (2 , 3) , (3 , 2) , (3 , 3)\}$

16

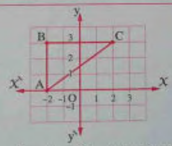


A Lies on the first quadrant
B Lies on the fourth quadrant
C Lies on the second quadrant
D Lies on the second quadrant
E Lies on the third quadrant
M Lies on y-axis **K** Lies on X-axis

17

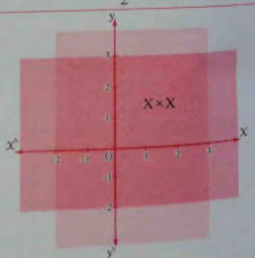
1 b	2 a	3 c	4 b	5 b
6 d	7 d	8 c	9 b	10 a
11 c	12 a	13 b	14 c	

18



$\because$ AB = 3 length unit , BC = 4 length unit
$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times AB \times BC$
$= \frac{1}{2} \times 3 \times 4 = 6$ square unit

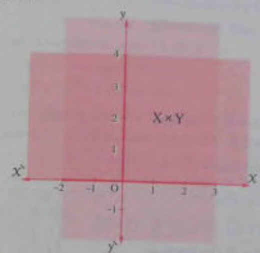
19



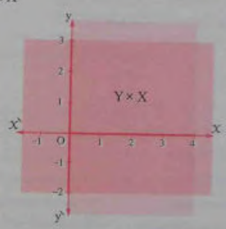
$A \in X \times X$ $B \in X \times X$
$C \notin X \times X$ $D \in X \times X$

20

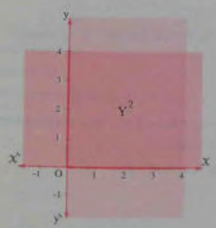
1 $X \times Y$



2 $Y \times X$



3 Y^2



21

1 a 2 a

22

$\because X = \{a , 2\} , Y = \{1 , 2 , 3\}$
$, \because X \subset Y$ $\therefore a = 1$ or $a = 3$

23

$X = \{4 , 1\}$, $Y = \{4 , 1 , 7\}$
$, X \times Y = \{(4 , 4) , (4 , 1) , (4 , 7) , (1 , 4) , (1 , 1) , (1 , 7)\}$

Answers of Exercise 2

1

1 b 2 b 3 c 4 a

2

R_1 is a function and its range = $\{1 , 9\}$
R_2 is a function and its range = $\{1 , 4 , 9\}$
R_3 is not a function.
R_4 is not a function.

3

f_1 is not a function because $3 \in X$ has no image
f_2 is not a function because $2 \in X$ has two images
f_3 is a function
$, f_3 = \{(a , 3) , (b , 3) , (c , 3) , (d , 3) , (e , 3)\}$, its range = $\{3\}$

4

1 R_1 is not a function because $c \in X$ has no image.

2 R_2 is not a function because $b \in X$ has two images.

3 R_3 is a function because each element of X appears only once as a first projection in an ordered pair of the relation.
and the range = $\{2 , 8 , 10\}$

5

1 $R = \{(-3 , 1) , (-3 , 8) , (1 , 2) , (4 , 4)\}$

2 R is not a function because $-3 \in X$ has two images.

3 $\because (X , 2) \in R$ $\therefore X = 1$

6

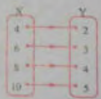
R = {(1 , 3) , (2 , 6) , (3 , 9)}

, R is a function from X to Y because each element of X has one image in Y

and the range = {3 , 6 , 9}

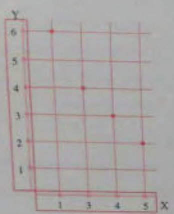
7

R = {(4 , 2) , (6 , 3) , (8 , 4)
, (10 , 5)}



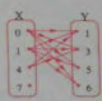
8

R = {(1 , 6) , (3 , 4) , (4 , 3) , (5 , 2)}



9

R = {(0 , 1) , (0 , 3) , (0 , 5)
, (0 , 6) , (1 , 1) , (1 , 3)
, (1 , 5) , (1 , 6)
, (4 , 1) , (4 , 3)}



R is not a function because 0 ∈ X , 1 ∈ X , 4 ∈ X each of them has more than one image in Y

also 7 ∈ X has no image in Y

10

R = {(2 , 4) , (2 , 5) , (2 , 6) , (2 , 7) ,
(2 , 9) , (4 , 4) , (4 , 5) , (4 , 6) ,
(4 , 7) , (4 , 9) , (5 , 5) , (5 , 6) ,
(5 , 7) , (5 , 9) , (7 , 7) , (7 , 9)}

Represent by yourself.

11

1 R = {(1 , 2) , (2 , 4)
, (3 , 6) , (4 , 8)}

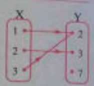
2 R is a function from X to Y because each element of X has one image in Y

, its range = {2 , 4 , 6 , 8}

12

1 R = {(1 , 2) , (2 , 3) , (3 , 2)}

Yes , R is a function.



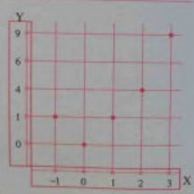
2 ∵ (2 , 3) ∈ R , ∵ 2 a R 3

∴ 2 a = 2 ∴ a = 1

13

1 R = {(−1 , 1)
, (0 , 0) , (1 , 1)
, (2 , 4) , (3 , 9)}

2 R is a function from X to Y because each element of X has one image in Y



14

R = {(1 , 1) , (2 , 8)} Represent by yourself.

15

R = {(−1 , −1) , (1 , 1) , (2 , 8)}

R is a function from X to Y because each element of X has one image in Y

, its range = {−1 , 1 , 8}

16

$R = \left\{\left(-2 , \frac{1}{4}\right) , \left(-1 , \frac{1}{2}\right) , (0 , 1) , (1 , 2) , (2 , 4)\right\}$

Represent by yourself.

, R is a function from X to Y because each element of X has one image in Y

The range = $\left\{\frac{1}{4} , \frac{1}{2} , 1 , 2 , 4\right\}$

17

R = {(2 , 10) , (2 , 16) , (2 , 24) , (2 , 30)
, (5 , 10) , (5 , 30) , (8 , 16) , (8 , 24)}

R is not a function because 2 ∈ X has more than one image in Y

also 5 ∈ X , 8 ∈ X each of them has two images in Y

Represent by yourself.

18

R = {(2 , 6) , (2 , 8) , (2 , 10) , (3 , 6) , (3 , 15)
, (4 , 8)}

19

1 a   2 c

20

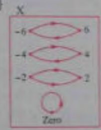
The arrow diagram number (2)

21

R = {(6 , −6) , (4 , −4) , (2 , −2) , (0 , 0)
, (−2 , 2) , (−4 , 4) , (−6 , 6)}

∴ R is a function on X because each element of X has a unique image in X

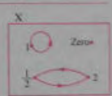
its range = X



22

$R = \left\{(1 , 1) , \left(2 , \frac{1}{2}\right) , \left(\frac{1}{2} , 2\right)\right\}$

∴ R is not a function because 0 ∈ X has no image in X



23

R = {(1 , 1) , (1 , 2) , (1 , 3) , (1 , 6)
, (1 , 11) , (3 , 1) , (3 , 2) , (3 , 3)
, (3 , 6) , (3 , 11) , (11 , 1) , (11 , 2)
, (11 , 3) , (11 , 6) , (11 , 11)}

R is not a function because each of 1 ∈ X , 3 ∈ X , 11 ∈ X has more than one image in X

also each of 2 ∈ X , 6 ∈ X has no image in X

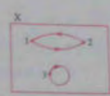
24

∵ X = {1 , 2 , 3}

R = {(1 , 2) , (2 , 1) , (3 , 3)}

∴ R is a function.

, its range = {1 , 2 , 3}



25

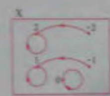
R = {(1 , 1) , (2 , 1) , (2 , 2) , (4 , 1) ,
(4 , 2) , (4 , 4) , (6 , 1) , (6 , 2) ,
(6 , 6) , (10 , 1) , (10 , 2) , (10 , 10)}

R is not a function because each of 2 ∈ X , 4 ∈ X , 6 ∈ X and 10 ∈ X has more than one image in X

26

R = {(−2 , 2) , (−1 , 1) , (0 , 0)
, (1 , 1) , (2 , 2)}

R is a function on X



27

1 ∵ R is a function from X to Y

∴ each element in X has only one image in Y

, ∵ the image of $-2 = (-2)^2 - 1 = 3$ ∈ Y

, the image of $2 = (2)^2 - 1 = 3$ ∈ Y

, the image of $5 = (5)^2 - 1 = 24$ ∈ Y

∴ $\ell = 24$

2 Represent by yourself.

28

1. $R_1 = \{(0, 0), (4, 2), (16, 4)\}$

   $\therefore R_1$ is a function from X to Y

2. $R_2 = \{(0, 0)\}$

   $\therefore R_2$ is not a function because each of $4 \in X$, $16 \in X$ has no image in Y

3. $R_3 = \{(0, 0), (4, 2)\}$

   $\therefore R_3$ is not a function because $16 \in X$ has no image in Y

29

$a R b$, $a \times b = 12$

1. $\because X R 4$ $\quad \therefore X \times 4 = 12$ $\quad \therefore X = 3$

2. $\because y R 3y$ $\quad \therefore y \times 3y = 12$

   $\therefore 3y^2 = 12$ $\quad \therefore y^2 = 4$

   $\therefore y = 2$ or $y = -2$ (refused because $y \in \mathbb{N}$)

30

$\because R_1 = \{(1, -1), (0, 0), (-1, 1)\}$

$, R_2 = \{(1, 1), (-1, -1)\}$ $, R = R_1 \cap R_2$

$\therefore R = \varnothing$, R is not a function

31

1. $R = \{(1, 13), (1, 31), (2, 23), (3, 13), (3, 31), (3, 23)\}$ (Represent by yourself)

2. 2 R 65 false (say why by yourself)

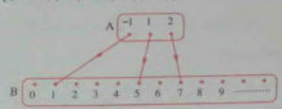
   1 R 31 true (say why by yourself)

   3 R 13 true (say why by yourself)

3. $M = \{(2, 23), (3, 23)\}$

32

$R = \{(-1, 1), (1, 5), (2, 7)\}$



33

1. L does not represent a relation because : $L \not\subset X \times Y$

2. M represents a relation because : $M \subset X \times Y$

34

1. The range $= \{3, 1, 5\}$

2. $\because f$ is a function on X

   $\therefore$ Each element in X has to appear only once as a first projection in R

   $\therefore a = 3, b = 5$ or $a = 5, b = 3$

   $\therefore a + b = 3 + 5 = 8$

35

$R = \{(-1, 1), (0, 0), (1, 1)\}$

R is not a function because $-2 \in X$, $2 \in X$ did not appear as a first projection in the ordered pairs of R

36

$\because$ a divides b

$, X \cup Y = \{2, 3, 5, 11, 14, 9, 35\}$

$, f$ is a function from X to Y

$, \because$ 2 divides 14 $\quad \therefore 2 \in X, 14 \in Y$

$, \because$ 3 divides 9 $\quad \therefore 3 \in X, 9 \in Y$

$, \because$ 5 divides 35 $\quad \therefore 5 \in X, 35 \in Y$

$, \because n(X) = 3$ $\quad \therefore X = \{2, 3, 5\}$

$, \because n(X \times Y) = 12$ $\quad \therefore n(Y) = 4$

$\therefore Y = \{14, 9, 35, 11\}$

$, R = \{(2, 14), (3, 9), (5, 35)\}$

, its range $= \{14, 9, 35\}$

37

$\because X \cup Y = \{4, 8, 9, 27\}$ $, n(X) = 4$

$\therefore X = \{4, 8, 9, 27\}$

$, \because$ a is a multiple of b $, f$ is a function from X to Y

$, n(Y) = 2$ $\quad \therefore Y = \{4, 9\}$

$\therefore R = \{(4, 4), (8, 4), (9, 9), (27, 9)\}$

, the range of the function $= \{4, 9\}$

Answers of Exercise 3

1

1 c	2 d	3 b	4 c	5 d
6 c	7 a	8 d	9 c	10 b
11 d	12 d	13 c	14 d	15 c
16 a	17 c	18 a	19 c	20 b
21 d	22 d			

2

	Degree	$f(-2)$	$f(0)$	$f\left(\frac{1}{2}\right)$
1	First	7	3	2
2	Second	zero	-4	$-3\frac{3}{4}$

3

$\because f(2) = 2 \times (2)^2 - 5 \times 2 + 2 = \text{zero}$

$, f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 - 5 \times \frac{1}{2} + 2 = \text{zero}$

$\therefore f(2) = f\left(\frac{1}{2}\right)$

4

$\because f(2) = 2 \times 2 - 1 = 3$ $, f(1) = 2 \times 1 - 1 = 1$

$\therefore f(2) - 3f(1) = 3 - 3 \times 1 = \text{zero}$

5

1. $f(\sqrt{2}) + 3g(\sqrt{2}) = (\sqrt{2})^2 - 3(\sqrt{2}) + 3(\sqrt{2} - 3)$
   $= 2 - 3\sqrt{2} + 3\sqrt{2} - 9 = -7$

2. $\because f(3) = (3)^2 - 3 \times 3 = 9 - 9 = \text{zero}$

   $, g(3) = 3 - 3 = \text{zero}$

   $\therefore f(3) = g(3) = \text{zero}$

6

$\because f(1 + \sqrt{6}) = (1 + \sqrt{6})^2 - 2(1 + \sqrt{6}) - 5$
$= 1 + 2\sqrt{6} + 6 - 2 - 2\sqrt{6} - 5 = \text{zero}$

$, f(1 - \sqrt{6}) = (1 - \sqrt{6})^2 - 2(1 - \sqrt{6}) - 5$
$= 1 - 2\sqrt{6} + 6 - 2 + 2\sqrt{6} - 5 = \text{zero}$

$\therefore f(1 + \sqrt{6}) = f(1 - \sqrt{6}) = \text{zero}$

7

1. $\because a = \text{zero}$ $\quad \therefore f(X) = bX + 5$

   $\therefore f$ is of the first degree.

2. $\because f(3) = 11$ $\quad \therefore b \times 3 + 5 = 11$

   $\therefore 3b = 6$ $\quad \therefore b = \frac{6}{3} = 2$

8

$f(1) = 5 \times 1 - b = 5 - b$ $, h(3) = 3 - 2b$

$, \because f(1) + h(3) = -7$ $\quad \therefore 5 - b + 3 - 2b = -7$

$\therefore 8 - 3b = -7$ $\quad \therefore 8 + 7 = 3b$

$\therefore 15 = 3b$ $\quad \therefore b = \frac{15}{3} = 5$

$\therefore f(X) = 5X - 5$

$\therefore f(3) = 5 \times 3 - 5 = 15 - 5 = 10$

$\therefore h(X) = X - 10$ $\quad \therefore h(1) = 1 - 10 = -9$

$\therefore f(3) + h(1) = 10 - 9 = 1$

9

$\because f(X) = t(X)$ $\quad \therefore (X - 3)^2 = X - 3$

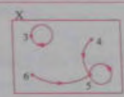
$\therefore X^2 - 6X + 9 - X + 3 = 0$

$\therefore X^2 - 7X + 12 = 0$ $\quad \therefore (X - 3)(X - 4) = 0$

$\therefore X = 3$ or $X = 4$

10

1.



2. $f = \{(3, 3), (4, 5), (5, 5), (6, 5)\}$

   , its range $= \{3, 5\}$

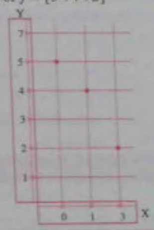
11

1. $\because f(0) = 5 - 0 = 5$

   also $, f(1) = 4$ $, f(3) = 2$

   $\therefore$ The range of $f = \{5, 4, 2\}$

2.



12

1. $t(0) = 2 \times 0 + 3 = 3$

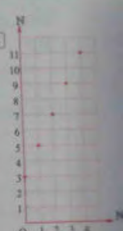
   also $t(1) = 5$

   $, t(2) = 7$ $, t(3) = 9$

   $, t(4) = 11$

   $, t(5) = 13$

2.



3. The range of t

   $= \{3, 5, 7, 9, 11, 13, \ldots\}$

   = The set of odd natural numbers except $\{1\}$

13

1 $f(4) = (4)^2 - 2 \times 4 - 3 = 16 - 8 - 3 = 5$
$, f(3) = 0 , f(2) = -3 , f(1) = -4$
$, f(0) = -3 , f(-1) = 0 , f(-2) = 5$

2

3 From 1 : $\because f(4) = 5 , f(-2) = 5$
$\therefore X = 4$ or -2

14

$\because f(a) = b$ $\therefore b = a^2 + b$ $\therefore a^2 = 0$
$\therefore a = 0$ $\therefore a b^2 + 5 = 0 \times b^2 + 5 = 5$

15

1 The domain = {1 , 2 , 3 , 4 , 5}
2 The range = {3 , 5 , 7 , 9 , 11}
3 The rule of the function f is : $f(X) = 2X + 1$

16

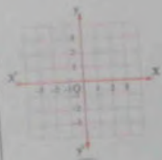
$\because f(0) = 0$ $\therefore 2 \times (0)^2 + b \times 0 + c = 0$
$\therefore c = 0$ $\therefore f(X) = 2X^2 + bX$
$\because f(3) = 0$ $\therefore 0 = 2(3)^2 + 3b$
$\therefore 0 = 18 + 3b$ $\therefore b = -6$

Answers of Exercise 4

1

1 a	2 b	3 c	4 c	5 b
6 a	7 a	8 c	9 b	10 b
11 b	12 c	13 c	14 a	15 a

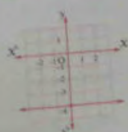
2



1

2

3

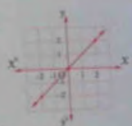


4

3

1 $f(X) = X$

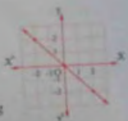
X	−2	zero	2
$f(X)$	−2	zero	2



The straight line representing the function intersects the two coordinate axes at the origin point O (0 , 0)

2 $f(X) = -X$

X	−2	zero	2
$f(X)$	2	zero	−2



The straight line representing the function intersects the two coordinate axes at the origin point O (0 , 0)

3 $f(X) = 3X$

X	−1	zero	1
$f(X)$	−3	zero	3

The straight line representing the function intersects the two coordinate axes at the origin point O (0 , 0)

4 $f(X) = -2X$

X	−2	zero	1
$f(X)$	4	zero	−2

The straight line representing the function intersects the two coordinate axes at the origin point O (0 , 0)

5 $f(X) = X + 2$

X	−2	zero	2
$f(X)$	zero	2	4

The straight line representing the function intersects the X-axis at the point (− 2 , 0) and the y-axis at the point (0 , 2)

6 $f(X) = 2 - X$

X	−2	zero	2
$f(X)$	4	2	zero

The straight line representing the function intersects the X-axis at the point (2 , 0) and the y-axis at the point (0 , 2)

7 $f(X) = 3X - 1$

X	zero	1	2
$f(X)$	−1	2	5

Represent by yourself.
From the graph we find that :
The straight line representing the function intersects the X-axis at the point $\left(\frac{1}{3} , 0\right)$ and the y-axis at the point (0 , − 1)

8 $f(X) = -2X + 3$

X	−1	zero	1
$f(X)$	5	3	1

Represent by yourself.
From the graph we find that :
The straight line representing the function intersects the X-axis at the point $(1.5 , 0)$ and the y-axis at (0 , 3)

9 $f(X) = \frac{1}{2} X$

X	−2	zero	2
$f(X)$	−1	zero	1

Represent by yourself.
From the graph we find that :
The straight line representing the function intersects the two coordinate axes at the origin point O (0 , 0)

10 $f(X) = 5 - \frac{1}{2} X$

X	zero	2	4
$f(X)$	5	4	3

Represent by yourself.
From the graph we find that :
The straight line representing the function intersects the X-axis at the point (10 , 0) and the y-axis at the point (0 , 5)

4

$\because f$ is a linear function
$\therefore a = 0$
$\therefore f(X) = 5X + 4$
$\therefore f(-2) = 5 \times -2 + 4 = -6$

5

$\because$ The straight line intersects the y-axis at (b , 2)
$\therefore b = 0$, $\because$ (0 , 2) satisfies the function
$\therefore 6 \times 0 - a = 2$ $\therefore a = -2$

6

$\because$ (a , 2 a) satisfies the function
$\therefore 2a = 3 \times a - 6$ $\therefore 2a = 3a - 6$
$\therefore 3a - 2a = 6$ $\therefore a = 6$
, at $X = 0$ $\therefore f(0) = 3 \times 0 - 6 = -6$
$\therefore$ The straight line intersects the y-axis at (0 , − 6)

7

1 $\because f(3) = 9$ $\therefore 2 \times 3 + a = 9$
$\therefore 6 + a = 9$ $\therefore a = 3$
2 At $y = 0$: $\therefore 2X + 3 = 0$ $\therefore X = -\frac{3}{2}$
$\therefore$ The straight line intersects the X-axis at $\left(-\frac{3}{2} , 0\right)$

8

$\because$ The straight line cuts a positive part of the y-axis of length 3 units.
$\therefore$ The straight line passes through (0 , 3)
$\therefore$ (0 , 3) satisfies the relation
$\therefore 3 = a \times 0 + b$ $\therefore b = 3$
$\therefore f(X) = aX + 3$

, ∵ (1 , 5) satisfies the relation

∴ $5 = a \times 1 + 3$ ∴ $a = 2$

9

∵ The point (0 , – 3) satisfies the relation $f(X) = aX + b$

∴ $-3 = a \times 0 + b$ ∴ $b = -3$

∴ $f(X) = aX - 3$

, ∵ the point (3 , 0) satisfies the relation $f(X) = aX - 3$

∴ $0 = 3 \times a - 3$ ∴ $3a = 3$ ∴ $a = 1$

∴ $f(X) = X - 3$ ∴ $f(1) = 1 - 3 = -2$

10

1. ∵ $r(2) = 9 - 2 = 7$, also $r(3) = 6$, $r(6) = 3$

   ∴ The set of images of elements of the set X with the function r = {7 , 6 , 3}

2. r is not a linear function because each of the domain and the codomain is not the set of real numbers.

11

1. Let A (X , 0)

   , ∵ A (X , 0) belongs to the straight line representing the function f

   ∴ $4 - 2X = 0$ ∴ $-2X = -4$

   ∴ $X = \frac{-4}{-2} = 2$ ∴ A (2 , 0)

   Let B (0 , y)

   , ∵ B (0 , y) belongs to the straight line representing the function f

   ∴ $4 - 2 \times 0 = y$ ∴ $y = 4$

   ∴ B (0 , 4)

2. The area of Δ AOB = $\frac{1}{2} \times 2 \times 4 = 4$ square unit

12

1. ∵ f is a constant function , passes through the point A (2 , 3) and is represented graphically by a straight line parallel to X-axis

   ∴ The rule of the function f is $f(X) = 3$

   , ∵ g is a linear function and passes through A (2 , 3) , O (0 , 0)

   ∴ The rule of the function g is $g(X) = bX + c$

   , ∵ (0 , 0) $\in \overleftrightarrow{OA}$

   ∴ $0 = b \times 0 + c$ ∴ $c = 0$

   ∴ $g(X) = bX$

   , ∵ (2 , 3) $\in \overleftrightarrow{OA}$ ∴ $3 = 2 \times b$

   ∴ $b = \frac{3}{2}$ ∴ $g(X) = \frac{3}{2}X$

2. $f(-10) + g(6) = 3 + \frac{3}{2} \times 6 = 12$

13

∵ $\overleftrightarrow{AB}$ represents the function $f : f(X) = 4$

, the point B ∈ y-axis

∴ B = (0 , 4) ∴ OB = 4 length unit

, ∵ the area of Δ ABO = 4 square unit

∴ $\frac{1}{2}$ AB × OB = 4 ∴ $\frac{1}{2}$ AB × 4 = 4

∴ $\frac{1}{2}$ AB = 1 ∴ AB = 2 length unit

∴ A = (2 , 4)

, ∵ the point O (0 , 0) belongs to the straight line representing the function g : $g(X) = nX + k$

∴ $0 = n \times 0 + k$ ∴ $k = 0$

∴ $g(X) = nX$

, ∵ the point A (2 , 4) belongs to the straight line representing the function g : $g(X) = nX$

∴ $4 = 2n$ ∴ $n = 2$

14

Let A (0 , y)

, ∵ A (0 , y) belongs to the straight line representing the function f

∴ $y = 0 + 3 = 3$ ∴ A (0 , 3)

, ∵ A (0 , 3) belongs to the straight line representing the function g

∴ $3 = m \times 0 + k$ ∴ $k = 3$

∴ $g(X) = mX + 3$

let C (X , 0)

, ∵ C (X , 0) belongs to the straight line representing the function f

∴ $0 = X + 3$ ∴ $X = -3$

∴ C (– 3 , 0) ∴ CO = 3 units

, ∵ BC = 7 units

∴ BO = 7 – 3 = 4 units ∴ B (4 , 0)

, ∵ B (4 , 0) belongs to the straight line representing the function g

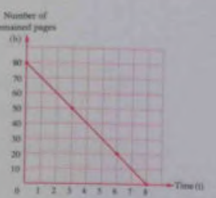
∴ $0 = 4m + 3$ ∴ $m = \frac{-3}{4}$

∴ $g(X) = \frac{-3}{4}X + 3$

∴ $g(8) = \frac{-3}{4} \times 8 + 3 = -6 + 3 = -3$

15

1.



You can find the algebraic relation easily after studying the equation of the straight line (the last lesson in geometry) as follows :

Taking the two points (3 , 50) and (6 , 20)

∴ The slope = $\frac{50 - 20}{3 - 6} = -10$

∴ $b = -10t + 80$

2. 8 hours.

3. 80 pages.

16

1. a 2. b 3. b 4. b 5. c

6. d 7. d 8. c 9. d 10. c

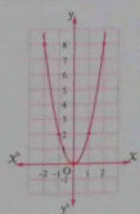
17

1. $f(X) = 2X^2$

X	– 2	– 1	0	1	2
$f(X)$	8	2	0	2	8

From the graph :

- The vertex of the curve is (0 , 0)
- The equation of the line of symmetry is $X = 0$
- The minimum value = zero

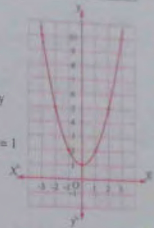


2. $f(X) = X^2 + 1$

X	– 3	– 2	– 1	0	1	2	3
$f(X)$	10	5	2	1	2	5	10

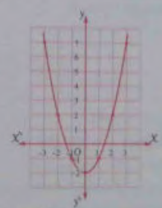
From the graph :

- The vertex of the curve is (0 , 1)
- The equation of the line of symmetry is $X = 0$
- The minimum value = 1



3. $f(X) = X^2 - 2$

X	– 3	– 2	– 1	0	1	2	3
$f(X)$	7	2	– 1	– 2	– 1	2	7



From the graph :

- The vertex of the curve is (0 , – 2)
- The equation of the line of symmetry is $X = 0$
- The minimum value = – 2

4. $f(X) = 2 - X^2$

X	– 3	– 2	– 1	0	1	2	3
$f(X)$	– 7	– 2	1	2	1	– 2	– 7

Represent by yourself.

From the graph , we find that :

- The vertex of the curve is (0 , 2)
- The equation of the line of symmetry is $X = 0$
- The maximum value = 2

5 $f(X) = X^2 - 2X$

X	-2	-1	0	1	2	3	4
$f(X)$	8	3	0	-1	0	3	8

Represent by yourself.

From the graph , we find that :

- The vertex of the curve is $(1, -1)$
- The equation of the line of symmetry is $X = 1$
- The minimum value $= -1$

6 $f(X) = X^2 + 2X + 1$

X	-4	-3	-2	-1	0	1	2
$f(X)$	9	4	1	0	1	4	9

Represent by yourself.

From the graph , we find that :

- The vertex of the curve is $(-1, 0)$
- The equation of the line of symmetry is $X = -1$
- The minimum value $= 0$

7 $f(X) = (X-2)^2 = X^2 - 4X + 4$

X	-1	0	1	2	3	4	5
$f(X)$	9	4	1	0	1	4	9

Represent by yourself.

From the graph , we find that :

- The vertex of the curve is $(2, 0)$
- The equation of the line of symmetry is $X = 2$
- The minimum value = zero

8 $f(X) = X^2 - 2X - 3$

X	-2	-1	0	1	2	3	4
$f(X)$	5	0	-3	-4	-3	0	5

Represent by yourself.

From the graph , we find that :

- The vertex of the curve is $(1, -4)$
- The equation of the line of symmetry is $X = 1$
- The minimum value $= -4$

9 $f(X) = 3 - 2X - X^2$

X	-4	-3	-2	-1	0	1	2
$f(X)$	-5	0	3	4	3	0	-5

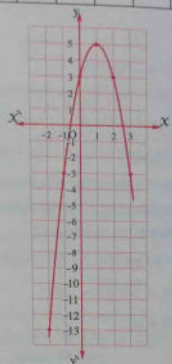
Represent by yourself.

From the graph , we find that :

- The vertex of the curve is $(-1, 4)$
- The equation of the line of symmetry is $X = -1$
- The maximum value $= 4$

10 $f(X) = 4X + 3 - 2X^2$

X	-2	-1	0	1	2	3
$f(X)$	-13	-3	3	5	3	-3

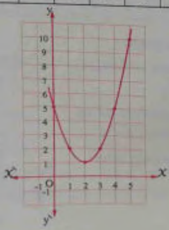


From the graph :

- The vertex of the curve is $(1, 5)$
- The equation of the axis of symmetry is $X = 1$
«notice that : the domain of f is $\mathbb{R}$ and the given interval is for facilitating representation only»
- The maximum value $= 5$

11 $f(X) = X^2 - 4X + 5$

X	0	1	2	3	4	5
$f(X)$	5	2	1	2	5	10



From the graph :

- The vertex of the curve is $(2, 1)$
- The equation of the axis of symmetry is $X = 2$
«notice that : the domain of f is $\mathbb{R}$ and the given interval is for facilitating representation only»
- The minimum value $= 1$

12 $f(X) = 1 - 3X + X^2$

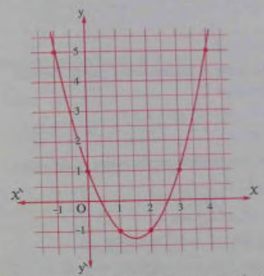
X	-1	0	1	2	3	4
$f(X)$	5	1	-1	-1	1	5

$\because$ The X-coordinate of the vertex of the curve

$= -\frac{b}{2a} = \frac{-(-3)}{2} = \frac{3}{2} = 1\frac{1}{2}$

, $f\left(\frac{3}{2}\right) = 1 - 3\left(\frac{3}{2}\right) + \frac{9}{4} = -\frac{5}{4} = -1\frac{1}{4}$

$\therefore$ The vertex of the curve is $\left(1\frac{1}{2}, -1\frac{1}{4}\right)$



- The equation of the axis of symmetry is $X = 1\frac{1}{2}$
- The minimum value $= -1\frac{1}{4}$

18

$\because$ The curve of the function f intersects the X-axis at the point $(-2, b)$

$\therefore b = 0$

$\therefore (-2, 0)$ satisfies the relation $f(X) = m - X^2$

$\therefore m - (-2)^2 = 0 \qquad \therefore m - 4 = 0$

$\therefore m = 4 \qquad \therefore m^b + 2m = 4^0 + 2 \times 4 = 9$

19

$\because 3f(2) + 3\ell(X) = 6 \qquad \therefore f(2) + \ell(X) = 2$

$\therefore a + 2^2 + c = 2 \qquad \therefore a + 4 + c = 2$

$\therefore a + c = -2$

$\therefore 2f(0) + 2\ell(7) = 2[f(0) + \ell(7)] = 2[a + (0)^2 + c]$
$= 2[a + c] = 2 \times (-2) = -4$

20

1 $\because$ The X – coordinate of the vertex of the curve $= \frac{-b}{2a} = -2$

$\therefore \frac{-(3k+2)}{2k} = -2 \qquad \therefore -3k - 2 = -4k$

$\therefore -3k + 4k = 2 \qquad \therefore k = 2$

$\therefore f(X) = 2X^2 + (3 \times 2 + 2)X + 6$

$\therefore f(X) = 2X^2 + 8X + 6$

2 $\because f(-2) = 2 \times (-2)^2 + 8 \times -2 + 6 = -2$

$\therefore$ The vertex of the curve is : $(-2, -2)$

, $\because$ The coefficient of X^2 is positive

$\therefore$ The minimum value $= -2$

21

1 Let $A = (X, 0)$ and $C = (-X, 0)$

$\because$ The curve of the function intersects the X-axis at the two points A and C

$\therefore 0 = 9 - X^2 \qquad \therefore X^2 = 9$

$\therefore X = 3$ or $X = -3$

$\therefore A = (3, 0)$, $C = (-3, 0)$

2 Let $B = (0, y)$

, $\because$ the point $B = (0, y)$ belongs to the curve of the function f

$\therefore y = 9 - (0)^2 \qquad \therefore y = 9$

$\therefore B = (0, 9) \qquad \therefore OB = 9$ length units.

, $\because A = (3, 0)$ and $C = (-3, 0)$

$\therefore AC = 6$ length units.

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 6 \times 9$
$= 27$ square units.

22

$\because AO = 4$ length units $\qquad \therefore A(0, 4)$

$\because A(0, 4)$ belongs to the curve of the function f

$\therefore$ A satisfies the equation of the curve

$\therefore 4 = m - (0)^2 \qquad \therefore m = 4$ (The first req.)

$\because$ The curve of the function intersects X-axis at the two points B and C

$\therefore 0 = 4 - X^2 \qquad \therefore X^2 = 4$

$\therefore X = 2$ or -2

$\therefore B = (2, 0)$, $C = (-2, 0)$ (The second req.)

$\therefore BC = 4$ length units

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 4 \times 4 = 8$ square units

(The third req.)

23

$\because A(0, -7)$ $\therefore OA = 7$ length units

$, \because$ the area of $\Delta ABC = \frac{1}{2} \times BC \times AO$

$\therefore 21 = \frac{1}{2} \times BC \times 7 \therefore BC = \frac{21 \times 2}{7} = 6$ length units

$\therefore OB = OC = \frac{6}{2} = 3$ length units

$\therefore B = (3, 0)$

$, \because B(3, 0) \in$ the curve of the function f

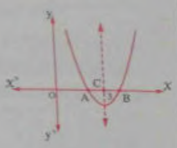
$\therefore \ell \times 3^2 - 7 = 0$ $\therefore 9\ell - 7 = 0$

$\therefore 9\ell = 7$ $\therefore \ell = \frac{7}{9}$

24

$\because$ The equation of the axis of symmetry is :

$X = \frac{-b}{2a} = \frac{6}{2} = 3$



$, \because$ the axis of symmetry bisects $\overline{AB}$

$\therefore CA = 1$ length unit

$\therefore AO = 3 - 1 = 2$ length units

$\therefore A(2, 0)$

$, \because A(2, 0)$ satisfies the equation

$\therefore 0 = 2^2 - 6 \times 2 + m$ $\therefore 4 - 12 + m = 0$

$\therefore m = 8$

, the minimum value $= f\left(\frac{-b}{2a}\right)$

$= f(3) = 3^2 - 6 \times 3 + 8 = -1$

25

1 The domain of the function $f = \mathbb{R}$

2 $\because$ The range of the function f = the set of images of the elements of the set $\mathbb{R}$ by the function f

$\therefore$ The range of the function $f = \left]-\infty, 4\frac{1}{2}\right]$

3 The equation of the line of symmetry of the curve of the function f is : $X = 2$

4 The maximum value of $f = 4\frac{1}{2}$

5 $f(1) = 4$

6 $\because \left(2, 4\frac{1}{2}\right) \in$ the curve of the function f

$\therefore a(2-2)^2 + k = 4\frac{1}{2}$ $\therefore k = 4\frac{1}{2}$

$, \because (5, 0) \in$ the curve of the function f

$\therefore a(5-2)^2 + 4\frac{1}{2} = 0$

$\therefore 9a = -4\frac{1}{2}$ $\therefore a = \frac{-4\frac{1}{2}}{9} = -\frac{1}{2}$

$\therefore a + k = -\frac{1}{2} + 4\frac{1}{2} = 4$

26

$\because$ The curve of the function intersects the X-axis at the two points $A(1, 0)$ and $B(4, 0)$

$\therefore f(1) = 0, f(4) = 0$ $\therefore f(1) = f(4)$

$, \because$ the function is symmetric

$\therefore$ The equation of the axis of symmetry is

$X = \frac{4+1}{2} = \frac{5}{2}$

$, \because \frac{-2+7}{2} = \frac{5}{2}$ $\therefore f(-2) = f(7)$

$\therefore f(-2) + f(-2) = 8$ $\therefore f(-2) = 4$

27

Let $C = (0, \ell)$

$, \because$ the curve of the function f passes through the point C

$\therefore \ell = 0^2 - (k-2) \times 0 - k + 4$

$\therefore \ell = 4 - k$

$, \because$ the X-coordinate of the vertex of the curve

$= \frac{-b}{2a} = \frac{k-2}{2}$

$\therefore AO = 2 \times \frac{k-2}{2} = k - 2$

$, \because \ell = AO$ $\therefore 4 - k = k - 2$

$\therefore 2k = 6$ $\therefore k = 3$

28

$\because OB = 5\,OA$ $\therefore \frac{OB}{OA} = \frac{5}{1}$

$\therefore OB = 5m, OA = m$

$\therefore B(5m, 0), A(-m, 0)$

$\therefore f(5m) = f(-m)$

$\therefore -25m^2 + 20m + k - 1 = -m^2 - 4m + k - 1$

$\therefore 24m^2 - 24m = 0$ $\therefore 24m(m-1) = 0$

$\therefore m = 0$ (refused) or $m = 1$

$\therefore B(5, 0)$

By substituting in the rule of the function f

$\therefore 0 = -25 + 20 + k - 1$ $\therefore k = 6$

Answers of unit two

Answers of Exercise 5

1

1 a	2 d	3 c	4 c	5 c
6 d	7 d	8 d	9 a	10 b
11 b	12 b	13 c	14 a	15 b
16 a	17 a	18 b	19 b	20 d
21 a	22 c	23 a	24 a	25 d
26 c				

2

1 Let the first proportional be X $\therefore \frac{X}{\sqrt{8}} = \frac{7}{14\sqrt{2}}$

$\therefore X = \frac{7 \times \sqrt{8}}{14\sqrt{2}} = \frac{7 \times 2\sqrt{2}}{14\sqrt{2}} = 1$

2 Let the third proportional be X

$\therefore \frac{a}{(a+b)} = \frac{X}{(a^2 - b^2)}$

$\therefore X = \frac{a(a^2 - b^2)}{(a+b)} = \frac{a(a+b)(a-b)}{(a+b)} = a(a-b)$

3 Let the fourth proportional be X

$\therefore \frac{(a+b)}{(a-b)} = \frac{(a-b)}{X}$ $\therefore X = \frac{(a-b)^2}{(a+b)}$

3

1 $\because \frac{2X-3}{X-5} = \frac{1}{4}$ $\therefore X - 5 = 4(2X - 3)$

$\therefore X - 5 = 8X - 12$ $\therefore 7X = 7$ $\therefore X = 1$

2 $\because \frac{X-5}{5X+3} = \frac{2}{3}$ $\therefore 3(X-5) = 2(5X+3)$

$\therefore 3X - 15 = 10X + 6$ $\therefore -7X = 21$

$\therefore X = -3$

3 $\because \frac{X^2 - 8}{2X^2 + 1} = \frac{1}{3}$ $\therefore 3X^2 - 24 = 2X^2 + 1$

$\therefore X^2 = 25$ $\therefore X = \pm 5$

4 $\because \frac{X^2 + 10X}{2X^2 - 3} = \frac{24}{5}$ $\therefore 5X^2 + 50X = 48X^2 - 72$

$\therefore 43X^2 - 50X - 72 = 0$

$\therefore (X - 2)(43X + 36) = 0$

$\therefore X = 2$ or $X = \frac{-36}{43}$ (refused)

4

$\because 3X - 6y = X + 3y$ $\therefore 2X = 9y$

$\therefore \frac{y}{X} = \frac{2}{9}$

5

$\because \frac{2X+3}{2X-3} = \frac{2y+5}{2y-5}$

$\therefore 4Xy + 6y - 10X - 15 = 4Xy - 6y + 10X - 15$

$\therefore 12y = 20X$ $\therefore \frac{X}{y} = \frac{12}{20} = \frac{3}{5}$

6

$\because X^2 - 3Xy - 4y^2 = 0$ $\therefore (X + y)(X - 4y) = 0$

$\therefore X + y = 0$ $\therefore X = -y$ $\therefore X : y = -1 : 1$

or $X - 4y = 0$ $\therefore X = 4y$ $\therefore X : y = 4 : 1$

7

$\because 3X^2 - 10Xy + 7y^2 = 0$ $\therefore (X - y)(3X - 7y) = 0$

$\therefore X = y$ (refused) or $3X - 7y = 0$ $\therefore 3X = 7y$

$\therefore X : y = 7 : 3$

8

$\because X^2 - 4Xy + 4y^2 = 0$ $\therefore (X - 2y)^2 = 0$

$\therefore X - 2y = 0$ $\therefore X = 2y$

$\therefore \frac{X}{y} = \frac{2}{1} = m$ $\therefore X = 2m, y = m$

$\therefore \frac{X + 3y}{3X - y} = \frac{2m + 3m}{6m - m} = \frac{5m}{5m} = 1$

9

$\because \frac{X}{y} = \frac{2}{3}$ $\therefore X = 2m, y = 3m$

$\therefore \frac{3X + 2y}{6y - X} = \frac{6m + 6m}{18m - 2m} = \frac{12m}{16m} = \frac{3}{4}$

10

$\because \frac{a}{b} = \frac{3}{5}$ $\therefore a = 3m, b = 5m$

$\therefore \frac{7a + 9b}{4a + 2b} = \frac{21m + 45m}{12m + 10m} = \frac{66m}{22m} = 3$

11

$\because \frac{a}{b} = \frac{3}{4}$ $\therefore a = 3m, b = 4m$

1 $\frac{4a + b}{2a - b} = \frac{12m + 4m}{6m - 4m} = \frac{16m}{2m} = 8$

2 $\frac{b^2 - a^2}{a^2 - b^2} = \frac{16m^2 - 9m^2}{9m^2 - 16m^2} = \frac{7m^2}{-7m^2} = -1$

$\frac{a}{b} = \frac{1}{3}$ $\quad \therefore a = 3m$, $b = 3m$

$\frac{c}{d} = \frac{7}{2}$ $\quad \therefore c = 7k$, $d = 2k$

$\frac{2ac+bd}{bc-3ad} = \frac{2 \times m \times 7k + 3m \times 2k}{3m \times 7k - 3 \times m \times 2k} = \frac{14mk + 6mk}{21mk - 6mk}$

$= \frac{20mk}{15mk} = \frac{4}{3}$

13

$\because \frac{7x-3y}{x+y} = \frac{3}{1}$ $\quad \therefore 7x - 3y = 3x + 3y$

$\therefore 4x = 6y$ $\quad \therefore \frac{x}{y} = \frac{6}{4} = \frac{3}{2}$

$\therefore x = 3m$, $y = 2m$

$\therefore \frac{12x+9y}{11x-3y} = \frac{36m+18m}{33m-6m} = \frac{54m}{27m} = 2:1$

14

$\because \frac{21x+a}{7x+b} = \frac{a}{b}$ $\quad \therefore 21bx + ab = 7ax + ab$

$\therefore 21bx = 7ax$ $\quad \therefore 3b = a$

$\therefore \frac{a+2b}{2a} = \frac{3b+2b}{2 \times 3b} = \frac{5b}{6b} = \frac{5}{6}$

15

Let the number be x

$\therefore 3+x$, $5+x$, $8+x$, $12+x$ are proportional.

$\therefore \frac{3+x}{5+x} = \frac{8+x}{12+x}$

$\therefore 40 + 13x + x^2 = 36 + 15x + x^2$

$\therefore 40 - 36 = 15x - 13x$ $\quad \therefore 4 = 2x$ $\quad \therefore x = 2$

$\therefore$ The required number = 2

16

Let the number be x

$\therefore 16-x$, $21-x$, $14-x$, $18-x$ are proportional.

$\therefore \frac{16-x}{21-x} = \frac{14-x}{18-x}$

$\therefore (21-x)(14-x) = (16-x)(18-x)$

$\therefore 294 - 35x + x^2 = 288 - 34x + x^2$

$\therefore x = 6$ $\quad \therefore$ The required number = 6

17

1 $\because \frac{a+b}{b} = \frac{c+d}{d}$ $\quad \therefore d(a+b) = b(c+d)$

$\therefore ad + bd = bc + bd$ $\quad \therefore ad = bc$

$\therefore \frac{a}{b} = \frac{c}{d}$

$\therefore a$, b , c , d are proportional.

Another solution :

$\frac{a+b}{b} = \frac{c+d}{d}$ $\quad \therefore \frac{a}{b} + \frac{b}{b} = \frac{c}{d} + \frac{d}{d}$

$\therefore \frac{a}{b} + 1 = \frac{c}{d} + 1$ $\quad \therefore \frac{a}{b} = \frac{c}{d}$

$\therefore a$, b , c , d are proportional.

2 $\because \frac{a}{b-a} = \frac{c}{d-c}$ $\quad \therefore a(d-c) = c(b-a)$

$\therefore ad - ac = cb - ca$ $\quad \therefore ad = cb$

$\therefore \frac{a}{b} = \frac{c}{d}$ $\quad \therefore a$, b , c , d are proportional.

Another solution :

$\because \frac{a}{b-a} = \frac{c}{d-c}$ $\quad \therefore \frac{b-a}{a} = \frac{d-c}{c}$

$\therefore \frac{b}{a} - 1 = \frac{d}{c} - 1$ $\quad \therefore \frac{b}{a} = \frac{d}{c}$

$\therefore \frac{a}{b} = \frac{c}{d}$ $\quad \therefore a$, b , c , d are proportional

3 $\because \frac{a-b}{a+b} = \frac{c-d}{c+d}$ $\quad \therefore (a-b)(c+d) = (a+b)(c-d)$

$\therefore ac + ad - bc - bd = ac - ad + bc - bd$

$\therefore 2ad = 2bc$ $\quad \therefore ad = bc$ $\quad \therefore \frac{a}{b} = \frac{c}{d}$

$\therefore a$, b , c , d are proportional.

4 $\because \frac{a^2-2c^2}{b^2-2d^2} = \frac{a^2}{b^2}$

$\therefore a^2(b^2-2d^2) = b^2(a^2-2c^2)$

$\therefore a^2b^2 - 2a^2d^2 = a^2b^2 - 2b^2c^2$

$\therefore a^2d^2 = b^2c^2$ $\quad \therefore ad = bc$

$\therefore \frac{a}{b} = \frac{c}{d}$ $\quad \therefore a$, b , c , d are proportional.

18

$\because a:b:c = 5:7:3$ $\quad \therefore a = 5m$, $b = 7m$, $c = 3m$

$\because a + b = 27.6$ $\quad \therefore 5m + 7m = 27.6$

$\therefore 12m = 27.6$ $\quad \therefore m = 2.3$

$a = 5 \times 2.3 = 11.5$, $b = 7 \times 2.3 = 16.1$, $c = 3 \times 2.3 = 6.9$

19

$\because a:b:c = 3:4:5$ $\quad \therefore a = 3m$, $b = 4m$, $c = 5m$

$\therefore \frac{a^2+b^2+c^2}{a(b+c)} = \frac{9m^2+16m^2+25m^2}{3m(4m+5m)} = \frac{50m^2}{27m^2} = \frac{50}{27}$

20

$\because 2a = 3b = 4c$ $\quad \therefore 2a = 3b$ $\quad \therefore a = \frac{3}{2}b$

, $3b = 4c$ $\quad \therefore c = \frac{3}{4}b$

$\therefore a:b:c = \frac{3}{2}b : b : \frac{3}{4}b$

multiplying by 4 $\quad \therefore a:b:c = 6b:4b:3b$

dividing by b $\quad \therefore a:b:c := 6:4:3$

Another solution :

$\because 2a = 3b = 4c$ $\quad$ (dividing by 12)

$\therefore \frac{2a}{12} = \frac{3b}{12} = \frac{4c}{12}$ $\quad \therefore \frac{a}{6} = \frac{b}{4} = \frac{c}{3}$

$\therefore a:b:c = 6:4:3$

21

$\because 4a = 3b = 6c$ $\quad \therefore 4a = 3b$ $\quad \therefore a = \frac{3}{4}b$

, $6c = 3b$ $\quad \therefore c = \frac{3}{6}b = \frac{1}{2}b$

, $\because a + b + c = 36$ $\quad \therefore \frac{3}{4}b + b + \frac{1}{2}b = 36$

$\therefore \frac{9}{4}b = 36$ $\quad \therefore b = 36 \times \frac{4}{9} = 16$

$\therefore a = \frac{3}{4} \times 16 = 12$, $c = \frac{1}{2} \times 16 = 8$

22

1 Let the number be x $\quad \therefore \frac{7+x}{11+x} = \frac{2}{3}$

$\therefore 21 + 3x = 22 + 2x$ $\quad \therefore x = 1$

$\therefore$ The required number is 1

2 Let the number be x $\quad \therefore \frac{49-3x}{69-3x} = \frac{2}{3}$

$\therefore 147 - 9x = 138 - 6x$ $\quad \therefore 3x = 9$

$\therefore x = 3$

$\therefore$ The required number = 3

3 Let the number be x $\quad \therefore \frac{7+x^2}{11+x^2} = \frac{4}{5}$

$\therefore 35 + 5x^2 = 44 + 4x^2$ $\quad \therefore x^2 = 9$

$\therefore x = \pm 3$

$\therefore$ The required number is 3 or -3

4 Let the number be x $\quad \therefore \frac{5+x^2}{11+x^2} = \frac{3}{5}$

$\therefore 25 + 5x^2 = 33 + 3x^2$ $\quad \therefore 2x^2 = 8$

$\therefore x^2 = 4$

$\therefore x = 2$ or $x = -2$ (refused)

$\therefore$ The required number = 2

5 Let the number be x $\quad \therefore \frac{15-x}{13+x} = \frac{3}{4}$

$\therefore 60 - 4x = 39 + 3x$ $\quad \therefore 7x = 21$

$\therefore x = 3$

$\therefore$ The required number = 3

6 Let the two numbers be a and b

$\therefore \frac{a}{b} = \frac{3}{7}$ $\quad \therefore a = 3m$, $b = 7m$

$\therefore \frac{3m-5}{7m-5} = \frac{1}{3}$ $\quad \therefore 9m - 15 = 7m - 5$

$\therefore 2m = 10$ $\quad \therefore m = 5$

$\therefore$ The two numbers are 15 and 35

7 Let the two numbers be a and b

$\therefore \frac{a}{b} = \frac{2}{3}$ $\quad \therefore a = 2m$, $b = 3m$

$\therefore \frac{2m+7}{3m-12} = \frac{5}{3}$ $\quad \therefore 6m + 21 = 15m - 60$

$\therefore 81 = 9m$ $\quad \therefore m = 9$

$\therefore$ The two numbers are 18 and 27

8 Let the two numbers be a and b

$\therefore \frac{a}{b} = \frac{4}{7}$ $\quad \therefore a = 4m$, $b = 7m$

$\therefore (4m)^2 - 5(7m) = 39$

$\therefore 16m^2 - 35m - 39 = 0$

$\therefore (m-3)(16m+13) = 0$

$\therefore m = 3$ or $m = \frac{-13}{16}$ (refused)

$\therefore$ The two numbers are 12 and 21

9 $\because$ The area of the unshaded part from the circle

$= 1 - \frac{5}{6} = \frac{1}{6}$ of the area of the circle

, the area of the unshaded part from the triangle

$= 1 - \frac{2}{3} = \frac{1}{3}$ of the area of the triangle.

$\therefore \frac{1}{6}$ of the area of the circle

$= \frac{1}{3}$ of the area of the triangle

$\therefore$ The area of the circle : the area of the triangle

$= \frac{1}{3} : \frac{1}{6}$ (multiply by 6) $= 2:1$

23

Let the costs of building the school be x

, the costs of building the medical unit = y

and the costs of building the youth centre = z

$\therefore x = \frac{3}{2}y$, $y = \frac{5}{6}z$ $\quad \therefore z = \frac{6}{5}y$

$\because x + y + z = 1.85 \times 10^6$

$\therefore \frac{3}{2}y + y + \frac{6}{5}y = 1.85 \times 10^6$ $\quad \therefore \frac{37}{10}y = 1.85 \times 10^6$

$\therefore 37y = 1.85 \times 10^7$ $\quad \therefore y = 5 \times 10^5$

$x = \frac{3}{2} \times 5 \times 10^5 = 7.5 \times 10^5$, $z = \frac{6}{5} \times 5 \times 10^5 = 6 \times 10^5$

24

Let the number of boys $= X$ and the number of girls $= y$

$\therefore$ The total number of pupils $= X + y$

The number of succeeded boys $= X \times \frac{79}{100} = 0.79\,X$

The number of succeeded girls $= y \times \frac{89}{100} = 0.89\,y$

The total number of succeeded pupils $= 0.79\,X + 0.89\,y$

$\therefore$ The ratio of success in 3rd grade preparatory

$\frac{0.79\,X + 0.89\,y}{X + y} = 0.83$

$\therefore (0.79)\,X + (0.89)\,y = (0.83)\,X + (0.83)\,y$

$\therefore (0.89)\,y - (0.83)\,y = (0.83)\,X - (0.79)\,X$

$\therefore (0.06)\,y = (0.04)\,X \qquad \therefore X : y = 6 : 4 = 3 : 2$

$\therefore$ The number of boys : the number of girls $= 3 : 2$

25

Let the circumference of the circle be a cm.
and the perimeter of the square be b cm.

$\therefore \frac{a}{b} = \frac{11}{8} \qquad \therefore a = 11\,m ، b = 8\,m$

$\therefore 11\,m + 8\,m = 152$

$\therefore 19\,m = 152 \qquad \therefore m = 8$

$\therefore$ The circumference of the circle $= 11 \times 8 = 88$ cm.

$\therefore 2 \times \frac{22}{7} \times r = 88 \qquad \therefore r = 14$ cm.

$\therefore$ The area of the circle $= \pi r^2 = \frac{22}{7} \times 14 \times 14 = 616\ \text{cm}^2$

، the perimeter of the square $= 8 \times 8 = 64$ cm.

$\therefore$ The side length of the square $= \frac{64}{4} = 16$ cm.

$\therefore$ The area of the square $= 16 \times 16 = 256\ \text{cm}^2$

$\therefore$ The area of the square : The area of the circle

$= \frac{256}{616} = \frac{32}{77}$

26

Let the second proportional be X

$\therefore$ The numbers are : $X - 2 ، X ، 8$ and X^2

$\therefore \frac{X-2}{X} = \frac{8}{X^2} \qquad \therefore X^3 - 2X^2 = 8X$

$\therefore X^3 - 2X^2 - 8X = 0 \qquad \therefore X(X^2 - 2X - 8) = 0$

$\therefore X(X-4)(X+2) = 0 \qquad \therefore X = 0$ (refused)

or $X = 4$ thus ، the numbers are : 2 ، 4 ، 8 and 16

or $X = -2$ thus ، the numbers are : -4 ، -2 ، 8 and 4

27

Let the number be X $\therefore$ Its multiplicative inverse $= \frac{1}{X}$

$\therefore \frac{2}{3 + \frac{1}{X}} = \frac{3}{5}$

Multiplying the two terms of the ratio in the left side by X

$\therefore \frac{2X}{3X+1} = \frac{3}{5} \qquad \therefore 10\,X = 9\,X + 3$

$\therefore X = 3 \qquad \therefore$ The number $= 3$

Answers of Exercise 6

1

1 d	2 b	3 c	4 b	5 b	6 d
7 b	8 c	9 a	10 c	11 d	12 b
13 d	14 d	15 d	16 c	17 b	

2

Let $\frac{a}{b} = \frac{c}{d} = m$ where $m > 0$

$\therefore a = b\,m ، c = d\,m$

1 L.H.S. $= \frac{3a + c}{5a - 2c} = \frac{3bm + dm}{5bm - 2dm}$

$= \frac{m(3b + d)}{m(5b - 2d)} = \frac{3b + d}{5b - 2d} =$ R.H.S.

2 L.H.S. $= \frac{3a - 2c}{5a + 3c} = \frac{3bm - 2dm}{5bm + 3dm}$

$= \frac{m(3b - 2d)}{m(5b + 3d)} = \frac{3b - 2d}{5b + 3d}$

$=$ R.H.S.

3 L.H.S. $= \frac{b^2 m^2 + d^2 m^2}{b^2 m + d^2 m} = \frac{m^2(b^2 + d^2)}{m(b^2 + d^2)} = m \qquad (1)$

، R.H.S. $= \frac{bm}{b} = m \qquad (2)$

From (1) and (2) : $\therefore$ The two sides are equal.

4 L.H.S. $= \frac{a^2 + c^2}{b^2 + d^2} = \frac{b^2 m^2 + d^2 m^2}{b^2 + d^2} = \frac{m^2(b^2 + d^2)}{b^2 + d^2}$

$= m^2 \qquad (1)$

R.H.S. $= \frac{ac}{bd} = \frac{bm \times dm}{bd} = m^2 \qquad (2)$

From (1) and (2) : $\therefore$ The two sides are equal.

5 L.H.S. $= \frac{ac}{bd} = \frac{bm \times dm}{bd} = m^2 \qquad (1)$

R.H.S. $= \left(\frac{a-c}{b-d}\right)^2 = \left(\frac{bm - dm}{b-d}\right)^2 = \left(\frac{m(b-d)}{b-d}\right)^2$

$= m^2 \qquad (2)$

From (1) and (2) : $\therefore$ The two sides are equal.

6 L.H.S. $= \left(\frac{a+b}{c+d}\right)^2 = \left(\frac{bm + b}{dm + d}\right)^2 = \left(\frac{b(m+1)}{d(m+1)}\right)^2$

$= \frac{b^2}{d^2} \qquad (1)$

R.H.S. $= \frac{2a^2 - 3b^2}{2c^2 - 3d^2} = \frac{2b^2 m^2 - 3b^2}{2d^2 m^2 - 3d^2}$

$= \frac{b^2(2m^2 - 3)}{d^2(2m^2 - 3)} = \frac{b^2}{d^2} \qquad (2)$

From (1) and (2) : $\therefore$ The two sides are equal.

7 L.H.S. $= \sqrt{\frac{3a^2 - 5c^2}{3b^2 - 5d^2}} = \sqrt{\frac{3b^2 m^2 - 5d^2 m^2}{3b^2 - 5d^2}}$

$= \sqrt{\frac{m^2(3b^2 - 5d^2)}{(3b^2 - 5d^2)}} = m \qquad (1)$

R.H.S. $= \frac{a}{b} = \frac{bm}{b} = m \qquad (2)$

From (1) and (2) : $\therefore$ The two sides are equal.

8 L.H.S. $= \sqrt[3]{\frac{5a^3 - 3c^3}{5b^3 - 3d^3}} = \sqrt[3]{\frac{5b^3 m^3 - 3d^3 m^3}{5b^3 - 3d^3}}$

$= \sqrt[3]{\frac{m^3(5b^3 - 3d^3)}{(5b^3 - 3d^3)}}$

$= \sqrt[3]{m^3} = m \qquad (1)$

R.H.S. $= \frac{bm + dm}{b + d} = \frac{m(b+d)}{(b+d)} = m \qquad (2)$

From (1) and (2) : $\therefore$ The two sides are equal.

9 L.H.S. $= \frac{a^2 - 2ac + c^2}{ac} = \frac{(a-c)^2}{ac}$

$= \frac{(bm - dm)^2}{bm \times dm} = \frac{\left(m(b-d)\right)^2}{bd\,m^2}$

$= \frac{m^2(b-d)^2}{bd\,m^2} = \frac{(b-d)^2}{bd} \qquad (1)$

R.H.S. $= \frac{b^2 - 2bd + d^2}{bd} = \frac{(b-d)^2}{bd} \qquad (2)$

From (1) and (2) : $\therefore$ The two sides are equal.

3

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = m$ where $m > 0$

$\therefore a = b\,m ، c = d\,m ، e = f\,m$

1 L.H.S. $= \frac{a + 5c}{b + 5d} = \frac{bm + 5dm}{b + 5d} = \frac{m(b + 5d)}{(b + 5d)} = m \quad (1)$

R.H.S. $= \frac{c - 3e}{d - 3f} = \frac{dm - 3fm}{d - 3f} = \frac{m(d - 3f)}{d - 3f} = m \quad (2)$

From (1) and (2) : $\therefore$ The two sides are equal.

2 L.H.S. $= \frac{2a + 7c - 4e}{2b + 7d - 4f} = \frac{2bm + 7dm - 4fm}{2b + 7d - 4f}$

$= \frac{m(2b + 7d - 4f)}{(2b + 7d - 4f)} = m \qquad (1)$

R.H.S. $= \frac{a - 8e}{b - 8f} = \frac{bm - 8fm}{b - 8f} = \frac{m(b - 8f)}{b - 8f} = m \quad (2)$

From (1) and (2) : $\therefore$ The two sides are equal.

3 L.H.S. $= \frac{2a^4 b^2 + 3a^2 e^2 - 5e^4 f}{2b^6 + 3b^2 f^2 - 5f^5}$

$= \frac{(2b^6 m^4 + 3b^2 f^2 m^4 - 5f^5 m^4)}{2b^6 + 3b^2 f^2 - 5f^5}$

$= \frac{m^4(2b^6 + 3b^2 f^2 - 5f^5)}{2b^6 + 3b^2 f^2 - 5f^5} = m^4 \qquad (1)$

R.H.S. $= \frac{a^4}{b^4} = \frac{b^4 m^4}{b^4} = m^4 \qquad (2)$

From (1) and (2) : $\therefore$ The two sides are equal.

4 L.H.S. $= \sqrt{\frac{5a^2 - 7ce}{5b^2 - 7df}} = \sqrt{\frac{5b^2 m^2 - 7df\,m^2}{5b^2 - 7df}}$

$= \sqrt{\frac{m^2(5b^2 - 7df)}{5b^2 - 7df}} = \sqrt{m^2} = m \qquad (1)$

R.H.S. $= \frac{2a + c}{2b + d} = \frac{2bm + dm}{2b + d} = \frac{m(2b + d)}{2b + d} = m \quad (2)$

From (1) and (2) : $\therefore$ The two sides are equal.

4

Let $\frac{X}{3} = \frac{y}{4} = \frac{z}{5} = m$ where $m > 0$

$\therefore X = 3\,m ، y = 4\,m ، z = 5\,m$

1 L.H.S. $= \frac{2y - z}{3X - 2y + z} = \frac{8m - 5m}{9m - 8m + 5m}$

$= \frac{3m}{6m} = \frac{1}{2} =$ R.H.S.

2 $\because \sqrt{3X^2 + 3y^2 + z^2} = \sqrt{27\,m^2 + 48\,m^2 + 25\,m^2}$

$= \sqrt{100\,m^2} = 10\,m \qquad (1)$

، $2X + y = 6\,m + 4\,m = 10\,m \qquad (2)$

From (1) and (2) : $\therefore \sqrt{3X^2 + 3y^2 + z^2} = 2X + y$

5

Let $\frac{X}{1} = \frac{y}{2} = \frac{z}{3} = m$

$\therefore X = m ، y = 2\,m ، z = 3\,m$

$\therefore$ L.H.S. $= \frac{X + y - 2z}{X - 3z} = \frac{m + 2m - 6m}{m - 9m} = \frac{-3m}{-8m}$

$= \frac{3}{8} =$ R.H.S.

Another solution :

$\because X = \frac{y}{2} = \frac{z}{3} \qquad \therefore y = 2X ، z = 3X$

$\therefore$ L.H.S $= \frac{x+y-2z}{x-3z} = \frac{x+2x-6x}{x-9x} = \frac{-3x}{-8x}$

$= \frac{3}{8} =$ R.H.S

6

$\because \frac{a}{2} = \frac{b}{3} = \frac{c}{4}$

, multiplying the two terms of the 1st ratio by 2 and the 2nd by − 5 and the 3rd by 3 and adding the antecedents and consequents of the three ratios.

$\therefore \frac{2a-5b+3c}{4-15+12} =$ one of the given ratios.

$\therefore 2a - 5b + 3c =$ one of the given ratios.

7

$\because \frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a-b+5c}{3x}$

, multiplying the two terms of the 1st ratio by 2 and the 2nd by − 1 and the 3rd by 5 and adding the antecedents and consequents of the three ratios.

$\therefore \frac{2a-b+5c}{4-3+20} =$ one of the given ratios.

$\therefore \frac{2a-b+5c}{21} = \frac{2a-b+5c}{3x}$

$\therefore 3x = 21 \qquad \therefore x = 7$

8

$\because \frac{a}{2} = \frac{b}{7} = \frac{c}{3}$

, multiplying the terms of the 2nd ratio by 2 and adding the antecedents and consequents of the 1st and 2nd ratios.

$\therefore \frac{a+2b}{2+14} = \frac{a+2b}{16} =$ one of the given ratios. (1)

Subtracting the antecedents and consequents of the 3rd ratio from the antecedents and consequents of the 2nd ratio.

$\therefore \frac{b-c}{7-3} = \frac{b-c}{4} =$ one of the given ratios. (2)

From (1) and (2) : $\therefore \frac{a+2b}{16} = \frac{b-c}{4}$

$\therefore \frac{a+2b}{b-c} = \frac{16}{4} = 4$

Another solution :

$\because \frac{a}{2} = \frac{b}{7} = \frac{c}{3} = m$

$\therefore a = 2m \ , \ b = 7m \ , \ c = 3m$

$\therefore \frac{a+2b}{b-c} = \frac{2m+14m}{7m-3m} = \frac{16m}{4m} = 4$

9

$\because \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{2}{3}$

multiplying the two terms of the 1st ratio by 5

, multiplying the two terms of the 2nd ratio by − 3

and adding the antecedents and consequents of the three ratios

$\therefore \frac{5a-3c+e}{5b-3d+f} =$ one of the given ratios

$\therefore \frac{5a-3c+e}{5b-3d+f} = \frac{2}{3}$

$, \because 5a - 3c + e = 18$

$\therefore \frac{18}{5b-3d+f} = \frac{2}{3}$

$\therefore 5b - 3d + f = \frac{3 \times 18}{2} = 27$

10

$\because \frac{a}{4x+y} = \frac{b}{x-4y}$

, adding the antecedents and consequents of the two ratios.

$\therefore \frac{a+b}{4x+y+x-4y} = \frac{a+b}{5x-3y}$

$=$ one of the given ratios. (1)

subtracting the antecedents and consequents of the 2nd ratio from the 1st ratio.

$\frac{a-b}{4x+y-x+4y} = \frac{a-b}{3x+5y}$

$=$ one of the given ratios. (2)

From (1) and (2) : $\therefore \frac{a+b}{5x-3y} = \frac{a-b}{3x+5y}$

11

$\because \frac{x+y}{19} = \frac{y+z}{7}$

, adding the antecedents and consequents of the two ratios.

$\therefore \frac{x+y+y+z}{19+7} = \frac{x+2y+z}{26}$

$=$ one of the given ratios. (1)

, subtracting the antecedent and consequent of the 2nd ratio from the antecedent and consequent of the 1st ratio.

$\therefore \frac{x+y-y-z}{19-7} = \frac{x-z}{12} =$ one of the given ratios. (2)

From (1) and (2) : $\therefore \frac{x+2y+z}{26} = \frac{x-z}{12}$

$\therefore \frac{x+2y+z}{13} = \frac{x-z}{6}$

12

$\because \frac{y}{x-z} = \frac{x}{y} = \frac{x+y}{z}$

, adding the antecedents and consequents of the three ratios $\therefore \frac{y+x+x+y}{x-z+y+z} = \frac{2(x+y)}{(x+y)} = 2$

$=$ one of the given ratios.

$\therefore$ Each ratio $= 2$ unless $x + y \neq 0$

$\therefore \frac{x}{y} = 2 \qquad \therefore x = 2y$

$, \frac{x+y}{z} = 2 \qquad \therefore x + y = 2z \qquad \therefore 2y + y = 2z$

$\therefore 3y = 2z \qquad \therefore z = \frac{3}{2} y$

$\therefore x : y : z = 2y : y : \frac{3}{2} y = 4 : 2 : 3$

13

$\because \frac{x}{a-b+c} = \frac{y}{b-c+a} = \frac{z}{c-a+b}$

, adding the antecedents and consequents of the 1st and 2nd ratios.

$\therefore \frac{x+y}{a-b+c+b-c+a} = \frac{x+y}{2a}$

$=$ one of the given ratios. (1)

, adding the antecedents and consequents of the 2nd and 3rd ratios.

$\therefore \frac{y+z}{b-c+a+c-a+b} = \frac{y+z}{2b}$

$=$ one of the given ratios. (2)

From (1) and (2) : $\therefore \frac{x+y}{2a} = \frac{y+z}{2b}$

$\therefore \frac{x+y}{a} = \frac{y+z}{b}$

14

$\because \frac{x}{2a+b} = \frac{y}{2b-c} = \frac{z}{2c-a}$

, multiplying the two terms of the 1st ratio by 2 and adding the antecedents and consequents of the 1st and the 2nd ratios.

$\therefore \frac{2x+y}{4a+2b+2b-c} = \frac{2x+y}{4a+4b-c}$

$=$ one of the given ratios. (1)

, multiplying the terms of the 1st ratio by 2 and the 2nd by 2 and adding the antecedents and consequents of the three ratios

$\therefore \frac{2x+2y+z}{4a+2b+4b-2c+2c-a} = \frac{2x+2y+z}{3a+6b}$

$=$ one of the given ratios. (2)

From (1) and (2) :

$\therefore \frac{2x+y}{4a+4b-c} = \frac{2x+2y+z}{3a+6b}$

15

$\because \frac{a}{2x-y} = \frac{b}{2y-x}$

, multiplying the terms of the 1st ratio by 2 and adding the antecedents and consequents of the two ratios.

$\therefore \frac{2a+b}{4x-2y+2y-x} = \frac{2a+b}{3x}$

$=$ one of the given ratios. (1)

, multiplying the terms of the 2nd ratio by 2 and adding the antecedents and consequents of the two ratios.

$\therefore \frac{a+2b}{2x-y+4y-2x} = \frac{a+2b}{3y}$

$=$ one of the given ratios. (2)

From (1) and (2) : $\therefore \frac{2a+b}{3x} = \frac{a+2b}{3y}$

$\therefore \frac{2a+b}{a+2b} = \frac{3x}{3y} = \frac{x}{y}$

16

$\because \frac{a}{2x+y} = \frac{b}{3y-x} = \frac{c}{4x+5y}$

, multiplying the terms of the 2nd ratio by 2 and adding the antecedents and consequents of the 1st and 2nd ratios

$\therefore \frac{a+2b}{2x+y+6y-2x} = \frac{a+2b}{7y}$

$=$ one of the given ratios. (1)

, multiplying the terms of the 2nd ratio by 4 and adding the antecedents and consequents of the 2nd and 3rd ratios.

$\therefore \frac{4b+c}{12y-4x+4x+5y} = \frac{4b+c}{17y}$

$=$ one of the given ratios. (2)

From (1) and (2) : $\therefore \frac{a+2b}{7y} = \frac{4b+c}{17y}$

$\therefore \frac{a+2b}{4b+c} = \frac{7y}{17y} = \frac{7}{17}$

17

$\because \frac{x+y}{7} = \frac{y+z}{5} = \frac{z+x}{8}$

, adding the antecedents and consequents of the three ratios.

$\therefore \frac{x+y+y+z+z+x}{7+5+8} = \frac{2(x+y+z)}{20} = \frac{x+y+z}{10}$

$=$ one of the given ratios. (1)

, multiplying the terms of the 2nd ratio by (− 1) and adding the antecedents and consequents of the 1st and 2nd ratios.

$\therefore \frac{x+y-y-z}{7-5}=\frac{x-z}{2}$ = one of the given ratios. (2)

From (1) and (2) : $\therefore \frac{x+y+z}{10}=\frac{x-z}{2}$

$\therefore \frac{x+y+z}{x-z}=\frac{10}{2}=5$

18

$\because \frac{a+b}{4}=\frac{b+c}{5}=\frac{c+a}{7}$

, adding the antecedents and consequents of the three ratios.

$\therefore \frac{a+b+b+c+c+a}{4+5+7}=\frac{2(a+b+c)}{16}=\frac{a+b+c}{8}$

= one of the given ratios. (1)

, multiplying the terms of the 2^{nd} ratio by (–1) and adding the antecedents and consequents of the three ratios.

$\therefore \frac{a+b-b-c+c+a}{4-5+7}=\frac{2a}{6}=\frac{a}{3}$

= one of the given ratios. (2)

From (1) and (2) :

$\therefore \frac{a+b+c}{8}=\frac{a}{3}$

19

$\because \frac{x+y}{3}=\frac{y+z}{8}=\frac{z+x}{6}$

, adding the antecedents and consequents of the three ratios.

$\therefore \frac{x+y+y+z+z+x}{3+8+6}=\frac{2x+2y+2z}{17}=\frac{2(x+y+z)}{17}$

= one of the given ratios. (1)

, multiplying the terms of the 2^{nd} ratio by 2 and adding the antecedents and consequents of the three ratios.

$\therefore \frac{x+y+2y+2z+z+x}{3+16+6}=\frac{2x+3y+3z}{25}$

= one of the given ratios.(2)

From (1) and (2) : $\therefore \frac{2(x+y+z)}{17}=\frac{2x+3y+3z}{25}$

$\therefore \frac{x+y+z}{2x+3y+3z}=\frac{17}{50}$

20

Multiplying the terms of the 2^{nd} ratio by (– 1) and adding the antecedents and consequents of the three ratios.

$\therefore \frac{x+y-y-z+z+x}{5-8+7}$

$=\frac{2x}{4}=\frac{x}{2}$ = one of the given ratios (1)

, multiplying the terms of the 3^{rd} ratio by (– 1) and adding the antecedents and consequents of the three ratios

$\therefore \frac{x+y+y+z-z-x}{5+8-7}=\frac{2y}{6}=\frac{y}{3}$

= one of the given ratios (2)

, multiplying the terms of the 1^{st} ratio by (– 1) and adding the antecedents and consequents of the three ratios.

$\therefore \frac{-x-y+y+z+z+x}{-5+8+7}=\frac{2z}{10}=\frac{z}{5}$

= one of the given ratios (3)

From (1) , (2) and (3) : $\therefore \frac{x}{2}=\frac{y}{3}=\frac{z}{5}$

21

$\because \frac{x+y}{25}=\frac{x-y}{11}=\frac{x+y-z}{8}$

, adding the antecedents and consequents of the 1^{st} and 2^{nd} ratios.

$\therefore \frac{2x}{36}=\frac{x}{18}$ = one of the given ratios. (1)

, subtracting the antecedent and consequent of the 3^{rd} ratio from the antecedent and consequent of the 1^{st} ratio.

$\therefore \frac{z}{17}$ = one of the given ratios. (2)

, subtracting the antecedent and consequent of the 2^{nd} ratio from the antecedent and consequent of the 1^{st} ratio.

$\therefore \frac{2y}{14}=\frac{y}{7}$ = one of the given ratios. (3)

From (1) , (2) , (3) :

$\therefore \frac{x}{18}=\frac{y}{7}=\frac{z}{17}$ $\qquad \therefore x : y : z = 18 : 7 : 17$

22

Multiplying the terms of the 2^{nd} ratio by (– 1) and adding the antecedents and consequents of the three ratios.

$\therefore \frac{a+3b-3b-5c+5c+a}{x+6y-6y-10z+10z+x}$

$=\frac{2a}{2x}=\frac{a}{x}$ = one of the given ratios (1)

, multiplying the terms of the 3^{rd} ratio by (– 1) and adding the antecedents and consequents of the three ratios.

$\therefore \frac{a+3b+3b+5c-5c-a}{x+6y+6y+10z-10z-x}$

$=\frac{6b}{12y}=\frac{b}{2y}$ = one of the given ratios (2)

From (1) and (2) : $\therefore \frac{a}{x}=\frac{b}{2y}$

$\therefore \frac{a}{b}=\frac{x}{2y}$

, multiplying the terms of the 1^{st} ratio by (– 1) and adding the antecedents and consequents of the three ratios.

$\therefore \frac{-a-3b+3b+5c+5c+a}{-x-6y+6y+10z+10z+x}=\frac{10c}{20z}=\frac{c}{2z}$

= one of the given ratios. (3)

From (1) , (2) and (3) :

$\therefore \frac{a}{x}=\frac{b}{2y}=\frac{c}{2z}$

$\therefore a : b : c = x : 2y : 2z$

23

Multiplying the terms of the 1^{st} ratio by (– 2) and the 3^{rd} ratio by 3 and adding the antecedents and consequents of the 1^{st} and 3^{rd} ratios.

$\therefore \frac{3c-2a}{3y+6x-6x-8y}=\frac{3c-2a}{-5y}$

= one of the given ratios (1)

, multiplying the terms of the 2^{nd} ratio by 2 and adding the antecedents and consequents of the 1^{st} and 2^{nd} ratios.

$\therefore \frac{a+2b}{3x+4y+10x-4y}=\frac{a+2b}{13x}$

= one of the given ratios (2)

From (1) and (2) : $\therefore \frac{3c-2a}{-5y}=\frac{a+2b}{13x}$

$\therefore 13x(3c-2a)=-5y(a+2b)$

$\therefore 13x(3c-2a)+5y(a+2b)=0$

24

$\because \frac{x}{7}=\frac{y}{3}=m$ $\qquad \therefore x=7m , y=3m$

$\therefore \frac{2x-3y}{x+2y}=\frac{2(7m)-3(3m)}{7m+2(3m)}=\frac{14m-9m}{7m+6m}$

$=\frac{5m}{13m}=\frac{5}{13}$ (1)

$\because \frac{10}{26}=\frac{5}{13}$ (2)

From (1) and (2) : $\therefore \frac{2x-3y}{x+2y}=\frac{10}{26}$

$\therefore (2x-3y) , (x+2y) , 10 , 26$ are proportional.

25

$\because \frac{a}{b}=\frac{3}{5}$ $\qquad \therefore b=\frac{5a}{3}$

$, \because \frac{a}{c}=\frac{3}{7}$ $\qquad \therefore c=\frac{7a}{3}$

$\therefore a+b+c=a+\frac{5a}{3}+\frac{7a}{3}=5a$

26

$\because \frac{a}{b}=\frac{2}{3}$ $\qquad \therefore b=\frac{3}{2}a$

$, \because \frac{a}{c}=\frac{3}{5}$ $\qquad \therefore c=\frac{5}{3}a$

$, \because a+b+c=75$ $\qquad \therefore a+\frac{3}{2}a+\frac{5}{3}a=75$

$\therefore \frac{25}{6}a=75$ $\qquad \therefore a=18$

$, b=\frac{3}{2}\times 18=27$ $\qquad , c=\frac{5}{3}\times 18=30$

27

$\because \Delta ABC \sim \Delta DEF$ $\qquad \therefore \frac{DE}{AB}=\frac{EF}{BC}=\frac{DF}{AC}=\frac{2}{3}$

Adding the antecedents and the consequents of the three ratios.

$\therefore \frac{DE+EF+DF}{AB+BC+AC}$ = one of the given ratios.

$\therefore \frac{22}{\text{perimeter of } \Delta ABC}=\frac{2}{3}$

$\therefore$ The perimeter of ΔABC = 33 cm.

28

$\because \frac{a}{x-y+z}=\frac{b}{x+y-z}=\frac{c}{y+z-x}$

, multiplying the terms of the 1^{st} ratio by x and the 2^{nd} by y and the 3^{rd} by z and adding the antecedents and consequents of the three ratios.

$\therefore$ Each ratio

$=\frac{ax+by+cz}{x^2-xy+zx+xy+y^2-zy+zy+z^2-xz}$

$=\frac{ax+by+cz}{x^2+y^2+z^2}$

29

$\because \frac{2x+y}{x}=\frac{4y+z}{y}=\frac{4z+3x}{z}$

, adding the antecedents and consequents of the three ratios

$\therefore \frac{5x+5y+5z}{x+y+z}=\frac{5(x+y+z)}{x+y+z}=5$

= one of the given ratios.

$\therefore$ Each ratio = 5 $\qquad \therefore \frac{2x+y}{x}=5$

$\therefore 2x+y=5x$ $\qquad \therefore y=3x$ (1)

$, \frac{4z+3X}{z}=5 \qquad \therefore 4z+3X=5z$

$\therefore z=3X \qquad (2)$

From (1) and (2)

$\therefore X:y:z=X:3X:3X=1:3:3$

$, \frac{2X+y+z}{3X-y+2z}=\frac{2X+3X+3X}{3X-3X+6X}=\frac{8X}{6X}=\frac{4}{3}$

30

$\because \frac{a+2b}{5}=\frac{3b-c}{3}=\frac{c-a}{2}=m$

$\therefore a+2b=5m \ (1) , 3b-c=3m \ (2) , c-a=2m \ (3)$

Adding (1) , (2) and (3) :

$\therefore 5b=10m \qquad \therefore b=2m$

From (1) : $\therefore a+4m=5m \qquad \therefore a=m$

From (3) : $\therefore c-m=2m \qquad \therefore c=3m$

1 $a+b-c=m+2m-3m=$ zero

2 $\frac{3b-a}{2b+c}=\frac{6m-m}{4m+3m}=\frac{5m}{7m}=\frac{5}{7}$

Answers of Exercise 7

1

1 The middle proportional $=\pm\sqrt{3\times 27}$

$=\pm\sqrt{81}=\pm 9$

2 The middle proportional $=\pm\sqrt{9\times 25}=\pm\sqrt{225}=\pm 15$

3 The middle proportional $=\pm\sqrt{-2\times -8}$

$=\pm\sqrt{16}=\pm 4$

4 The middle proportional $=\pm\sqrt{\frac{1}{5}\times 125}$

$=\pm\sqrt{25}=\pm 5$

5 The middle proportional $=\pm\sqrt{2a\times 8ab^2}$

$=\pm\sqrt{16a^2b^2}=\pm 4ab$

6 The middle proportional $=\pm\sqrt{(l^2-m^2)^2}$

$=\pm(l^2-m^2)$

2

1 Let the third proportional be c

$\therefore \frac{6}{12}=\frac{12}{c} \qquad \therefore c=\frac{12\times 12}{6}=24$

2 Let the third proportional be c

$\therefore \frac{X^2}{-5X}=\frac{-5X}{c} \qquad \therefore c=\frac{-5X\times -5X}{X^2}=25$

3 Let the third proportional be c

$\therefore \frac{X^2}{-3X^2}=\frac{-3X^2}{c} \qquad \therefore c=\frac{-3X^2\times -3X^2}{X^2}=9X^2$

3

1 Let $\frac{a}{b}=\frac{b}{c}=m \qquad \therefore b=cm , a=cm^2$

$\therefore \frac{a}{c}=\frac{cm^2}{c}=m^2 \quad (1) \quad , \frac{b^2}{c^2}=\frac{c^2m^2}{c^2}=m^2 \quad (2)$

From (1) and (2) : $\therefore \frac{a}{c}=\frac{b^2}{c^2}$

Another solution :

$\because b^2=ac \qquad \therefore \frac{b^2}{c^2}=\frac{ac}{c^2}=\frac{a}{c}=$ L.H.S

2 Let $\frac{a}{b}=\frac{b}{c}=m \qquad \therefore b=cm , a=cm^2$

$\therefore \frac{2a+3b}{2b+3c}=\frac{2cm^2+3cm}{2cm+3c}=\frac{cm(2m+3)}{c(2m+3)}=m \quad (1)$

$, \frac{a}{b}=\frac{cm^2}{cm}=m \quad (2)$

From (1) and (2) : $\therefore \frac{2a+3b}{2b+3c}=\frac{a}{b}$

3 Let $\frac{a}{b}=\frac{b}{c}=m \qquad \therefore b=cm , a=cm^2$

$\therefore \frac{a-b}{b-c}=\frac{cm^2-cm}{cm-c}=\frac{cm(m-1)}{c(m-1)}=m \quad (1)$

$, \frac{a+3b}{3c+b}=\frac{cm^2+3cm}{3c+cm}=\frac{cm(m+3)}{c(3+m)}=m \quad (2)$

From (1) and (2) : $\therefore \frac{a-b}{b-c}=\frac{a+3b}{3c+b}$

4 Let $\frac{a}{b}=\frac{b}{c}=m \qquad \therefore b=cm , a=cm^2$

$\therefore \frac{a^2+b^2}{b^2+c^2}=\frac{c^2m^4+c^2m^2}{c^2m^2+c^2}=\frac{c^2m^2(m^2+1)}{c^2(m^2+1)}=m^2 \quad (1)$

$, \frac{a}{c}=\frac{cm^2}{c}=m^2 \quad (2)$

From (1) and (2) : $\therefore \frac{a^2+b^2}{b^2+c^2}=\frac{a}{c}$

Another solution :

$\because b^2=ac$

$\therefore \frac{a^2+b^2}{b^2+c^2}=\frac{a^2+ac}{ac+c^2}=\frac{a(a+c)}{c(a+c)}=\frac{a}{c}=$ R.H.S.

5 Let $\frac{a}{b}=\frac{b}{c}=m \qquad \therefore b=cm , a=cm^2$

$\therefore \left(\frac{b-c}{a-b}\right)^2=\left(\frac{cm-c}{cm^2-cm}\right)^2=\left(\frac{c(m-1)}{cm(m-1)}\right)^2$

$=\frac{1}{m^2} \quad (1)$

$, \frac{c}{a}=\frac{c}{cm^2}=\frac{1}{m^2} \quad (2)$

From (1) and (2) : $\therefore \left(\frac{b-c}{a-b}\right)^2=\frac{c}{a}$

6 Let $\frac{a}{b}=\frac{b}{c}=m \qquad \therefore b=cm , a=cm^2$

$\therefore \frac{a^3+b^3}{b^3+c^3}=\frac{c^3m^6+c^3m^3}{c^3m^3+c^3}=\frac{c^3m^3(m^3+1)}{c^3(m^3+1)}=m^3 \quad (1)$

$, \frac{a^2}{cb}=\frac{c^2m^4}{c\times cm}=m^3 \quad (2)$

From (1) and (2) : $\therefore \frac{a^3+b^3}{b^3+c^3}=\frac{a^2}{cb}$

7 Let $\frac{a}{b}=\frac{b}{c}=m \qquad \therefore b=cm , a=cm^2$

$\therefore \frac{a^3-4b^3}{b^3-4c^3}=\frac{c^3m^6-4c^3m^3}{c^3m^3-4c^3}$

$=\frac{c^3m^3(m^3-4)}{c^3(m^3-4)}=m^3 \quad (1)$

$, \frac{b^3}{c^3}=\frac{c^3m^3}{c^3}=m^3 \quad (2)$

From (1) and (2) : $\therefore \frac{a^3-4b^3}{b^3-4c^3}=\frac{b^3}{c^3}$

8 Let $\frac{a}{b}=\frac{b}{c}=m \qquad \therefore b=cm , a=cm^2$

$\therefore \frac{2c^2-3b^2}{2b^2-3a^2}=\frac{2c^2-3c^2m^2}{2c^2m^2-3c^2m^4}$

$=\frac{c^2(2-3m^2)}{c^2m^2(2-3m^2)}=\frac{1}{m^2} \quad (1)$

$, \frac{c}{a}=\frac{c}{cm^2}=\frac{1}{m^2} \quad (2) \quad , \frac{c^2}{b^2}=\frac{c^2}{c^2m^2}=\frac{1}{m^2} \quad (3)$

From (1) , (2) and (3) : $\therefore \frac{2c^2-3b^2}{2b^2-3a^2}=\frac{c}{a}=\frac{c^2}{b^2}$

Another solution : $\because b^2=ac$

$\therefore \frac{2c^2-3b^2}{2b^2-3a^2}=\frac{2c^2-3ac}{2ac-3a^2}=\frac{c(2c-3a)}{a(2c-3a)}=\frac{c}{a}$

$, \frac{c^2}{b^2}=\frac{c^2}{ac}=\frac{c}{a}$

$\therefore \frac{2c^2-3b^2}{2b^2-3a^2}=\frac{c}{a}=\frac{c^2}{b^2}$

9 Let $\frac{a}{b}=\frac{b}{c}=m \qquad \therefore b=cm , a=cm^2$

$\therefore \frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{c^2m^4+c^2m^3+c^2m^2}{c^2m^2+c^2m+c^2}$

$=\frac{c^2m^2(m^2+m+1)}{c^2(m^2+m+1)}=m^2 \quad (1)$

$, \frac{a^2-b^2}{b^2-c^2}=\frac{c^2m^4-c^2m^2}{c^2m^2-c^2}=\frac{c^2m^2(m^2-1)}{c^2(m^2-1)}=m^2 \quad (2)$

From (1) and (2) : $\therefore \frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{a^2-b^2}{b^2-c^2}$

10 Let $\frac{a}{b}=\frac{b}{c}=m \qquad \therefore b=cm , a=cm^2$

$\therefore \frac{2a}{c}=\frac{2cm^2}{c}=2m^2 \quad (1)$

$, \frac{a^2}{b^2}+\frac{b^2}{c^2}=m^2+m^2=2m^2 \quad (2)$

From (1) and (2) : $\therefore \frac{2a}{c}=\frac{a^2}{b^2}+\frac{b^2}{c^2}$

Another solution : $\because b^2=ac$

$\therefore \frac{a^2}{b^2}+\frac{b^2}{c^2}=\frac{a^2}{ac}+\frac{ac}{c^2}=\frac{a}{c}+\frac{a}{c}=\frac{2a}{c}=$ R.H.S

11 Let $\frac{a}{b}=\frac{b}{c}=m \qquad \therefore b=cm , a=cm^2$

$\therefore \frac{a+b+c}{a^{-1}+b^{-1}+c^{-1}}=\frac{cm^2+cm+c}{c^{-1}m^{-2}+c^{-1}m^{-1}+c^{-1}}$

$=\frac{c(m^2+m+1)}{c^{-1}m^{-2}(1+m+m^2)}$

$=c\times cm^2=c^2m^2=b^2$

12 Let $\frac{a}{b}=\frac{b}{c}=m \qquad \therefore b=cm , a=cm^2$

$\therefore \frac{ac}{b(b+c)}=\frac{cm^2\times c}{cm(cm+c)}=\frac{c^2m^2}{c^2m(m+1)}=\frac{m}{m+1} \quad (1)$

$, \frac{a}{a+b}=\frac{cm^2}{cm^2+cm}=\frac{cm^2}{cm(m+1)}=\frac{m}{m+1} \quad (2)$

From (1) and (2) : $\therefore \frac{ac}{b(b+c)}=\frac{a}{a+b}$

Another solution : $\because b^2=ac$

$\therefore \frac{ac}{b(b+c)}=\frac{ac}{b^2+bc}=\frac{ac}{ac+bc}=\frac{ac}{c(a+b)}$

$=\frac{a}{a+b}=$ R.H.S

13 Let $\frac{a}{b}=\frac{b}{c}=m$

$\therefore b=cm , a=cm^2$

$\therefore \frac{a-b}{a-c}=\frac{cm^2-cm}{cm^2-c}=\frac{cm(m-1)}{c(m^2-1)}$

$=\frac{m(m-1)}{(m-1)(m+1)}=\frac{m}{m+1} \quad (1)$

$, \frac{b}{b+c}=\frac{cm}{cm+c}=\frac{cm}{c(m+1)}=\frac{m}{m+1} \quad (2)$

From (1) and (2) : $\therefore \frac{a-b}{a-c}=\frac{b}{b+c}$

4

1 Let $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$

$\therefore c=dm , b=dm^2 , a=dm^3$

$\therefore \frac{a-2b}{b-2c}=\frac{dm^3-2dm^2}{dm^2-2dm}=\frac{dm^2(m-2)}{dm(m-2)}=m \quad (1)$

$, \frac{3b+4c}{3c+4d}=\frac{3dm^2+4dm}{3dm+4d}=\frac{dm(3m+4)}{d(3m+4)}=m \quad (2)$

From (1) and (2) : $\therefore \frac{a-2b}{b-2c}=\frac{3b+4c}{3c+4d}$

2 Let $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$

$\therefore c=dm , b=dm^2 , a=dm^3$

$\therefore \frac{3a+5c}{3b+5d}=\frac{3dm^3+5dm}{3dm^2+5d}=\frac{dm(3m^2+5)}{d(3m^2+5)}=m \quad (1)$

$\frac{a-4c}{b-4d}=\frac{dm^3-4dm}{dm^2-4d}=\frac{dm(m^2-4)}{d(m^2-4)}=m$ (2)

From (1) and (2) : $\therefore \frac{3a+5c}{3b+5d}=\frac{a-4c}{b-4d}$

[3] Let $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$

$\therefore c=dm \ , b=dm^2 \ , a=dm^3$

$\therefore \frac{3a-5c}{a-b+c}=\frac{3dm^3-5dm}{dm^3-dm^2+dm}$

$=\frac{dm(3m^2-5)}{dm(m^2-m+1)}=\frac{3m^2-5}{m^2-m+1}$ (1)

$, \frac{3b-5d}{b-c+d}=\frac{3dm^2-5d}{dm^2-dm+d}$

$=\frac{d(3m^2-5)}{d(m^2-m+1)}=\frac{3m^2-5}{m^2-m+1}$ (2)

From (1) and (2) : $\therefore \frac{3a-5c}{a-b+c}=\frac{3b-5d}{b-c+d}$

[4] Let $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$

$\therefore c=dm \ , b=dm^2 \ , a=dm^3$

$\therefore \frac{a-d}{a+b+c}=\frac{dm^3-d}{dm^3+dm^2+dm}$

$=\frac{d(m^3-1)}{dm(m^2+m+1)}=\frac{(m-1)(m^2+m+1)}{m(m^2+m+1)}$

$=\frac{m-1}{m}$ (1)

$, \frac{a-2b+c}{a-b}=\frac{dm^3-2dm^2+dm}{dm^3-dm^2}$

$=\frac{dm(m^2-2m+1)}{dm^2(m-1)}=\frac{(m-1)^2}{m(m-1)}=\frac{m-1}{m}$ (2)

From (1) and (2) : $\therefore \frac{a-d}{a+b+c}=\frac{a-2b+c}{a-b}$

[5] Let $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$

$\therefore c=dm \ , b=dm^2 \ , a=dm^3$

$\therefore \frac{c^2-d^2}{a-c}=\frac{d^2m^2-d^2}{dm^3-dm}=\frac{d^2(m^2-1)}{dm(m^2-1)}=\frac{d}{m}$ (1)

$, \frac{bd}{a}=\frac{d^2m^2}{dm^3}=\frac{d}{m}$ (2)

From (1) and (2) : $\therefore \frac{c^2-d^2}{a-c}=\frac{bd}{a}$

[6] Let $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$

$\therefore c=dm \ , b=dm^2 \ , a=dm^3$

$\therefore \frac{a^2-3c^2}{b^2-3d^2}=\frac{d^2m^6-3d^2m^2}{d^2m^4-3d^2}=\frac{d^2m^2(m^4-3)}{d^2(m^4-3)}$

$=m^2$ (1)

$, \frac{b}{d}=\frac{dm^2}{d}=m^2$ (2)

From (1) and (2) : $\therefore \frac{a^2-3c^2}{b^2-3d^2}=\frac{b}{d}$

[7] Let $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$

$\therefore c=dm \ , b=dm^2 \ , a=dm^3$

$\therefore \frac{ab-cd}{b^2-c^2}=\frac{dm^3\times dm^2-dm\times d}{d^2m^4-d^2m^2}=\frac{d^2m^5-d^2m}{d^2m^2(m^2-1)}$

$=\frac{d^2m(m^4-1)}{d^2m^2(m^2-1)}=\frac{(m^2-1)(m^2+1)}{m(m^2-1)}=\frac{m^2+1}{m}$ (1)

$, \frac{a+c}{b}=\frac{dm^3+dm}{dm^2}=\frac{dm(m^2+1)}{dm^2}=\frac{m^2+1}{m}$ (2)

From (1) and (2) : $\therefore \frac{ab-cd}{b^2-c^2}=\frac{a+c}{b}$

[8] Let $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$

$\therefore c=dm \ , b=dm^2 \ , a=dm^3$

$\therefore \frac{a}{b+d}=\frac{dm^3}{dm^2+d}=\frac{dm^3}{d(m^2+1)}=\frac{m^3}{m^2+1}$ (1)

$, \frac{c^3}{c^2d+d^3}=\frac{d^3m^3}{d^2m^2\times d+d^3}=\frac{d^3m^3}{d^3(m^2+1)}=\frac{m^3}{m^2+1}$ (2)

From (1) and (2) : $\therefore \frac{a}{b+d}=\frac{c^3}{c^2d+d^3}$

[9] Let $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$

$\therefore c=dm \ , b=dm^2 \ , a=dm^3$

$\therefore \frac{a^2+b^2+c^2}{b^2+c^2+d^2}=\frac{d^2m^6+d^2m^4+d^2m^2}{d^2m^4+d^2m^2+d^2}$

$=\frac{d^2m^2(m^4+m^2+1)}{d^2(m^4+m^2+1)}=m^2$ (1)

$, \frac{ac}{bd}=\frac{dm^3\times dm}{dm^2\times d}=m^2$ (2)

From (1) and (2) : $\therefore \frac{a^2+b^2+c^2}{b^2+c^2+d^2}=\frac{ac}{bd}$

[10] Let $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$

$\therefore c=dm \ , b=dm^2 \ , a=dm^3$

$\therefore \frac{2a+3d}{3a-4d}=\frac{2dm^3+3d}{3dm^3-4d}$

$=\frac{d(2m^3+3)}{d(3m^3-4)}=\frac{2m^3+3}{3m^3-4}$ (1)

$, \frac{2a^3+3b^3}{3a^3-4b^3}=\frac{2d^3m^9+3d^3m^6}{3d^3m^9-4d^3m^6}$

$=\frac{d^3m^6(2m^3+3)}{d^3m^6(3m^3-4)}=\frac{2m^3+3}{3m^3-4}$ (2)

From (1) and (2) : $\therefore \frac{2a+3d}{3a-4d}=\frac{2a^3+3b^3}{3a^3-4b^3}$

[11] Let $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$ where $m>0$

$\therefore c=dm \ , b=dm^2 \ , a=dm^3$

$\therefore \frac{a+5b}{b+5c}=\frac{dm^3+5dm^2}{dm^2+5dm}=\frac{dm^2(m+5)}{dm(m+5)}=m$ (1)

$, \sqrt{\frac{b}{d}}=\sqrt{\frac{dm^2}{d}}=\sqrt{m^2}=m$ (2)

From (1) and (2) : $\therefore \frac{a+5b}{b+5c}=\sqrt{\frac{b}{d}}$

[12] Let $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$

$\therefore c=dm \ , b=dm^2 \ , a=dm^3$

$\therefore \sqrt[3]{\frac{5a^3-3c^3}{5b^3-3d^3}}=\sqrt[3]{\frac{5d^3m^9-3d^3m^3}{5d^3m^6-3d^3}}$

$=\sqrt[3]{\frac{d^3m^3(5m^6-3)}{d^3(5m^6-3)}}=\sqrt[3]{m^3}=m$ (1)

$, \frac{a+c}{b+d}=\frac{dm^3+dm}{dm^2+d}=\frac{dm(m^2+1)}{d(m^2+1)}=m$ (2)

From (1) and (2) : $\therefore \sqrt[3]{\frac{5a^3-3c^3}{5b^3-3d^3}}=\frac{a+c}{b+d}$

[13] Let $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$

where m is a positive real number.

$\therefore c=dm \ , b=dm^2 \ , a=dm^3$

$\therefore \left(\frac{a+b}{b+c}\right)^3=\left(\frac{dm^3+dm^2}{dm^2+dm}\right)^3=\left(\frac{dm^2(m+1)}{dm(m+1)}\right)^3$

$=m^3$ (1)

$, \frac{a}{d}=\frac{dm^3}{d}=m^3$ (2)

From (1) and (2) : $\therefore \left(\frac{a+b}{b+c}\right)^3=\frac{a}{d}$

[14] Let $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$

$\therefore c=dm \ , b=dm^2 \ , a=dm^3$

$\therefore \frac{a^2+d^2}{c(a+c)}=\frac{d^2m^6+d^2}{dm(dm^3+dm)}=\frac{d^2(m^6+1)}{d^2m^2(m^2+1)}$

$=\frac{(m^2+1)(m^4-m^2+1)}{m^2(m^2+1)}=\frac{m^4-m^2+1}{m^2}$ (1)

$, \frac{b}{d}+\frac{d}{b}-1=\frac{dm^2}{d}+\frac{d}{dm^2}-1=m^2+\frac{1}{m^2}-1$

$=\frac{m^4+1-m^2}{m^2}=\frac{m^4-m^2+1}{m^2}$ (2)

From (1) and (2) : $\therefore \frac{a^2+d^2}{c(a+c)}=\frac{b}{d}+\frac{d}{b}-1$

5

[1] c [2] c [3] c [4] c [5] b [6] a

[7] b [8] c [9] c [10] c [11] c [12] a

6

$\because a , 3 , 9 , b$ are in continued proportion

$\therefore \frac{a}{3}=\frac{3}{9}=\frac{9}{b}$

$\therefore a=\frac{3\times 3}{9}=1 \ , b=\frac{9\times 9}{3}=27$

7

$\because 3 , \ell , 12 , m$ are in continued proportion

$\therefore \frac{3}{\ell}=\frac{\ell}{12}=\frac{12}{m}$ $\therefore \ell=\pm\sqrt{3\times 12}=\pm\sqrt{36}=\pm 6$

$, m=\frac{12\times 12}{\pm 6}=\pm 24$

8

$\because 2 , a , b , 54$ are in continued proportion

$\therefore \frac{2}{a}=\frac{a}{b}=\frac{b}{54}=m$

$\therefore b=54m , a=54m^2 , 2=54m^3$

$\therefore m^3=\frac{1}{27}$ $\therefore m=\frac{1}{3}$

$\therefore b=\frac{1}{3}\times 54=18 , a=54\times\left(\frac{1}{3}\right)^2=6$

$\therefore a+b=6+18=24$

9

Let the number be X $\therefore \frac{3-X}{7-X}=\frac{7-X}{19-X}$

$\therefore (3-X)(19-X)=(7-X)^2$

$\therefore 57-22X+X^2=49-14X+X^2$

$\therefore 57-49=-14X+22X$ $\therefore 8=8X$

$\therefore X=1$ $\therefore$ The number is 1

10

$\because a=4c=4$ $\therefore a=4 \ , c=1$

$, \because$ b is the middle proportional between a and c

$\therefore b^2=ac$ $\therefore b^2=4\times 1=4$

$\therefore a^2+b^2+c^2=4^2+4+1^2=16+4+1=21$

11

Let $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$

$\therefore c=dm \ , b=dm^2 \ , a=dm^3$

$\therefore \frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{d^2}=m^2+m^2+m^2=3m^2$ (1)

$, \frac{a}{c}+\frac{b}{d}+\frac{ac}{bd}=\frac{dm^3}{dm}+\frac{dm^2}{d}+\frac{dm^3\times dm}{dm^2\times d}$

$=m^2+m^2+m^2=3m^2$ (2)

From (1) and (2) : $\therefore \frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{d^2}=\frac{a}{c}+\frac{b}{d}+\frac{ac}{bd}$

12

$\because y^2=Xz$ $\therefore X , y , z$ are proportional

$\therefore \frac{X}{y}=\frac{y}{z}=m$ $\therefore y=zm , X=zm^2$

$\therefore \frac{X(X-y)}{y(y-z)}=\frac{zm^2(zm^2-zm)}{zm(zm-z)}=\frac{zm^2\times zm(m-1)}{zm\times z(m-1)}$

$=\frac{z^2m^3(m-1)}{z^2m(m-1)}=m^2$ (1)

[illegible] $= m^3$ (2)

From (1) and (2) : $\therefore \frac{x(x-y)}{y(y-z)} = $ [illegible]

13

$\because b^2 = a\,c \quad \therefore \frac{a}{b} = \frac{b}{c} \quad \because c^2 = b\,d \quad \therefore \frac{b}{c} = \frac{c}{d}$

$\therefore \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = d\,m \text{ , } b = d\,m^2 \text{ , } a = d\,m^3$

$, \frac{2a+3d}{3a-4d} = \frac{2\,d\,m^3+3\,d}{3\,d\,m^3-4\,d} = \frac{d\,(2\,m^3+3)}{d\,(3\,m^3-4)} = \frac{2\,m^3+3}{3\,m^3-4}$ (1)

$, \frac{2a^3+3b^3}{3a^3-4b^3} = \frac{2\,d^3m^9+3\,d^3m^6}{3\,d^3m^9-4\,d^3m^6}$

$= \frac{d^3m^6\,(2\,m^3+3)}{d^3m^6\,(3\,m^3-4)} = \frac{2\,m^3+3}{3\,m^3-4}$ (2)

From (1) and (2) : $\therefore \frac{2a+3d}{3a-4d} = \frac{2a^3+3b^3}{3a^3-4b^3}$

14

$\because \frac{a^2+b^2}{b^2} = \frac{b^2+c^2}{c^2} \quad \therefore a^2c^2 + b^2c^2 = b^4 + b^2c^2$

$\therefore a^2c^2 = b^4 \quad \therefore a\,c = b^2$

$\therefore$ b is the middle proportional between a and c

15

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = d\,m \text{ , } b = d\,m^2 \text{ , } a = d\,m^3$

$\therefore (b+c)^2 = (d\,m^2 + d\,m)^2 = \left(d\,m\,(m+1)\right)^2$

$= d^2\,m^2\,(m+1)^2$ (1)

$, (a+b)(c+d) = (d\,m^3 + d\,m^2)(d\,m + d)$

$= d\,m^2\,(m+1) \times d\,(m+1) = d^2\,m^2\,(m+1)^2$ (2)

From (1) and (2) : $\therefore (b+c)^2 = (a+b)(c+d)$

$\therefore$ (b + c) is the middle proportional between (a + b) and (c + d)

16

Let $\frac{5a}{6b} = \frac{6b}{7c} = \frac{7c}{8d} = m$ where $m > 0$

$\therefore 7\,c = 8\,d\,m \text{ , } 6\,b = 8\,d\,m^2 \text{ , } 5\,a = 8\,d\,m^3$

$\therefore \sqrt[3]{\frac{5a}{8d}} = \sqrt[3]{\frac{8\,d\,m^3}{8\,d}} = \sqrt[3]{m^3} = m$ (1)

$, \sqrt{\frac{5a+6b}{7c+8d}} = \sqrt{\frac{8\,d\,m^3+8\,d\,m^2}{8\,d\,m+8\,d}}$

$= \sqrt{\frac{8\,d\,m^2\,(m+1)}{8\,d\,(m+1)}} = \sqrt{m^2} = m$ (2)

From (1) and (2) : $\therefore \sqrt[3]{\frac{5a}{8d}} = \sqrt{\frac{5a+6b}{7c+8d}}$

17

$\because \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm \text{ , } a = cm^2$

$\therefore \frac{a^4+b^4+c^4}{a^{-4}+b^{-4}+c^{-4}} = \frac{c^4m^8+c^4m^4+c^4}{c^{-4}m^{-8}+c^{-4}m^{-4}+c^{-4}}$

$= \frac{c^4m^8\,(1+m^{-4}+m^{-8})}{c^{-4}\,(m^{-8}+m^{-4}+1)} = c^8m^8$

$, \because b^8 = c^8m^8 \quad \therefore \frac{a^4+b^4+c^4}{a^{-4}+b^{-4}+c^{-4}} = b^8$

18

Let $\frac{x}{y} = \frac{y}{z} = m \quad \therefore y = z\,m \text{ , } x = z\,m^2$

$\because x + y = 15 \quad \therefore z\,m + z\,m^2 = 15$

$\therefore z\,m\,(1+m) = 15 \quad \therefore 1 + m = \frac{15}{z\,m}$ (1)

$\because y + z = 22.5 \quad \therefore z\,m + z = 22.5$

$\therefore z\,(m+1) = 22.5 \quad \therefore m + 1 = \frac{22.5}{z}$ (2)

From (1) and (2) : $\therefore \frac{15}{z\,m} = \frac{22.5}{z}$

$\therefore 22.5\,m = 15 \quad \therefore m = \frac{15}{22.5} = \frac{2}{3}$

$\therefore x : y = 2 : 3$

19

Let $\frac{m(\angle A)}{m(\angle B)} = \frac{m(\angle B)}{m(\angle C)} = e$

$\therefore m(\angle B) = m(\angle C) \times e$

$, m(\angle A) = m(\angle C) \times e^2$

$\because m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$

$\therefore m(\angle C) \times e^2 + m(\angle C) \times e + m(\angle C) = 180^\circ$

$\therefore 60^\circ\,e^2 + 60^\circ\,e + 60^\circ = 180^\circ$

$\therefore e^2 + e + 1 = 3 \quad \therefore e^2 + e - 2 = 0$

$\therefore (e+2)(e-1) = 0 \quad \therefore e = -2$ (refused) or $e = 1$

$\therefore m(\angle A) = 60^\circ \times 1^2 = 60^\circ$

$, m(\angle B) = 60^\circ \times 1 = 60^\circ$

20

$\because \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = 2$

$\therefore c = 2\,d \text{ , } b = 4\,d \text{ , } a = 8\,d$

$\because a\,x^2 - 2\,b\,x + c = 0$

$\therefore 8\,d\,x^2 - 8\,d\,x + 2\,d = 0$

dividing by 2 d $\quad \therefore 4\,x^2 - 4\,x + 1 = 0$

$\therefore (2\,x - 1)^2 = 0 \quad \therefore 2\,x - 1 = 0$

$\therefore x = \frac{1}{2} \quad \therefore$ The S.S $= \left\{\frac{1}{2}\right\}$

21

$\because$ The number 5 is the middle proportional between x and y

$\therefore x\,y = 25$ (1)

Let the middle proportional between $\left(x + \frac{1}{y}\right)$ and $\left(y + \frac{1}{x}\right)$ be z

$\therefore z^2 = \left(x + \frac{1}{y}\right)\left(y + \frac{1}{x}\right) = x\,y + 1 + 1 + \frac{1}{x\,y}$

$= x\,y + \frac{1}{x\,y} + 2$

and from (1) : $\therefore z^2 = 25 + \frac{1}{25} + 2 = 27.04$

$\therefore z = \pm\sqrt{27.04} = \pm 5.2$

Answers of Exercise 8

1

1 d	2 a	3 a	4 a	5 b
6 c	7 d	8 d	9 d	10 b
11 c	12 c	13 c	14 d	15 d
16 c	17 b	18 b	19 a	20 d
21 a	22 d			

2

$\because y \propto x \quad \therefore \frac{y_1}{y_2} = \frac{x_1}{x_2}$

$\therefore \frac{20}{40} = \frac{7}{x_2} \quad \therefore x_2 = \frac{40 \times 7}{20} = 14$

3

$\because a \propto \frac{1}{b} \quad \therefore \frac{a_1}{a_2} = \frac{b_2}{b_1}$

1 $\frac{12}{a_2} = \frac{1.5}{8} \quad \therefore a_2 = \frac{8 \times 12}{1.5} = 64$

2 $\frac{12}{2} = \frac{b_2}{8} \quad \therefore b_2 = \frac{8 \times 12}{2} = 48$

4

1 $\because y \propto x \quad \therefore y = m\,x$

$\therefore 14 = 42\,m \quad \therefore m = \frac{1}{3}$

$\therefore y = \frac{1}{3}\,x$ (The relation between x , y)

2 As $x = 60 \quad \therefore y = \frac{1}{3} \times 60 = 20$

5

1 $\because y \propto \frac{1}{x} \quad \therefore x\,y = m$

$\therefore 3 \times 2 = m \quad \therefore m = 6$

$\therefore x\,y = 6$ (The relation between x , y)

2 As $x = 1.5 \quad \therefore y = \frac{6}{1.5} = 4$

6

$\because y \propto \frac{1}{x} \quad \therefore x\,y = m$

$\therefore m = 3 \times 10 = 30 \quad \therefore x\,y = 30 \quad \therefore y = \frac{30}{x}$

As $x = 1 \quad \therefore y = \frac{30}{1} = 30$

As $x = 2 \quad \therefore y = \frac{30}{2} = 15$

As $x = 3 \quad \therefore y = \frac{30}{3} = 10$

As $x = 4 \quad \therefore y = \frac{30}{4} = 7.5$

As $x = 5 \quad \therefore y = \frac{30}{5} = 6$

7

$\because y \propto x^2 \quad \therefore y = m\,x^2$

$\therefore 4 = m\,(3)^2 \quad \therefore m = \frac{4}{9}$

$\therefore y = \frac{4}{9}\,x^2$ (The relation between x and y)

As $x = 9 \quad \therefore y = \frac{4}{9} \times 9^2 = 36$

8

$\because y \propto x^3 \quad \therefore y = m\,x^3$

$\therefore 64 = m\,(2)^3 \quad \therefore m = 8$

$\therefore y = 8\,x^3$ (The relation between x and y)

As $x = \frac{1}{2} \quad \therefore y = 8\left(\frac{1}{2}\right)^3 = 1$

9

$\because y \propto \frac{1}{\sqrt[4]{x}} \quad \therefore \frac{y_1}{y_2} = \frac{\sqrt[4]{x_2}}{\sqrt[4]{x_1}}$

$\therefore \frac{2}{y_2} = \frac{\sqrt[4]{32}}{\sqrt[4]{16}} \quad \therefore y_2 = \frac{2 \times 4}{4\sqrt[4]{2}} = \sqrt[4]{2}$

10

$\because y^2 \propto x^3 \quad \therefore y^2 = m\,x^3 \quad \because y = 3$ as $x = 2$

$\therefore 9 = 8\,m \quad \therefore m = \frac{9}{8} \quad \therefore y^2 = \frac{9}{8}\,x^3$

11

$\because y^2 \propto \frac{1}{\sqrt[3]{x}} \quad \therefore \left(\frac{y_1}{y_2}\right)^2 = \sqrt[3]{\frac{x_2}{x_1}}$

$\therefore \left(\frac{3}{1.5}\right)^2 = \sqrt[3]{\frac{x_2}{8}} \quad \therefore 4 = \frac{\sqrt[3]{x_2}}{2}$

$\therefore \sqrt[3]{x_2} = 8 \quad \therefore x_2 = 512$

12

$\because y \propto (x+1) \quad \therefore y = m\,(x+1)$

$\because y = 2 \text{ , } x = 3 \quad \therefore 2 = m\,(3+1)$

$\therefore m = \frac{1}{2} \quad \therefore y = \frac{1}{2}\,(x+1)$

13

$\because \dfrac{5x-3y}{3x+5y}=1$ $\quad \therefore 5x-3y=3x+5y$

$\therefore 2x=8y$

$\therefore y=\frac{1}{4}x$ $\quad \therefore y \propto x$

14

$\because \dfrac{a+2b}{6}=\dfrac{b+3c}{3}$ $\quad \therefore 3a+6b=6b+18c$

$\therefore 3a=18c$ $\quad \therefore a=6c$ $\quad \therefore a \propto c$

15

$\because \dfrac{21x-y}{7x-z}=\dfrac{y}{z}$ $\quad \therefore 21xz-zy=7xy-zy$

$\therefore 21xz=7xy$ $\quad \therefore 3z=y$ $\quad \therefore y \propto z$

16

$\because x^2y^2-6xy+9=0$ $\quad \therefore (xy-3)^2=0$

$\therefore xy=3$ $\quad \therefore y \propto \dfrac{1}{x}$

17

$\because 4a^2-12ab+9b^2=0$

$\therefore (2a-3b)^2=0$ $\quad \therefore 2a-3b=0$ $\quad \therefore 2a=3b$

$\therefore a=\frac{3}{2}b$ $\quad \therefore a \propto b$

18

$\because x^4y^2-14x^2y+49=0$

$\therefore (x^2y-7)^2=0$ $\quad \therefore x^2y-7=0$

$\therefore x^2y=7$ $\quad \therefore y \propto \dfrac{1}{x^2}$

19

$\because (4x+7y) \propto (x+2y)$

$\therefore 4x+7y=m(x+2y)$

$\therefore 4x+7y=mx+2my$

$\therefore 7y-2my=mx-4x$

$\therefore y(7-2m)=x(m-4)$

$\therefore y=\dfrac{m-4}{7-2m}x$

putting $\dfrac{m-4}{7-2m}=k \in \mathbb{R}^*$

$\therefore y=kx$ $\quad \therefore y \propto x$

20

$\because \left(\frac{a}{y}-\frac{a}{x}\right) \propto x-y$ $\quad \therefore \frac{a}{y}-\frac{a}{x}=m(x-y)$

$\therefore \dfrac{ax-ay}{xy}=m(x-y)$ $\quad \therefore \dfrac{a(x-y)}{xy}=m(x-y)$

$\therefore xy=\frac{a}{m}$ (constant) $\quad \therefore x$ varies inversely as y

21

The first table represents an inverse variation

because : $3 \times 20=60$, $5 \times 12=60$, $4 \times 15=60$

, $6 \times 10=60$ $\quad$ **i.e.** $xy=m$

The second table represents a direct variation

because : $\frac{9}{2}=\frac{18}{4}=\frac{54}{12}=\frac{72}{16}$ $\quad$ **i.e.** $\frac{y}{x}=m$

The third table represents a direct variation

because : $\frac{9}{5}=\frac{18}{10}=\frac{27}{15}=\frac{45}{25}$ $\quad$ **i.e.** $\frac{y}{x}=m$

The fourth table does not represent a direct variation nor an inverse variation because :

$3 \times 6 \neq -18 \times 1$ or $\frac{6}{3} \neq \frac{-9}{-2}$

i.e. $xy \neq m$ $\quad \therefore$ The variation is not inverse

or $\frac{y}{x} \neq m$ $\quad \therefore$ The variation is not direct

22

[1] The variation is inverse.

[2] $\because y \propto \frac{1}{x}$ $\quad \therefore yx=m$ $\quad \therefore m=12$

[3] As $x=3$ $\quad \therefore 3y=12$ $\quad \therefore y=4$

[4] As $y=2\frac{2}{5}$ $\quad \therefore \left(2\frac{2}{5}\right)x=12$

$\therefore \frac{12}{5}x=12$ $\quad \therefore x=12 \times \frac{5}{12}=5$

23

[1] Direct variation

[2] $\because y \propto x$ $\quad \therefore \frac{y}{x}=m$ $\quad \therefore m=12$

$\therefore \frac{a}{2}=12$ $\quad \therefore a=24$

, $\frac{36}{b}=12$ $\quad \therefore b=3$

24

$\because y=z+5$, $z \propto \frac{1}{x}$ $\quad \therefore z=\frac{m}{x}$

$\therefore y=\frac{m}{x}+5$

At $y=6$, $x=2$

$\therefore 6=\frac{m}{2}+5$ $\quad \therefore 1=\frac{m}{2}$ $\quad \therefore m=2$

$\therefore y=\frac{2}{x}+5$

At $x=1$ $\quad \therefore y=\frac{2}{1}+5=7$

25

$\because y=a+b$ $\quad$, $b \propto x$ $\quad \therefore b=mx$

$\therefore y=a+mx$ $\quad$ At $y=3$, $x=0$

$\therefore 3=a+m \times 0$ $\quad \therefore a=3$

$\therefore y=3+mx$ $\quad$ At $y=5$, $x=3$

$\therefore 5=3+m \times 3$ $\quad \therefore m=\frac{2}{3}$

$\therefore y=3+\frac{2}{3}x$

At $x=7$ $\quad \therefore y=3+\frac{2}{3} \times 7=7\frac{2}{3}$

26

$\because y=a-9$ $\quad$, $y \propto \frac{1}{x^2}$ $\quad \therefore y=\frac{m}{x^2}$

$\therefore \frac{m}{x^2}=a-9$ $\quad \therefore m=x^2(a-9)$

, $\because a=18$ as $x=\frac{2}{3}$ $\quad \therefore m=\frac{4}{9}(18-9)$

$\therefore m=\frac{4}{9} \times 9=4$ $\quad \therefore y=\frac{4}{x^2}$

As $x=1$ $\quad \therefore y=4$

27

[1] $\because y=2+a$ $\quad$, $a \propto \frac{1}{x}$ $\quad \therefore a=\frac{m}{x}$

At $a=5$ $\quad$, $x=2$

$\therefore 5=\frac{m}{2}$ $\quad \therefore m=10$

$\therefore a=\frac{10}{x}$ $\quad \therefore y=2+\frac{10}{x}$

[2] At $x=5$ $\quad \therefore y=2+\frac{10}{5}=4$

28

$\because x=\ell+9$ $\quad$, $\because \ell \propto y$ $\quad \therefore \ell=my$

$\therefore x=my+9$ $\quad$ As $x=24$, $y=5$

$\therefore 24=5m+9$ $\quad \therefore 5m=15$

$\therefore m=3$ $\quad \therefore \ell=3y$

As $\ell=12$ $\quad \therefore 12=3y$ $\quad \therefore y=4$

29

$\because h \propto \frac{1}{r^2}$ $\quad \therefore \frac{h_1}{h_2}=\frac{r_2^2}{r_1^2}$

$\therefore \frac{27}{h_2}=\frac{(15.75)^2}{(10.5)^2}$ $\quad \therefore h_2=\frac{27 \times (10.5)^2}{(15.75)^2}=12$ cm.

30

$\because d \propto t$ $\quad \therefore \frac{d_1}{d_2}=\frac{t_1}{t_2}$

$\therefore \frac{150}{d_2}=\frac{6}{10}$ $\quad \therefore d_2=\frac{150 \times 10}{6}=250$ km.

31

$\because W \propto R$ $\quad \therefore \frac{W_1}{W_2}=\frac{R_1}{R_2}$

$\therefore \frac{14}{W_2}=\frac{84}{144}$ $\quad \therefore W_2=\frac{14 \times 144}{84}=24$ kg.

32

$\because n \propto \frac{1}{x}$ $\quad \therefore \frac{n_1}{n_2}=\frac{x_2}{x_1}$

$\therefore \frac{4}{n_2}=\frac{8}{6}$ $\quad \therefore n_2=\frac{4 \times 6}{8}=3$ hours.

33

$\because d \propto t^2$ $\quad \therefore \frac{d_1}{d_2}=\frac{t_1^2}{t_2^2}$

$\therefore \dfrac{\frac{81}{16}}{144}=\dfrac{\frac{1}{16}}{t_2^2}$ $\quad \therefore t_2^2=\dfrac{144 \times \frac{1}{16}}{\frac{81}{16}}=\frac{16}{9}$

$\therefore t_2=\frac{4}{3}=1\frac{1}{3}$ hour.

34

$\because v \propto \frac{1}{r^2}$ $\quad \therefore \frac{v_1}{v_2}=\frac{r_2^2}{r_1^2}$

$\therefore \frac{5}{v_2}=\frac{(2.5)^2}{3^2}$ $\quad \therefore v_2=\frac{5 \times 3^2}{(2.5)^2}=7.2$ cm./s.

35

Let the weight of the body = w , and the distance from the centre of the earth = d

$\therefore w \propto \frac{1}{d^2}$ $\quad \therefore \frac{w_1}{w_2}=\frac{d_2^2}{d_1^2}$

$\therefore \frac{500}{w_2}=\frac{(640+6390)^2}{(6390)^2}$ $\quad \therefore w_2=413$ w.kg.

36

$\because x \propto y$, $z \propto \ell$ $\quad \therefore x=my$, $z=k\ell$

$\therefore (x+y)(z+\ell)=(my+y)(k\ell+\ell)$

$=y\ell(m+1)(k+1)$ $\qquad (1)$

, $(x-y)(z-\ell)=(my-y)(k\ell-\ell)$

$=y\ell(m-1)(k-1)$ $\qquad (2)$

Dividing (1) by (2) :

$\dfrac{(x+y)(z+\ell)}{(x-y)(z-\ell)}=\dfrac{y\ell(m+1)(k+1)}{y\ell(m-1)(k-1)}=\dfrac{(m+1)(k+1)}{(m-1)(k-1)}$

$=$ constant

$\therefore (x+y)(z+\ell) \propto (x-y)(z-\ell)$

37

$\because (a+b) \propto \frac{a}{b}$ $\quad \therefore (a+b)=\frac{ma}{b}$ $\qquad (1)$

, $\because (a^2-ab+b^2) \propto \frac{b}{a}$ $\quad \therefore (a^2-ab+b^2)=\frac{\ell b}{a}$ $\qquad (2)$

, multiplying (1) by (2) :

$\therefore (a+b)(a^2-ab+b^2)=\frac{ma}{b} \times \frac{\ell b}{a}$

$\therefore a^3+b^3=\ell m=$ constant

Answers of unit three

Answers of Exercise 9

1

1 c 2 c 3 d
4 b 5 b 6 b

2

The primary sources : 1 and 2

The secondary sources : 3 , 4 and 5

3

Side of comparison / The method	Mass population	Samples
Its definition	It is setup collecting data related to the phenomenon from all the individuals of the statistical society.	It is setup collecting data about the phonomenon under study from some individuals of the statistical society not all the individuals , this by selecting a sample representing all statistical society.
Advantages	Accuracy , perfect representation of all statistical society	1 It is faster and less cost 2 It is the unique method for collecting data from the large societies (infinite) 3 It is the unique method for collecting data from some limited societies
Disadvantages	Sometimes it needs a long time and more costs	The results are not accurated specially if the sample does not represent the statistical society very well.

4

The method of mass population : 1 and 5

The method of samples : 2 , 3 and 4

5 , **6** Answer by yourself.

7

The total number of students

$= 4\,000 + 3\,000 + 2\,000 + 1\,000 = 10\,000$ students

The number of the individuals of the first layer in the sample $= \frac{4\,000}{10\,000} \times 500 = 200$ students

The number of the individuals of the second layer in the sample $= \frac{3\,000}{10\,000} \times 500 = 150$ students

The number of the individuals of the third layer in the sample $= \frac{2\,000}{10\,000} \times 500 = 100$ students

The number of the individuals of the fourth layer in the sample $= \frac{1\,000}{10\,000} \times 500 = 50$ students

8

The total number of cars $= 300 + 500 + 200$
$= 1000$ cars

The number of individuals of the sample $= 1000 \times 5\%$
$= 50$ cars

The number of the first model in the sample
$= \frac{300}{1000} \times 50 = 15$ cars

The number of the second model in the sample
$= \frac{500}{1000} \times 50 = 25$ cars

The number of the third model in the sample
$= \frac{200}{1000} \times 50 = 10$ cars

9

The number of the second layer
$= 5\,000 - 1\,500 = 3\,500$ individuals

The number of individuals of all the sample
$= \frac{5\,000 \times 140}{3\,500} = 200$ individuals

10

The size of the whole sample $= \frac{40\,000 \times 240}{12\,000}$
$= 800$ individuals

Answers of Exercise 10

1

1 c 2 a 3 a 4 c 5 b
6 a 7 b 8 c 9 c 10 b
11 d 12 c 13 c 14 b 15 d
16 c 17 c 18 a 19 c 20 d
21 a 22 c

2

1 The mean $(\bar{x}) = \frac{16 + 32 + 5 + 20 + 27}{5} = 20$

x	$x - \bar{x}$	$(x - \bar{x})^2$
16	$16 - 20 = -4$	16
32	$32 - 20 = 12$	144
5	$5 - 20 = -15$	225
20	$20 - 20 = 0$	0
27	$27 - 20 = 7$	49
Total		434

The standard deviation $(\sigma) = \sqrt{\frac{434}{5}} \simeq 9.3$

2 The mean $(\bar{x}) = \frac{72 + 53 + 61 + 70 + 59}{5} = 63$

x	$x - \bar{x}$	$(x - \bar{x})^2$
72	$72 - 63 = 9$	81
53	$53 - 63 = -10$	100
61	$61 - 63 = -2$	4
70	$70 - 63 = 7$	49
59	$59 - 63 = -4$	16
Total		250

The standard deviation $(\sigma) = \sqrt{\frac{250}{5}} \simeq 7.1$

3 The mean $(\bar{x}) = \frac{15 + (-12) + (-9) + 27 + (-6)}{5} = 3$

x	$x - \bar{x}$	$(x - \bar{x})^2$
15	$15 - 3 = 12$	144
-12	$-12 - 3 = -15$	225
-9	$-9 - 3 = -12$	144
27	$27 - 3 = 24$	576
-6	$-6 - 3 = -9$	81
Total		1170

The standard deviation $(\sigma) = \sqrt{\frac{1170}{5}} \simeq 15.3$

4 The mean $(\bar{x}) = \frac{22 + 20 + 20 + 20 + 18}{5} = 20$

x	$x - \bar{x}$	$(x - \bar{x})^2$
22	$22 - 20 = 2$	4
20	$20 - 20 = 0$	0
20	$20 - 20 = 0$	0
20	$20 - 20 = 0$	0
18	$18 - 20 = -2$	4
Total		8

The standard deviation $(\sigma) = \sqrt{\frac{8}{5}} \simeq 1.3$

3

- The mean of the set (A) $= \frac{7 + 8 + 9 + 10 + 11}{5} = 9$

x	$x - \bar{x}$	$(x - \bar{x})^2$
7	$7 - 9 = -2$	4
8	$8 - 9 = -1$	1
9	$9 - 9 = 0$	0
10	$10 - 9 = 1$	1
11	$11 - 9 = 2$	4
Total		10

The standard deviation (σ) of the set (A) $= \sqrt{\frac{10}{5}} \simeq 1.4$

- The mean of the set (B) $= \frac{21 + 20 + 11 + 19}{4} = 17.75$

x	$x - \bar{x}$	$(x - \bar{x})^2$
21	$21 - 17.75 = 3.25$	10.5625
20	$20 - 17.75 = 2.25$	5.0625
11	$11 - 17.75 = -6.75$	45.5625
19	$19 - 17.75 = 1.25$	1.5625
Total		62.75

The standard deviation of the set (B) $= \sqrt{\frac{62.75}{4}} \simeq 4$

- The mean of the set (C) $= \frac{29 + 30 + 30 + 35}{4} = 31$

x	$x - \bar{x}$	$(x - \bar{x})^2$
29	$29 - 31 = -2$	4
30	$30 - 31 = -1$	1
30	$30 - 31 = -1$	1
35	$35 - 31 = 4$	16
Total		22

The standard deviation of the set (C) $= \sqrt{\frac{22}{4}} \simeq 2.3$

$\therefore$ The set B has more dispersion

4

1 The mean $(\bar{x}) = \frac{73 + 54 + 62 + 71 + 60}{5} = 64$

x	$x - \bar{x}$	$(x - \bar{x})^2$
73	$73 - 64 = 9$	81
54	$54 - 64 = -10$	100
62	$62 - 64 = -2$	4
71	$71 - 64 = 7$	49
60	$60 - 64 = -4$	16
Total		250

The standard deviation $(\sigma) = \sqrt{\frac{250}{5}} \simeq 7.07$

2 The mean $\left(\overline{x}\right) = \frac{13+14+17+19+22}{5} = 17$

x	$x-\overline{x}$	$(x-\overline{x})^2$
13	$13-17=-4$	16
14	$14-17=-3$	9
17	$17-17=0$	0
19	$19-17=2$	4
22	$22-17=5$	25
Total		54

The standard deviation $(\sigma) = \sqrt{\frac{54}{5}} \simeq 3.286$

3 The mean $\left(\overline{x}\right)$

$= \frac{65+61+70+64+70+76+70}{7} = 68$

x	$x-\overline{x}$	$(x-\overline{x})^2$
65	$65-68=-3$	9
61	$61-68=-7$	49
70	$70-68=2$	4
64	$64-68=-4$	16
70	$70-68=2$	4
76	$76-68=8$	64
70	$70-68=2$	4
Total		150

The standard deviation $(\sigma) = \sqrt{\frac{150}{7}} \simeq 4.6$

4 The mean $\left(\overline{x}\right)$

$= \frac{23+12+17+13+15+16+8+9+37+10}{10} = 16$

x	$x-\overline{x}$	$(x-\overline{x})^2$
23	$23-16=7$	49
12	$12-16=-4$	16
17	$17-16=1$	1
13	$13-16=-3$	9
15	$15-16=-1$	1
16	$16-16=0$	0
8	$8-16=-8$	64
9	$9-16=-7$	49
37	$37-16=21$	441
10	$10-16=-6$	36
Total		666

The standard deviation $(\sigma) = \sqrt{\frac{666}{10}} \simeq 8.2$

5

1 The mean of the marks of pupils $= \frac{8+9+6+12+10}{5} = 9$

2

x	$x-\overline{x}$	$(x-\overline{x})^2$
8	$8-9=-1$	1
9	$9-9=0$	0
6	$6-9=-3$	9
12	$12-9=3$	9
10	$10-9=1$	1
Total		20

The standard deviation (σ) of the marks of pupils $= \sqrt{\frac{20}{5}} = 2$

6

1 The mean of the maximum degrees $\left(\overline{x}\right)$

$= \frac{25+26+24+24+22+26+27+26}{8} = 25$ degrees

x	$x-\overline{x}$	$(x-\overline{x})^2$
25	$25-25=0$	0
26	$26-25=1$	1
24	$24-25=-1$	1
24	$24-25=-1$	1
22	$22-25=-3$	9
26	$26-25=1$	1
27	$27-25=2$	4
26	$26-25=1$	1
Total		18

The standard deviation $(\sigma) = \sqrt{\frac{18}{8}} = 1.5$ degrees

2 The mean of the minimum degrees $\left(\overline{x}\right)$

$= \frac{11+12+10+6+7+16+15+11}{8} = 11$ degrees

x	$x-\overline{x}$	$(x-\overline{x})^2$
11	$11-11=0$	0
12	$12-11=1$	1
10	$10-11=-1$	1
6	$6-11=-5$	25
7	$7-11=-4$	16
16	$16-11=5$	25
15	$15-11=4$	16
11	$11-11=0$	0
Total		84

The standard deviation $(\sigma) = \sqrt{\frac{84}{8}} \simeq 3.2$ degrees

7

Number of children (X)	Number of families (k)	$X \times k$
0	8	0
1	16	16
2	50	100
3	20	60
4	6	24
Total	100	200

The mean $\left(\overline{x}\right) = \frac{200}{100} = 2$ children

x	k	$x-\overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
0	8	$0-2=-2$	4	32
1	16	$1-2=-1$	1	16
2	50	$2-2=0$	0	0
3	20	$3-2=1$	1	20
4	6	$4-2=2$	4	24
Total	100			92

The standard deviation $(\sigma) = \sqrt{\frac{92}{100}} \simeq 1$ child

8

Number of defective units (X)	Number of boxes (k)	$X \times k$
0	3	0
1	16	16
2	17	34
3	25	75
4	20	80
5	19	95
Total	100	300

The mean $\left(\overline{x}\right) = \frac{300}{100} = 3$ units

x	k	$x-\overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
0	3	$0-3=-3$	9	27
1	16	$1-3=-2$	4	64
2	17	$2-3=-1$	1	17
3	25	$3-3=0$	0	0
4	20	$4-3=1$	1	20
5	19	$5-3=2$	4	76
Total	100			204

The standard deviation $(\sigma) = \sqrt{\frac{204}{100}} \simeq 1.4$ units

9

Number of goals (X)	Number of players (k)	$X \times k$
0	2	0
1	4	4
2	5	10
3	8	24
4	7	28
5	4	20
Total	30	86

The mean $\left(\overline{x}\right) = \frac{86}{30} \simeq 2.9$ goals

x	k	$x-\overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
0	2	$0-2.9=-2.9$	8.41	16.82
1	4	$1-2.9=-1.9$	3.61	14.44
2	5	$2-2.9=-0.9$	0.81	4.05
3	8	$3-2.9=0.1$	0.01	0.08
4	7	$4-2.9=1.1$	1.21	8.47
5	4	$5-2.9=2.1$	4.41	17.64
Total	30			61.5

The standard deviation $(\sigma) = \sqrt{\frac{61.5}{30}} \simeq 1.4$ goals

10

Age (X)	Number of children (k)	$X \times k$
5	1	5
8	2	16
9	3	27
10	3	30
12	1	12
Total	10	90

The mean $\left(\overline{x}\right) = \frac{90}{10} = 9$ years

x	k	$x-\overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
5	1	$5-9=-4$	16	16
8	2	$8-9=-1$	1	2
9	3	$9-9=0$	0	0
10	3	$10-9=1$	1	3
12	1	$12-9=3$	9	9
Total	10			30

The standard deviation $(\sigma) = \sqrt{\frac{30}{10}} \simeq 1.7$ years

11

Number of students (X)	Number of classes (k)	X × k
0	1	0
1	3	3
2	5	10
3	6	18
4	3	12
5	2	10
Total	20	53

The mean $\left(\overline{x}\right) = \frac{53}{20} = 2.65$ students

x	k	$x - \overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
0	1	$0 - 2.65 = -2.65$	7.0225	7.0225
1	3	$1 - 2.65 = -1.65$	2.7225	8.1675
2	5	$2 - 2.65 = -0.65$	0.4225	2.1125
3	6	$3 - 2.65 = 0.35$	0.1225	0.735
4	3	$4 - 2.65 = 1.35$	1.8225	5.4675
5	2	$5 - 2.65 = 2.35$	5.5225	11.045
Total	20			34.55

The standard deviation $(\sigma) = \sqrt{\frac{34.55}{20}} \simeq 1.3$ student

12

Sets	Centres of sets (X)	Frequency (k)	X × k
0 –	2	3	6
4 –	6	4	24
8 –	10	7	70
12 –	14	2	28
16 – 20	18	9	162
Total		25	290

The mean $\left(\overline{x}\right) = \frac{290}{25} = 11.6$

x	k	$x - \overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
2	3	$2 - 11.6 = -9.6$	92.16	276.48
6	4	$6 - 11.6 = -5.6$	31.36	125.44
10	7	$10 - 11.6 = -1.6$	2.56	17.92
14	2	$14 - 11.6 = 2.4$	5.76	11.52
18	9	$18 - 11.6 = 6.4$	40.96	368.64
Total	25			800

The standard deviation $(\sigma) = \sqrt{\frac{800}{25}} \simeq 5.7$

13

Sets	Centres of sets (X)	Frequency (k)	X × k
20 –	25	10	250
30 –	35	12	420
40 –	45	8	360
50 –	55	6	330
60 –	65	3	195
70 –	75	1	75
Total		40	1630

The mean $\left(\overline{x}\right) = \frac{1630}{40} = 40.75$ pounds

x	k	$x - \overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
25	10	$25 - 40.75 = -15.75$	248.0625	2480.625
35	12	$35 - 40.75 = -5.75$	33.0625	396.75
45	8	$45 - 40.75 = 4.25$	18.0625	144.5
55	6	$55 - 40.75 = 14.25$	203.0625	1218.375
65	3	$65 - 40.75 = 24.25$	588.0625	1764.1875
75	1	$75 - 40.75 = 34.25$	1173.0625	1173.0625
Total	40			7177.5

The standard deviation $(\sigma) = \sqrt{\frac{7177.5}{40}} \simeq 13.4$ pounds

14

Sets	Centres of sets (X)	Frequency (k)	X × k
5 –	6	3	18
7 –	8	6	48
9 –	10	10	100
11 –	12	12	144
13 –	14	5	70
15 – 17	16	4	64
Total		40	444

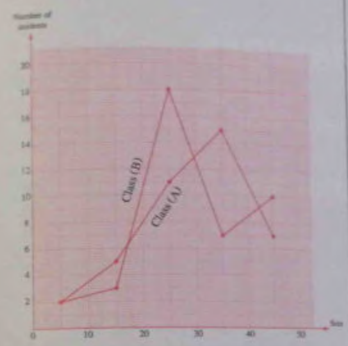
The mean $\left(\overline{x}\right) = \frac{444}{40} = 11.1$ km./litre

x	k	$x - \overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
6	3	$6 - 11.1 = -5.1$	26.01	78.03
8	6	$8 - 11.1 = -3.1$	9.61	57.66
10	10	$10 - 11.1 = -1.1$	1.21	12.1
12	12	$12 - 11.1 = 0.9$	0.81	9.72
14	5	$14 - 11.1 = 2.9$	8.41	42.05
16	4	$16 - 11.1 = 4.9$	24.01	96.04
Total	40			295.6

The standard deviation $(\sigma) = \sqrt{\frac{295.6}{40}} \simeq 2.7$ km./litre

15

1



2 With respect to class (A)

Sets	Centres of sets (X)	Frequency (k)	X × k
0 –	5	2	10
10 –	15	5	75
20 –	25	11	275
30 –	35	15	525
40 – 50	45	7	315
Total		40	1200

The mean $\left(\overline{x}\right)$ of class (A) $= \frac{1200}{40} = 30$ marks

x	k	$x - \overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
5	2	$5 - 30 = -25$	625	1250
15	5	$15 - 30 = -15$	225	1125
25	11	$25 - 30 = -5$	25	275
35	15	$35 - 30 = 5$	25	375
45	7	$45 - 30 = 15$	225	1575
Total	40			4600

The standard deviation (σ) of class (A)

$= \sqrt{\frac{4600}{40}} \simeq 10.7$ marks

With respect to class (B)

Sets	Centres of sets (X)	Frequency (k)	X × k
0 –	5	2	10
10 –	15	3	45
20 –	25	18	450
30 –	35	7	245
40 – 50	45	10	450
Total		40	1200

The mean $\left(\overline{x}\right)$ of class (B) $= \frac{1200}{40} = 30$ marks

x	k	$x - \overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
5	2	$5 - 30 = -25$	625	1250
15	3	$15 - 30 = -15$	225	675
25	18	$25 - 30 = -5$	25	450
35	7	$35 - 30 = 5$	25	175
45	10	$45 - 30 = 15$	225	2250
Total	40			4800

The standard deviation (σ) of class (B)

$= \sqrt{\frac{4800}{40}} \simeq 11$ marks

3 Class A is the most homogeneous in getting marks.

Answers of accumulative basic skills

1 d	2 c	3 b	4 d	5 c
6 d	7 c	8 d	9 b	10 c
11 b	12 b	13 c	14 a	15 b
16 d	17 c	18 c	19 a	20 a
21 c	22 d	23 d	24 c	25 c
26 c	27 c	28 c	29 b	30 a
31 d	32 c	33 a	34 c	35 c
36 c				

Guide Answers

Of Trigonometry and Geometry Exercises

Answers of unit four

Answers of Exercise 1

1

1 $\frac{15}{17}$, $\frac{8}{17}$ 2 $\frac{8}{17}$, $\frac{15}{17}$ 3 $\frac{15}{8}$, $\frac{8}{15}$

2

1 a	2 b	3 c	4 d	5 c
6 a	7 d	8 d	9 c	10 b
11 b	12 b	13 c		

3

Let the measures of the two angles be $3X$ and $5X$

$\therefore 3X + 5X = 180^\circ$ $\therefore 8X = 180^\circ$

$\therefore X = \frac{180^\circ}{8} = 22.5^\circ$

The measure of the first angle $= 3 \times 22.5^\circ = 67.5^\circ$

$= 67^\circ\ 30'$

The measure of the second angle $= 5 \times 22.5^\circ$

$= 112.5^\circ = 112^\circ\ 30'$

4

Let the measures of the two angles be $3X$ and $4X$

$\therefore 3X + 4X = 90^\circ$ $\therefore 7X = 90^\circ$

$\therefore X = \frac{90^\circ}{7} = 12\frac{6^\circ}{7}$

$\therefore$ The measure of the greater angle

$= 4 \times 12\frac{6^\circ}{7} \simeq 51^\circ\ 25'\ 43''$

5

Let the measures of the interior angles of the triangle be $3X$, $4X$, $7X$

$\therefore 3X + 4X + 7X = 180^\circ$ $\therefore 14X = 180^\circ$

$\therefore X = \frac{180^\circ}{14}$

The measure of the first angle

$= 3 \times \frac{180^\circ}{14} \simeq 38^\circ\ 34'\ 17''$

The measure of the second angle

$= 4 \times \frac{180^\circ}{14} \simeq 51^\circ\ 25'\ 43''$

The measure of the third angle $= 7 \times \frac{180^\circ}{14} = 90^\circ$

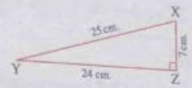
6

$\because m(\angle A) = 90^\circ$ $\therefore (BC)^2 = (20)^2 + (15)^2 = 625$

$\therefore BC = 25$ cm.

$\therefore \cos C \cos B - \sin C \sin B = \frac{15}{25} \times \frac{20}{25} - \frac{20}{25} \times \frac{15}{25} = 0$

7



$\because m(\angle Z) = 90^\circ$

$\therefore (ZY)^2 = (25)^2 - (7)^2$

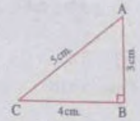
$= 576$

$\therefore ZY = 24$ cm.

1 $\tan X \times \tan Y = \frac{24}{7} \times \frac{7}{24} = 1$

2 $\sin^2 X + \sin^2 Y = \left(\frac{24}{25}\right)^2 + \left(\frac{7}{25}\right)^2 = \frac{625}{625} = 1$

8



$\because m(\angle B) = 90^\circ$

$\therefore (AB)^2 = (5)^2 - (4)^2 = 9$

$\therefore AB = 3$ cm.

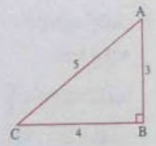
$\therefore \sin^2 A - \cos^2 A$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

$2\sin^2 A - 1 = 2 \times \left(\frac{4}{5}\right)^2 - 1 = 2 \times \frac{16}{25} - 1 = \frac{7}{25}$

$\therefore \sin^2 A - \cos^2 A = 2\sin^2 A - 1$

9



$\because \frac{AB}{AC} = \frac{3}{5}$

$\therefore$ Let AB = 3 length unit

, AC = 5 length unit

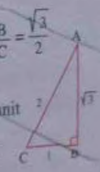
, $\because m(\angle B) = 90^\circ$

$\therefore (BC)^2 = 5^2 - 3^2 = 16$ $\therefore$ BC = 4 length unit

$\therefore \sin A = \frac{BC}{AC} = \frac{4}{5}$, $\cos A = \frac{AB}{AC} = \frac{3}{5}$

, $\tan A = \frac{BC}{AB} = \frac{4}{3}$

10



$\because 2\,AB = \sqrt{3}\,AC$ $\therefore \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

Let AB $= \sqrt{3}$ length unit

, AC = 2 length unit $\therefore$ BC = 1 length unit

$\therefore \sin C = \frac{\sqrt{3}}{2}$, $\cos C = \frac{1}{2}$, $\tan C = \sqrt{3}$

Guide Answers

Of Trigonometry and Geometry Exercises

Answers of unit four

Answers of Exercise 1

1

1) $\frac{15}{17}$, $\frac{8}{17}$ 2) $\frac{8}{17}$, $\frac{15}{17}$ 3) $\frac{15}{8}$, $\frac{8}{15}$

2

1 a	2 b	3 c	4 d	5 c
6 a	7 d	8 d	9 c	10 b
11 b	12 b	13 c		

3

Let the measures of the two angles be $3X$ and $5X$

$\therefore 3X + 5X = 180^\circ$ $\therefore 8X = 180^\circ$

$\therefore X = \frac{180^\circ}{8} = 22.5^\circ$

The measure of the first angle $= 3 \times 22.5^\circ = 67.5^\circ$
$= 67^\circ\ 30'$

The measure of the second angle $= 5 \times 22.5^\circ$
$= 112.5^\circ = 112^\circ\ 30'$

4

Let the measures of the two angles be $3X$ and $4X$

$\therefore 3X + 4X = 90^\circ$ $\therefore 7X = 90^\circ$

$\therefore X = \frac{90^\circ}{7} = 12\frac{6^\circ}{7}$

$\therefore$ The measure of the greater angle

$= 4 \times 12\frac{6^\circ}{7} \simeq 51^\circ\ 25'\ 43''$

5

Let the measures of the interior angles of the triangle be $3X$, $4X$, $7X$

$\therefore 3X + 4X + 7X = 180^\circ$ $\therefore 14X = 180^\circ$

$\therefore X = \frac{180^\circ}{14}$

The measure of the first angle

$= 3 \times \frac{180^\circ}{14} \simeq 38^\circ\ 34'\ 17''$

The measure of the second angle

$= 4 \times \frac{180^\circ}{14} \simeq 51^\circ\ 25'\ 43''$

The measure of the third angle $= 7 \times \frac{180^\circ}{14} = 90^\circ$

6

$\because m(\angle A) = 90^\circ$ $\therefore (BC)^2 = (20)^2 + (15)^2 = 625$

$\therefore BC = 25$ cm.

$\therefore \cos C \cos B - \sin C \sin B = \frac{15}{25} \times \frac{20}{25} - \frac{20}{25} \times \frac{15}{25} = 0$

7

$\because m(\angle Z) = 90^\circ$

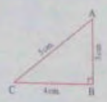
$\therefore (ZY)^2 = (25)^2 - (7)^2$
$= 576$

$\therefore ZY = 24$ cm.

1) $\tan X \times \tan Y = \frac{24}{7} \times \frac{7}{24} = 1$

2) $\sin^2 X + \sin^2 Y = \left(\frac{24}{25}\right)^2 + \left(\frac{7}{25}\right)^2 = \frac{625}{625} = 1$

8



$\because m(\angle B) = 90^\circ$

$\therefore (AB)^2 = (5)^2 - (4)^2 = 9$

$\therefore AB = 3$ cm.

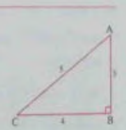
$\therefore \sin^2 A - \cos^2 A$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

$2\sin^2 A - 1 = 2 \times \left(\frac{4}{5}\right)^2 - 1 = 2 \times \frac{16}{25} - 1 = \frac{7}{25}$

$\therefore \sin^2 A - \cos^2 A = 2\sin^2 A - 1$

9



$\because \frac{AB}{AC} = \frac{3}{5}$

$\therefore$ Let $AB = 3$ length unit

, $AC = 5$ length unit

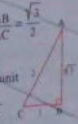
, $\because m(\angle B) = 90^\circ$

$\therefore (BC)^2 = 5^2 - 3^2 = 16$ $\therefore BC = 4$ length unit

$\therefore \sin A = \frac{BC}{AC} = \frac{4}{5}$, $\cos A = \frac{AB}{AC} = \frac{3}{5}$

, $\tan A = \frac{BC}{AB} = \frac{4}{3}$

10



$\because 2AB = \sqrt{3}\,AC$ $\therefore \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

Let $AB = \sqrt{3}$ length unit

, $AC = 2$ length unit $\therefore BC = 1$ length unit

$\therefore \sin C = \frac{\sqrt{3}}{2}$, $\cos C = \frac{1}{2}$, $\tan C = \sqrt{3}$

11

1 $\because \tan C = \frac{AB}{BC}$ $\qquad \therefore \frac{3}{4} = \frac{6}{BC}$

$\therefore BC = 8$ cm.

, $\because (AC)^2 = (AB)^2 + (BC)^2$

$\therefore (AC)^2 = 36 + 64 = 100$ $\qquad \therefore AC = 10$ cm.

2 $\sin A + \cos A = \frac{8}{10} + \frac{6}{10} = \frac{14}{10} = \frac{7}{5}$

12

1 In ΔABC : $\because m(\angle BAC) = 90^\circ$

$\therefore (BC)^2 = 36 + 64 = 100$ $\quad \therefore BC = 10$ cm.

, $\because \overline{AD} \perp \overline{BC}$ $\qquad \therefore AD = \frac{6 \times 8}{10} = 4.8$ cm.

, $(AB)^2 = BD \times BC$ $\qquad \therefore 36 = BD \times 10$

$\therefore BD = 3.6$ cm.

$\therefore \tan(\angle BAD) = \frac{3.6}{4.8} = \frac{3}{4}$

2 $\cos(\angle DAC) + \cos(\angle DAB)$

$= \frac{4.8}{8} + \frac{4.8}{6} = \frac{7}{5}$

13

From ΔABD : $\because \cos B = \frac{BD}{AB}$

, from ΔACD : $\because \cos C = \frac{CD}{AC}$

$\therefore AB \cos B + AC \cos C = AB \times \frac{BD}{AB} + AC \times \frac{CD}{AC}$

$= BD + CD = 8$ cm.

14

In ΔABD : $\because m(\angle A) = 90^\circ$

$\therefore (AD)^2 = (BD)^2 - (AB)^2 = 100 - 36 = 64$

$\therefore AD = 8$ cm.

$\therefore \tan(\angle ADB) = \frac{6}{8} = \frac{3}{4}$

, $\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{BD}$ is a transversal to them

$\therefore m(\angle ADB) = m(\angle DBC)$ (alternate angles)

$\therefore \tan(\angle ADB) = \tan(\angle DBC)$

$\therefore \tan(\angle DBC) = \frac{3}{4}$ $\qquad \therefore \frac{DC}{10} = \frac{3}{4}$

$\therefore DC = \frac{10 \times 3}{4} = 7.5$ cm.

15

In ΔBCD : $\because BD = CD$, $\overline{DH} \perp \overline{BC}$

$\therefore$ H is the midpoint of $\overline{BC}$

$\therefore CH = \frac{1}{2} BC = \frac{1}{2} \times 24 = 12$ cm.

, $\because m(\angle DHC) = 90^\circ$

$\therefore (DH)^2 = (DC)^2 - (CH)^2 = 169 - 144 = 25$

$\therefore DH = 5$ cm.

In ΔDHC : $\therefore \tan(\angle DCB) = \frac{DH}{CH} = \frac{5}{12}$

In ΔABC : $\because m(\angle A) = 90^\circ$

$\therefore m(\angle ABC) + m(\angle ACB) = 90^\circ$

$\therefore \cos(\angle ABC) = \sin(\angle ACB) = \frac{DH}{DC}$

$\therefore \cos(\angle ABC) = \frac{5}{13}$

16

$\because CE = 5$ cm , $AE = 3$ cm.

$\therefore AC = 8$ cm.

, $\because$ In the square the two diagonals bisect each other

$\therefore$ M is the midpoint of $\overline{AC}$

$\therefore AM = 4$ cm.

$\therefore EM = 4 - 3 = 1$ cm.

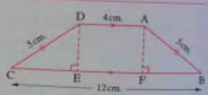
, $\because MD = \frac{1}{2} AC = \frac{1}{2} \times 8 = 4$ cm.

, $\because \overline{AC} \perp \overline{BD}$ (properties of the square)

$\therefore m(\angle AMD) = 90^\circ$

$\therefore$ From ΔDME : $\tan(\angle DEC) = \frac{DM}{EM} = \frac{4}{1} = 4$

17



Draw $\overline{AF} \perp \overline{BC}$,

$\overline{DE} \perp \overline{BC}$

, $\because \overline{AD} \parallel \overline{BC}$,

$\therefore$ AFED is a rectangle , $FE = 4$ cm.

$\therefore BF + EC = 8$ cm.

, $\because BF = EC = 4$ cm.

(ΔABF and ΔDCE are congruent)

$\therefore$ from ΔABF which is right-angled at F :

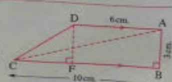
$(AF)^2 = (5)^2 - (4)^2 = 9$

$\therefore AF = 3$ cm. $\qquad \therefore DE = AF = 3$ cm.

(AFED is a rectangle)

$\therefore \frac{5 \tan B \cos C}{\sin^2 C + \cos^2 B} = \frac{5 \times \frac{3}{4} \times \frac{4}{5}}{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = 3$

18



Draw $\overline{DF} \perp \overline{BC}$

$\therefore \overline{AD} \parallel \overline{BC}$, $\overline{AB} \perp \overline{BC}$

, $\overline{DF} \perp \overline{BC}$

$\therefore$ ABFD is a rectangle

$\therefore BF = AD = 6$ cm.

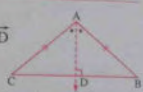
$\therefore FC = 4$ cm. , $DF = AB = 3$ cm.

$\therefore$ From ΔDFC which is right-angled at F :

$(DC)^2 = 3^2 + 4^2 = 25$ $\qquad \therefore DC = 5$ cm.

$\therefore \cos(\angle DCB) - \tan(\angle ACB) = \frac{4}{5} - \frac{3}{10} = \frac{1}{2}$

19



Bisect $\angle A$ by the bisector $\overrightarrow{AD}$

, $\because \Delta ABC$ is an isosceles triangle

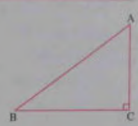
$\therefore \overline{AD} \perp \overline{BC}$ $\qquad \therefore \sin \frac{A}{2} = \sin(\angle BAD)$

$\therefore \sin(\angle BAD) = \frac{4}{5}$

$\because \angle B$, $\angle BAD$ are acute angles

$\therefore \cos B = \sin(\angle BAD)$ $\qquad \therefore \cos B = \frac{4}{5}$

20



$\because \sin B = \frac{AC}{AB}$

, $\cos B = \frac{BC}{AB}$

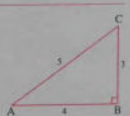
$\therefore \sin B + \cos B = \frac{AC}{AB} + \frac{BC}{AB}$

$= \frac{AC + BC}{AB}$

From triangle inequality

$\because AC + BC > AB$ $\qquad \therefore \sin B + \cos B > 1$

21



$\because \sin A = \frac{6}{10} = \frac{3}{5}$

$\therefore \frac{BC}{AC} = \frac{3}{5}$

Assuming that :

$BC = 3$ length unit , $AC = 5$ length unit

$\therefore AB = 4$ length unit

$\therefore \sin A \cos C + \cos A \sin C$

$= \frac{3}{5} \times \frac{3}{5} + \frac{4}{5} \times \frac{4}{5} = \frac{9}{25} + \frac{16}{25} = 1$

22

$\because 7 \tan A - 24 = 0$

$\therefore 7 \tan A = 24$

$\therefore \tan A = \frac{24}{7}$

$\therefore \frac{BC}{AB} = \frac{24}{7}$

Assuming that $BC = 24$ length unit

$\therefore AB = 7$ length unit $\qquad \therefore AC = 25$ length unit

$\therefore 1 - \tan A \sin C = 1 - \frac{24}{7} \times \frac{7}{25} = \frac{1}{25}$

23

1 $\because m(\angle 1) = X$ (corresponding angles)

, $\because \tan(\angle 1) = \frac{2}{5}$

$\therefore \tan X = \frac{2}{5}$

2 $\because m(\angle 1) = X$ (alternate angles)

, $\because \tan(\angle 1) = \frac{3}{2}$

$\therefore \tan X = \frac{3}{2}$

3 $\because m(\angle 1) = X$ (corresponding angles)

$\because \Delta ABC$ is right-angled at B

$\therefore (AC)^2 = 3^2 + 4^2 = 25$

$\therefore AC = 5$ length units

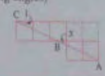
$\therefore \cos(\angle 1) = \frac{4}{5}$

$\therefore \cos X = \frac{4}{5}$

4 $\because m(\angle 1) = X$ (corresponding angles)

, $\because \tan(\angle 1) = \frac{2}{5}$

$\therefore \tan X = \frac{2}{5}$



24

From ΔABC : $\cos B = \frac{18}{BC}$

, $\because$ H is the midpoint of $\overline{BC}$

$\therefore BC = 2\,BH$

$\therefore \cos B = \frac{18}{2\,BH} = \frac{9}{BH}$ (1)

, from ΔBDH : $\cos B = \frac{BH}{13}$ (2)

From (1) and (2) : $\therefore \frac{9}{BH} = \frac{BH}{13}$

$\therefore (BH)^2 = 9 \times 13$ $\qquad \therefore BH = 3\sqrt{13}$ cm.

, $\because m(\angle BHD) = 90^\circ$

$\therefore (DH)^2 = (BD)^2 - (BH)^2 = 169 - 117 = 52$

$\therefore DH = 2\sqrt{13}$ cm. $\qquad \therefore \tan B = \frac{2\sqrt{13}}{3\sqrt{13}} = \frac{2}{3}$

Another Solution :

Construction : Draw $\overline{CD}$

Proof : In ΔBCD :

$\because \overline{DH} \perp \overline{BC}$, $BH = CH$

$\therefore BD = CD = 13$ cm.

In ΔACD : $\because m(\angle A) = 90^\circ$

$\therefore (AC)^2 = (CD)^2 - (AD)^2 = 169 - 25 = 144$

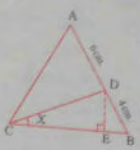
$\therefore AC = 12$ cm.

In ΔABC : $\tan B = \frac{AC}{AB} = \frac{12}{18} = \frac{2}{3}$

Construction :

Draw $\overline{DE} \perp \overline{BC}$

and intersects it at E



Proof :

$\because$ ABC is an equilateral triangle

$\therefore m(\angle B) = 60^\circ$

In Δ BDE : $\because \overline{DE} \perp \overline{BC}$

$\therefore m(\angle DEB) = 90^\circ$

$\therefore m(\angle BDE) = 180^\circ - (60^\circ + 90^\circ) = 30^\circ$

$\therefore BE = \frac{1}{2} BD = 2$ cm.

, $\because (DE)^2 = (DB)^2 - (BE)^2$ (Pythagoras' theorem)

$\therefore (DE)^2 = 16 - 4 = 12$

$\therefore DE = \sqrt{12} = 2\sqrt{3}$ cm.

, $\because BC = BA = 10$ cm.

$\therefore EC = BC - BE = 10 - 2 = 8$ cm.

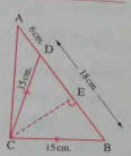
$\therefore \tan X = \frac{DE}{EC} = \frac{2\sqrt{3}}{8} = \frac{\sqrt{3}}{4}$, $\because k \tan X = \sqrt{3}$

$\therefore k \times \frac{\sqrt{3}}{4} = \sqrt{3}$

$\therefore k = \sqrt{3} \times \frac{4}{\sqrt{3}} = 4$

26

Draw : $\overline{CE} \perp \overline{BD}$



In Δ CBD : $\because CB = CD$

, $\overline{CE} \perp \overline{BD}$

$\therefore$ E is the midpoint of $\overline{BD}$

$\therefore ED = 9$ cm.

In Δ EDC which is right-angled at E

$(CE)^2 = (15)^2 - (9)^2 = 144$

$\therefore CE = 12$ cm.

$\therefore \tan(\angle BAC) = \frac{EC}{AE} = \frac{12}{15} = \frac{4}{5}$

27

Let $DE = \ell$ cm. $\qquad \therefore AE = (5 - \ell)$ cm.

, $\because m(\angle AEB) = m(\angle DCE)$

$\therefore \tan(\angle AEB) = \tan(\angle DCE)$

$\therefore \frac{2}{5-\ell} = \frac{\ell}{2}$ $\qquad \therefore 5\ell - \ell^2 = 4$

$\therefore \ell^2 - 5\ell + 4 = 0$ $\qquad \therefore (\ell - 4)(\ell - 1) = 0$

$\therefore \ell = 4$ or $\ell = 1$

, $\because AE < ED$ $\qquad \therefore ED = 4$ cm.

$\therefore \tan(\angle CED) = \frac{2}{4} = \frac{1}{2}$

28

$\because \angle BAD$ and $\angle CAD$ are two acute angles

, $\sin(\angle BAD) = \cos(\angle CAD)$ $\quad \therefore m(\angle CAB) = 90^\circ$

, $\because \sin(\angle BAD) = \frac{DB}{AB} = \frac{3}{5}$ $\quad \therefore \frac{9}{AB} = \frac{3}{5}$

$\therefore AB = 15$ cm.

, $\because (AD)^2 = (15)^2 - (9)^2 = 144$

$\therefore AD = 12$ cm.

, $\because \overline{AD} \perp \overline{BC}$, $\overline{CA} \perp \overline{AB}$

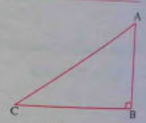
$\therefore (AB)^2 = BD \times BC$ (Euclidean theorem)

$\therefore (15)^2 = 9 \times BC$ $\qquad \therefore BC = 25$ cm.

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 25 \times 12 = 150$ cm^2.

29

$\because \Delta$ ABC is right-angled at B



$\therefore \sin A = \frac{BC}{AC}$

$\therefore \sin^2 A = \frac{(BC)^2}{(AC)^2}$

, $\sin C = \frac{AB}{AC}$ $\qquad \therefore \sin^2 C = \frac{(AB)^2}{(AC)^2}$

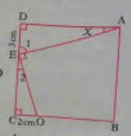
$\therefore \sin^2 A + \sin^2 C = \frac{(BC)^2}{(AC)^2} + \frac{(AB)^2}{(AC)^2} = \frac{(BC)^2 + (AB)^2}{(AC)^2}$

, $\because (BC)^2 + (AB)^2 = (AC)^2$ (Pythagoras)

$\therefore \sin^2 A + \sin^2 C = \frac{(AC)^2}{(AC)^2} = 1$

30

$\because m(\angle AEO) = 90^\circ$



$\therefore m(\angle 1) + m(\angle 2) = 90^\circ$

In Δ ADE which is right-angled at D

$\therefore X + m(\angle 1) = 90^\circ$

$\therefore X = m(\angle 2)$

Let the side length of the square be ℓ

$\therefore EC = \ell - 3$ $\qquad \therefore \tan X = \tan(\angle 2)$

$\therefore \frac{3}{\ell} = \frac{2}{\ell - 3}$ $\qquad \therefore 2\ell = 3\ell - 9$

$\therefore \ell = 9$ $\qquad \therefore \tan X = \frac{DE}{AD} = \frac{3}{9} = \frac{1}{3}$

Answers of Exercise 2

1

1. $\sin 45^\circ - \cos 45^\circ = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$

2. $\cos 60^\circ + \sin 30^\circ = \frac{1}{2} + \frac{1}{2} = 1$

3. $\sin 30^\circ + \cos 60^\circ - \tan 45^\circ = \frac{1}{2} + \frac{1}{2} - 1 = 0$

4. $\sin 60^\circ + \cos 30^\circ + \tan 60^\circ = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \sqrt{3} = 2\sqrt{3}$

5. $\sin^2 45^\circ + \cos^2 45^\circ = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$

6. $4 \cos 30^\circ \tan 60^\circ = 4 \times \frac{\sqrt{3}}{2} \times \sqrt{3} = 6$

7. $\tan^2 60^\circ - 2 \sin 45^\circ \cos 45^\circ$
$= \left(\sqrt{3}\right)^2 - 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 3 - 1 = 2$

8. $\sin^2 60^\circ - \tan 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$
$= \left(\frac{\sqrt{3}}{2}\right)^2 - \sqrt{3} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{3}{2} + \frac{1}{4} = -\frac{1}{2}$

9. $2 \sin 30^\circ \cos 60^\circ + \sqrt{2} \sin 45^\circ$
$= 2 \times \frac{1}{2} \times \frac{1}{2} + \sqrt{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2} + 1 = 1\frac{1}{2}$

10. $(\cos 30^\circ - \cos 60^\circ)(\sin 30^\circ + \sin 60^\circ)$
$= \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$
$= \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

11. $\frac{\sin 30^\circ}{\cos 60^\circ} - \cos 30^\circ \sin 60^\circ = \frac{\frac{1}{2}}{\frac{1}{2}} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$
$= 1 - \frac{3}{4} = \frac{1}{4}$

12. $\frac{\cos^2 60^\circ + \cos^2 30^\circ + \tan^2 45^\circ}{\sin 60^\circ \tan 60^\circ - \sin 30^\circ}$
$= \frac{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + 1^2}{\frac{\sqrt{3}}{2} \times \sqrt{3} - \frac{1}{2}} = \frac{\frac{1}{4} + \frac{3}{4} + 1}{\frac{3}{2} - \frac{1}{2}} = 2$

2

1. The left side $= \sin 60^\circ = \frac{\sqrt{3}}{2}$
The right side $= 2 \sin 30^\circ \cos 30^\circ$
$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$
$\therefore$ The two sides are equal.

2. The left side $= \cos 60^\circ = \frac{1}{2}$
The right side $= 2 \cos^2 30^\circ - 1$
$= 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1 = 2 \times \frac{3}{4} - 1 = \frac{1}{2}$
$\therefore$ The two sides are equal.

3. The left side $= 2 \cos^2 30^\circ - 1 = 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1$
$= 2 \times \frac{3}{4} - 1 = \frac{1}{2}$
The right side $= 1 - 2 \sin^2 30^\circ = 1 - 2 \times \left(\frac{1}{2}\right)^2$
$= 1 - 2 \times \frac{1}{4} = \frac{1}{2}$
$\therefore$ The two sides are equal.

4. The left side $= \cos 60^\circ = \frac{1}{2}$
The right side $= \cos^2 30^\circ - \sin^2 30^\circ$
$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$
$\therefore$ The two sides are equal.

5. The left side $= \tan 60^\circ = \sqrt{3}$
The right side $= \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$
$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \sqrt{3}$
$\therefore$ The two sides are equal.

6. The left side $= \cos^2 60^\circ = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$
The right side $= 5 \sin^2 30^\circ - \tan^2 45^\circ$
$= 5\left(\frac{1}{2}\right)^2 - 1^2 = 5 \times \frac{1}{4} - 1 = \frac{1}{4}$
$\therefore$ The two sides are equal.

7. The left side $= \sin^3 30^\circ = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$
The right side $= 9 \cos^3 60^\circ - \tan^2 45^\circ$
$= 9 \times \left(\frac{1}{2}\right)^3 - 1^2$
$= 9 \times \frac{1}{8} - 1 = \frac{9}{8} - \frac{8}{8} = \frac{1}{8}$
$\therefore$ The two sides are equal.

8 The left side $= \dfrac{\sin 30^\circ \cos 45^\circ + \sin 45^\circ \cos 30^\circ}{\sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ}$

$$= \frac{\frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}}$$

$$= \frac{\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}}{\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}} = 1$$

The right side $= \tan^2 45^\circ = 1^2 = 1$

$\therefore$ The two sides are equal.

9 The left side $= \sin 30^\circ = \frac{1}{2}$

The right side $= \sqrt{\dfrac{1 - \cos 60^\circ}{2}} = \sqrt{\dfrac{1 - \frac{1}{2}}{2}} = \sqrt{\dfrac{\frac{1}{2}}{2}}$

$= \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\therefore$ The two sides are equal.

3

1 b	2 b	3 d	4 c	5 b
6 c	7 d	8 a	9 b	10 d
11 c	12 c	13 d	14 a	15 a
16 d	17 b	18 c	19 d	20 a

4

1 $X \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\sqrt{3}\right)^2 \quad \therefore \frac{1}{2} X = 3 \quad \therefore X = 6$

2 $X \times \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$

$\therefore \frac{1}{4} X = \frac{3}{4} \qquad \therefore X = 3$

3 $X \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \sqrt{3} = (1)^2 - \left(\frac{1}{2}\right)^2$

$\therefore \frac{\sqrt{3}}{2} X = \frac{3}{4} \qquad \therefore X = \frac{\sqrt{3}}{2}$

4 $4 X = \left(\frac{\sqrt{3}}{2}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)^2 \times (1)^2$

$\therefore 4 X = \frac{1}{4} \qquad \therefore X = \frac{1}{16}$

5

1 $\because \tan X = 4 \times \frac{1}{2} \times \frac{1}{2} \quad \therefore \tan X = 1 \quad \therefore X = 45^\circ$

2 $\because \sin X = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} \quad \therefore \sin X = \frac{1}{2}$

$\therefore X = 30^\circ$

3 $\because 2 \sin X = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \quad \therefore 2 \sin X = 1$

$\therefore \sin X = \frac{1}{2} \qquad \therefore X = 30^\circ$

4 $\because 6 \times \sin X \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1 - \frac{1}{4}$

$\therefore 6 \times \frac{1}{2} \times \sin X = \frac{3}{4} \qquad \therefore 3 \times \sin X = \frac{3}{4}$

$\therefore \sin X = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4} \qquad \therefore X \simeq 14^\circ\, 28'\, 39''$

5 $\because \cos X = \dfrac{\frac{\sqrt{3}}{2} \times \frac{1}{2}}{1 \times \left(\frac{1}{\sqrt{2}}\right)^2} \quad \therefore \cos X = \frac{\sqrt{3}}{2} \quad \therefore X = 30^\circ$

6 $\because \cos (3 X + 6^\circ) = \frac{1}{2} \qquad \therefore 3 X + 6^\circ = 60^\circ$

$\therefore 3 X = 54^\circ \qquad \therefore X = \frac{54}{3} = 18^\circ$

7 $\because \sqrt{3} \times \sin X \times \frac{1}{\sqrt{3}} = 1 \times \cos 2 X$

$\therefore \sin X = \cos 2 X \qquad \therefore X + 2 X = 90^\circ$

$\therefore 3 X = 90^\circ \qquad \therefore X = 30^\circ$

6

1 $\because \left(\frac{1}{\sqrt{2}}\right)^2 = \cos E \times \frac{1}{\sqrt{3}}$

$\therefore \cos E = \frac{1}{2} \div \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{2} \qquad \therefore E = 30^\circ$

2 $\because \sin E \times \left(\frac{\sqrt{3}}{2}\right)^2 = 3 \times \left(\frac{1}{\sqrt{2}}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 \times \frac{1}{2}$

$\therefore \sin E \times \frac{3}{4} = \frac{3}{8} \qquad \therefore \sin E = \frac{1}{2}$

$\therefore E = 30^\circ$

3 $\because 3 \tan E - 4 \times \left(\frac{1}{2}\right)^2 = 8 \times \left(\frac{1}{2}\right)^2$

$\therefore 3 \tan E - 1 = 2$

$\therefore 3 \tan E = 3 \quad \therefore \tan E = 1 \quad \therefore E = 45^\circ$

7

$\because \tan X = \frac{1}{\sqrt{3}} \qquad \therefore X = 30^\circ$

$\therefore \sin X \tan \left(\frac{3 X}{2}\right) + \cos (2 X)$

$= \sin 30^\circ \tan \left(\frac{3 \times 30^\circ}{2}\right) + \cos (2 \times 30^\circ)$

$= \sin 30^\circ \tan 45^\circ + \cos 60^\circ = \frac{1}{2} \times 1 + \frac{1}{2} = 1$

8

$\because \sin X = \tan 30^\circ \sin 60^\circ \qquad \therefore \sin X = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$

$\therefore \sin X = \frac{1}{2} \qquad \therefore X = 30^\circ$

$\therefore 4 \cos X \sin X = 4 \cos 30^\circ \sin 30^\circ$

$= 4 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \sqrt{3}$

9

$\because \frac{\cos 5 X}{\sin X} = 1 \qquad \therefore \cos 5 X = \sin X$

$\therefore 5 X + X = 90^\circ \qquad \therefore 6 X = 90^\circ$

$\therefore X = \frac{90^\circ}{6} = 15^\circ$

$\therefore \sin 2 X = \sin 2 \times 15^\circ = \sin 30^\circ = \frac{1}{2}$

10

$\because \cos X \tan X + \frac{1}{2} = 1 \qquad \therefore \cancel{\cos X} \times \frac{\sin X}{\cancel{\cos X}} = \frac{1}{2}$

$\therefore \sin X = \frac{1}{2} \qquad \therefore X = 30^\circ$

11

1 $\because \tan 32^\circ = \frac{6}{BC}$

$\therefore BC = 6 \div \tan 32^\circ \simeq 9.6$ cm.

2 $\because \cos 50^\circ = \frac{AB}{8}$

$\therefore AB = 8 \times \cos 50^\circ \simeq 5.14$ cm.

3 $\because \sin 65^\circ = \frac{BC}{12}$

$\therefore BC = 12 \times \sin 65^\circ \simeq 10.88$ cm.

12

1 $\because \sin A = \frac{4}{6} \qquad \therefore m(\angle A) \simeq 41^\circ\, 48'\, 37''$

2 $\because \tan C = \frac{10}{6} \qquad \therefore m(\angle C) \simeq 59^\circ\, 2'\, 10''$

3 $\because \cos C = \frac{5}{7} \qquad \therefore m(\angle C) \simeq 44^\circ\, 24'\, 55''$

13

In Δ ACD :

$\because m(\angle DAC) = 90^\circ$, $m(\angle D) = 30^\circ$

$\therefore AC = \frac{1}{2} CD = \frac{1}{2} \times 8 = 4$ cm.

$\therefore \tan B = \frac{AC}{BC} = \frac{4}{3}$ (First req.)

In Δ ABC :

$\because \tan (\angle BAC) = \frac{3}{4} \qquad \therefore m(\angle BAC) \simeq 36^\circ\, 52'\, 12''$

$\therefore m(\angle BAD) \simeq 90^\circ + 36^\circ\, 52'\, 12''$

$\simeq 126^\circ\, 52'\, 12''$ (Second req.)

14

Draw $\overline{AD} \perp \overline{BC}$ to cut it at D

$\because \overline{AD} \perp \overline{BC}$, AB = AC

$\therefore$ BD = DC = 5 cm.

In Δ ABD : $\cos B = \frac{5}{7}$

$\therefore m(\angle B) \simeq 44^\circ\, 24'\, 55''$ (First req.)

In Δ ABD : $\because (AD)^2 = (AB)^2 - (BD)^2$

$\therefore (AD)^2 = 49 - 25 = 24 \qquad \therefore AD = 2\sqrt{6}$ cm.

$\therefore$ The area of Δ ABC $= \frac{1}{2} \times 10 \times 2\sqrt{6}$

$= 10\sqrt{6}$ cm². (Second req.)

15

Draw $\overline{AD} \perp \overline{BC}$ to cut it at D

$\because \overline{AD} \perp \overline{BC}$, AB = AC

$\therefore$ BD = DC

In Δ ADC : $\cos C = \frac{DC}{AC}$

$\therefore \cos 84^\circ\, 24' = \frac{DC}{12.6}$

$\therefore DC = 12.6 \times \cos 84^\circ\, 24' \simeq 1.23$ cm.

$\therefore BC = 2 \times 1.23 = 2.46 \simeq 2.5$ cm. (The req.)

16

$\because \Delta$ ABC is a right-angled triangle at B

$\therefore m(\angle A) + m(\angle C) = 90^\circ$

, $\because m(\angle A) = 2\, m(\angle C)$

$\therefore 2\, m(\angle C) + m(\angle C) = 90^\circ$

$\therefore 3\, m(\angle C) = 90^\circ \qquad \therefore m(\angle C) = 30^\circ$

$\therefore m(\angle A) = 60^\circ$

$\therefore \cos^2 A + \tan^2 C = \cos^2 60^\circ + \tan^2 30^\circ$

$= \frac{1}{4} + \frac{1}{3} = \frac{3+4}{12} = \frac{7}{12}$

17

$\because$ ABCD is a rectangle $\qquad \therefore m(\angle B) = 90^\circ$

In Δ ABC : $\therefore \sin (\angle ACB) = \frac{15}{25}$

$\therefore m(\angle ACB) \simeq 36^\circ\, 52'\, 12''$ (First req.)

In Δ ABC :

$\because (BC)^2 = (AC)^2 - (AB)^2$

$\therefore (BC)^2 = 625 - 225 = 400$ $\quad \therefore BC = 20$ cm.

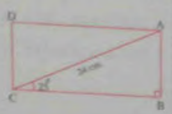
$\therefore$ The area of the rectangle $= 15 \times 20 = 300\ cm^2$.

(Second req.)

18

$\because$ ABCD is a rectangle

$\therefore m(\angle B) = 90°$



In Δ ABC :

$\because \cos(\angle ACB) = \frac{BC}{AC}$

$\therefore \cos 25° = \frac{BC}{24}$

$\therefore BC = 24 \cos 25° \simeq 21.8$ cm. (The req.)

19

$BC = 96 \div 8 = 12$ cm.

$\because AD = BC$ $\quad \therefore AD = 12$ cm. (First req.)

$\because BC = 12$ cm. , $BE = \frac{1}{4} BC$ $\therefore BE = 3$ cm.

In Δ ABE :

$\because m(\angle AEB) = 90°$ $\quad \therefore \tan B = \frac{AE}{BE} = \frac{8}{3}$

$\therefore m(\angle B) \simeq 69° \ 26' \ 38''$ (Second req.)

In Δ AEB :

$\because \sin B = \frac{AE}{AB}$ $\quad \therefore \sin 69° \ 26' \ 38'' = \frac{8}{AB}$

$\therefore AB = \frac{8}{\sin 69° \ 26' \ 38''} \simeq 8.5$ cm. (Third req.)

Another solution :

In Δ AEB : $\because \cos B = \frac{BE}{AB}$

$\therefore \cos 69° \ 26' \ 38'' = \frac{3}{AB}$

$\therefore AB = \frac{3}{\cos 69° \ 26' \ 38''} \simeq 8.5$ cm. (Third req.)

A third solution :

$\because \Delta$ ABE is right-angled at E

$\therefore (AB)^2 = (AE)^2 + (BE)^2$

$= 8^2 + 3^2 = 64 + 9 = 73$

$\therefore AB = \sqrt{73} \simeq 8.5$ cm. (Third req.)

20

Draw $\overline{AF} \perp \overline{BC}$, $\overline{DE} \perp \overline{BC}$

$\because \overline{AD} \parallel \overline{BC}$, $\overline{AF} \perp \overline{BC}$,

$\overline{DE} \perp \overline{BC}$

$\therefore$ AFED is a rectangle $\quad \therefore FE = 5$ cm.

$\therefore BF + EC = 6$ cm. $\quad \therefore BF = 3$ cm.

In Δ ABF :

$\because \cos B = \frac{BF}{AB}$ $\quad \therefore \cos B = \frac{3}{5}$

$\therefore m(\angle B) \simeq 53° \ 7' \ 48''$

$\therefore m(\angle A) = 180° - 53° \ 7' \ 48''$

$= 126° \ 52' \ 12''$ (First req.)

In Δ ABF :

$\because (AF)^2 = (AB)^2 - (BF)^2$

$\therefore (AF)^2 = (5)^2 - (3)^2 = 16$ $\quad \therefore AF = 4$ cm.

$\therefore$ The area of the trapezium $= \frac{1}{2}(5 + 11) \times 4 = 32\ cm^2$

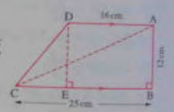
(Second req.)

21

Draw $\overline{DE} \perp \overline{BC}$

, $\because \overline{AD} \parallel \overline{BC}$, $\overline{AB} \perp \overline{BE}$

, $\overline{DE} \perp \overline{BE}$



$\therefore$ ABED is a rectangle.

$\therefore BE = 16$ cm.

, $EC = 25 - 16 = 9$ cm. $\quad$, $DE = 12$ cm.

In Δ DEC : $\because m(\angle DEC) = 90°$

$\therefore (DC)^2 = (12)^2 + (9)^2 = 225$

$\therefore DC = 15$ cm. (First req.)

, $\because \tan C = \frac{12}{9}$

$\therefore m(\angle C) \simeq 53° \ 7' \ 48''$ (Second req.)

, $\sin(\angle DCB) - \tan(\angle ACB) = \frac{12}{15} - \frac{12}{25} = \frac{8}{25}$

(Third req.)

22

In Δ ABC :

$\because m(\angle C) = 90°$

$\therefore \sin B = \frac{AC}{AB}$

$\therefore \sin 60° = \frac{AC}{6}$

$\therefore AC = 6 \sin 60° = 3\sqrt{3}$ m.

(The req.)

23

In Δ ABC :

$\because m(\angle B) = 90°$

$\therefore \sin C = \frac{AB}{AC}$ $\quad \therefore \sin 22° = \frac{AB}{500}$

$\therefore AB = 500 \sin 22° \simeq 187$ m. (The req.)

24

In Δ ABC :

$\because m(\angle B) = 90°$

$\therefore \cos A = \frac{AB}{AC}$

$\therefore \cos 60° = \frac{4}{AC}$

$\therefore AC = \frac{4}{\cos 60°} = 8$ m.

$\therefore (BC)^2 = 8^2 - 4^2 = 48$ $\quad \therefore BC = 4\sqrt{3}$ m.

$\therefore$ The height of the tree $= 8 + 4\sqrt{3} \simeq 15$ m.

25

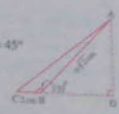
1 b $\quad$ 2 d $\quad$ 3 c

26

Draw $\overline{AD} \perp \overleftrightarrow{BC}$

$\therefore m(\angle ABD) = 180° - 135° = 45°$

, $\because \tan(\angle ABD) = \frac{AD}{DB}$



$\therefore \tan 45° = \frac{AD}{DB} = 1$

$\therefore AD = DB = X$ cm.

$\therefore$ In Δ ADB which is right-angled at D :

$(AB)^2 = X^2 + X^2$ $\quad \therefore 72 = 2X^2$

$\therefore X^2 = 36$ $\quad \therefore X = 6$ cm.

$\therefore \tan C = \frac{AD}{DC} = \frac{6}{8} = \frac{3}{4}$

Answers of unit five

Answers of Exercise 3

1

(1) $AB = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{9+16} = 5$ length unit.

(2) $AB = \sqrt{(5-2)^2 + (-5+1)^2} = \sqrt{9+16} = 5$ length unit.

(3) $AB = \sqrt{(3+2)^2 + (-5-7)^2}$
$= \sqrt{25+144} = 13$ length unit.

(4) $AB = \sqrt{(3+2)^2 + (0-5)^2}$
$= \sqrt{25+25} = 5\sqrt{2}$ length unit.

(5) $AB = \sqrt{(15-6)^2 + (0-0)^2} = \sqrt{9^2} = 9$ length unit.

(6) $AB = \sqrt{(6-0)^2 + (0+8)^2}$
$= \sqrt{36+64} = 10$ length unit.

2

(1) d	(2) c	(3) c
(4) a	(5) d	(6) b
(7) d	(8) c	(9) d
(10) a	(11) c	

3

$AB = \sqrt{(1-3)^2 + (2-1)^2} = \sqrt{4+1} = \sqrt{5}$ length unit.

, $BC = \sqrt{(5-1)^2 + (4-2)^2} = \sqrt{16+4} = \sqrt{20}$
$= 2\sqrt{5}$ length unit.

$\therefore BC = 2\,AB$

4

$AB = \sqrt{(1-4)^2 + (1-3)^2} = \sqrt{9+4} = \sqrt{13}$ length unit

, $BC = \sqrt{(-5-1)^2 + (-3-1)^2} = \sqrt{36+16} = \sqrt{52}$
$= 2\sqrt{13}$ length unit

, $AC = \sqrt{(4+5)^2 + (3+3)^2} = \sqrt{81+36} = \sqrt{117}$
$= 3\sqrt{13}$ length unit

$\therefore AC = AB + BC$

$\therefore$ A , B and C are collinear.

5

$\because CA = \sqrt{(3+2)^2 + (4-2)^2} = \sqrt{25+4} = \sqrt{29}$ length unit

, $CB = \sqrt{(3-1)^2 + (4+1)^2} = \sqrt{4+25} = \sqrt{29}$ length unit

$\therefore CA = CB$

$\therefore$ C lies on the axis of symmetry of $\overline{AB}$

6

(1) $AB = \sqrt{(3-1)^2 + (-2-4)^2}$
$= \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$ length unit.

, $BC = \sqrt{(-3-3)^2 + (16+2)^2}$
$= \sqrt{36+324} = \sqrt{360} = 6\sqrt{10}$ length unit.

, $AC = \sqrt{(-3-1)^2 + (16-4)^2}$
$= \sqrt{16+144} = \sqrt{160} = 4\sqrt{10}$ length unit.

$\therefore BC = AB + AC$

$\therefore$ A , B and C are collinear points.

(2) $AB = \sqrt{(-3-7)^2 + (6-0)^2} = \sqrt{100+36}$
$= 2\sqrt{34}$ length unit.

, $BC = \sqrt{(22+3)^2 + (9-6)^2} = \sqrt{625+9}$
$= \sqrt{634}$ length unit.

, $AC = \sqrt{(22-7)^2 + (9-0)^2} = \sqrt{225+81}$
$= 3\sqrt{34}$ length unit.

$\therefore BC \neq AB + AC$

$\therefore$ A , B and C are non-collinear points

(3) $AB = \sqrt{(3+1)^2 + (-14-4)^2}$
$= \sqrt{16+324} = 2\sqrt{85}$ length unit.

, $BC = \sqrt{(-5-3)^2 + (-6+14)^2}$
$= \sqrt{64+64} = 8\sqrt{2}$ length unit.

, $AC = \sqrt{(-5+1)^2 + (-6-4)^2} = \sqrt{16+100}$
$= 2\sqrt{29}$ length unit

$\therefore AB \neq BC + AC$

$\therefore$ A , B and C are non-collinear points.

7

$\because AB = \sqrt{(-2-3)^2 + (4+1)^2} = \sqrt{25+25} = \sqrt{50}$
$= 5\sqrt{2}$ length unit

, $BC = \sqrt{(3-4)^2 + (-1-5)^2} = \sqrt{1+36}$
$= \sqrt{37}$ length unit

, $AC = \sqrt{(-2-4)^2 + (4-5)^2} = \sqrt{36+1}$
$= \sqrt{37}$ length unit

$\therefore BC = AC$

$\therefore \Delta ABC$ is an isosceles triangle.

8

(1) $AB = \sqrt{(4-2)^2 + (-2-1)^2}$
$= \sqrt{4+9} = \sqrt{13}$ length unit.

, $BC = \sqrt{(7-4)^2 + (5+2)^2} = \sqrt{9+49}$
$= \sqrt{58}$ length unit.

, $AC = \sqrt{(7-2)^2 + (5-1)^2}$
$= \sqrt{25+16} = \sqrt{41}$ length unit.

, $\because (BC)^2 > (AB)^2 + (AC)^2$

$\therefore$ A , B and C are the vertices of an obtuse-angled triangle at A

(2) $AB = \sqrt{(-1-3)^2 + (1-5)^2} = \sqrt{16+16} = \sqrt{32}$ length unit.

, $BC = \sqrt{(5+1)^2 + (-5-1)^2} = \sqrt{36+36}$
$= \sqrt{72}$ length unit.

, $AC = \sqrt{(5-3)^2 + (-5-5)^2} = \sqrt{4+100}$
$= \sqrt{104}$ length unit.

, $\because (AC)^2 = (AB)^2 + (BC)^2$

$\therefore$ A , B and C are the vertices of a right-angled triangle at B

(3) $AB = \sqrt{(3-4)^2 + (-1-4)^2} = \sqrt{1+25} = \sqrt{26}$ length unit.

, $BC = \sqrt{(-2-3)^2 + (4+1)^2}$
$= \sqrt{25+25} = \sqrt{50}$ length unit.

, $AC = \sqrt{(-2-4)^2 + (4-4)^2} = \sqrt{36} = 6$ length units.

, $\because \overline{BC}$ is the longest side ,

$\because (BC)^2 < (AB)^2 + (AC)^2$

$\therefore$ A , B and C are the vertices of an acute-angled triangle.

(4) $AB = \sqrt{(6-0)^2 + (0-0)^2} = \sqrt{36+0} = 6$ length units.

, $BC = \sqrt{(0-6)^2 + (8-0)^2} = \sqrt{36+64} = \sqrt{100}$
$= 10$ length units

, $AC = \sqrt{(0-0)^2 + (8-0)^2} = \sqrt{0+64} = 8$ length units

, $\because (BC)^2 = (AB)^2 + (AC)^2$

$\therefore$ A , B and C are the vertices of a right-angled triangle at A

(5) $AB = \sqrt{(2-1)^2 + (1+1)^2} = \sqrt{1+4} = \sqrt{5}$ length unit.

, $BC = \sqrt{(-3-2)^2 + (-2-1)^2} = \sqrt{25+9}$
$= \sqrt{34}$ length unit.

, $AC = \sqrt{(1+3)^2 + (-1+2)^2} = \sqrt{16+1}$
$= \sqrt{17}$ length unit.

, $\because (BC)^2 > (AB)^2 + (AC)^2$

$\therefore$ A , B and C are the vertices of an obtuse-angled triangle at A

9

$AB = \sqrt{(-1-5)^2 + (7+5)^2} = \sqrt{36+144}$
$= \sqrt{180}$ length unit

, $BC = \sqrt{(15+1)^2 + (15-7)^2} = \sqrt{256+64}$
$= \sqrt{320}$ length unit

and $CA = \sqrt{(5-15)^2 + (-5-15)^2} = \sqrt{100+400}$
$= \sqrt{500}$ length unit

, $\because (CA)^2 = (AB)^2 + (BC)^2$

$\therefore \Delta ABC$ is right angled at B

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times AB \times BC$
$= \frac{1}{2} \times \sqrt{180} \times \sqrt{320}$
$= 120$ square units

10

$\because AB = \sqrt{(7-5)^2 + (2\sqrt{3}-0)^2}$
$= \sqrt{4+12} = 4$ length unit.

, $BC = \sqrt{(3-7)^2 + (2\sqrt{3}-2\sqrt{3})^2}$
$= \sqrt{16} = 4$ length unit.

, $AC = \sqrt{(3-5)^2 + (2\sqrt{3}-0)^2}$
$= \sqrt{4+12} = 4$ length unit.

$\therefore \Delta ABC$ is an equilateral triangle

Let M be the midpoint of the base $\overline{AB}$

$\therefore$ The height $MC = \sqrt{(4)^2 - (2)^2} = \sqrt{16-4} = \sqrt{12}$
$= 2\sqrt{3}$ length unit.

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times AB \times MC$
$= \frac{1}{2} \times 4 \times 2\sqrt{3}$
$= 4\sqrt{3}$ square unit.

11

(1) $AB = \sqrt{(0+1)^2 + (5-1)^2} = \sqrt{1+16} = \sqrt{17}$ length unit.

, $BC = \sqrt{(5-0)^2 + (6-5)^2} = \sqrt{25+1} = \sqrt{26}$ length unit.

, $CD = \sqrt{(4-5)^2 + (2-6)^2} = \sqrt{1+16} = \sqrt{17}$ length unit.

, $DA = \sqrt{(-1-4)^2 + (1-2)^2} = \sqrt{25+1} = \sqrt{26}$ length unit.

$\therefore AB = CD$, $BC = DA$

$\therefore$ ABCD is a parallelogram.

(2) $AB = \sqrt{(5+2)^2 + (-3-4)^2} = \sqrt{49+49}$
$= \sqrt{98} = 7\sqrt{2}$ length unit.

$, BC = \sqrt{(7-5)^2 + (1+3)^2} = \sqrt{4+16}$

$= \sqrt{20} = 2\sqrt{5}$ length unit.

$, CD = \sqrt{(0-7)^2 + (8-1)^2} = \sqrt{49+49}$

$= \sqrt{98} = 7\sqrt{2}$ length unit.

$, DA = \sqrt{(0+2)^2 + (8-4)^2} = \sqrt{4+16}$

$= \sqrt{20} = 2\sqrt{5}$ length unit.

$\therefore AB = CD , BC = DA$

$\therefore$ ABCD is a parallelogram.

12

$\because AB = \sqrt{(0-4)^2 + (1-5)^2} = \sqrt{16+16}$

$= \sqrt{32} = 4\sqrt{2}$ length unit.

$, BC = \sqrt{(4-1)^2 + (5-8)^2} = \sqrt{9+9}$

$= \sqrt{18} = 3\sqrt{2}$ length unit.

$, CD = \sqrt{(1+3)^2 + (8-4)^2} = \sqrt{16+16}$

$= \sqrt{32} = 4\sqrt{2}$ length unit.

$, AD = \sqrt{(0+3)^2 + (1-4)^2} = \sqrt{9+9}$

$= \sqrt{18} = 3\sqrt{2}$ length unit.

$\because AB = CD , BC = AD$

$\therefore$ The figure ABCD is a parallelogram

$, \because AC = \sqrt{(0-1)^2 + (1-8)^2} = \sqrt{1+49}$

$= \sqrt{50} = 5\sqrt{2}$ length unit.

$, BD = \sqrt{(4+3)^2 + (5-4)^2} = \sqrt{49+1}$

$= \sqrt{50} = 5\sqrt{2}$ length unit.

$\therefore AC = BD = 5\sqrt{2}$ length unit.

$\therefore$ The figure ABCD is a rectangle its diagonal length $= 5\sqrt{2}$ length unit.

13

$\because AB = \sqrt{(3-0)^2 + (3-3)^2} = \sqrt{9+0} = 3$ length unit.

$, BC = \sqrt{(0-0)^2 + (3-0)^2} = \sqrt{0+9} = 3$ length unit.

$, CD = \sqrt{(0-3)^2 + (0-0)^2} = \sqrt{9+0} = 3$ length unit

$, DA = \sqrt{(3-3)^2 + (0-3)^2} = \sqrt{0+9} = 3$ length unit.

$\because AB = BC = CD = DA$ $\qquad \therefore$ ABCD is a rhombus

$\because AC = \sqrt{(3-0)^2 + (3-0)^2} = \sqrt{9+9} = \sqrt{18}$

$= 3\sqrt{2}$ length unit.

$, BD = \sqrt{(0-3)^2 + (3-0)^2} = \sqrt{9+9} = \sqrt{18}$

$= 3\sqrt{2}$ length unit.

$\therefore AC = BD$

$\therefore$ The figure ABCD is a square , the length of its diagonal $= 3\sqrt{2}$ length unit ,

its area $= 3 \times 3 = 9$ square unit.

14

$AB = \sqrt{(6-5)^2 + (-2-3)^2} = \sqrt{1+25} = \sqrt{26}$ length unit.

$, BC = \sqrt{(1-6)^2 + (-1+2)^2}$

$= \sqrt{25+1} = \sqrt{26}$ length unit.

$, CD = \sqrt{(0-1)^2 + (4+1)^2} = \sqrt{1+25} = \sqrt{26}$ length unit.

$, DA = \sqrt{(5-0)^2 + (3-4)^2} = \sqrt{25+1} = \sqrt{26}$ length unit.

$\therefore AB = BC = CD = DA$

$\therefore$ The figure ABCD is a rhombus

$, \because AC = \sqrt{(1-5)^2 + (-1-3)^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ length unit.

$, BD = \sqrt{(0-6)^2 + (4+2)^2}$

$= \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$ length unit.

$\therefore$ The area of the rhombus $= \frac{1}{2} AC \times BD$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

$= 24$ square units

15

$AB = \sqrt{(3+2)^2 + (3-5)^2} = \sqrt{25+4} = \sqrt{29}$ length unit.

$, BC = \sqrt{(-4-3)^2 + (2-3)^2} = \sqrt{49+1} = \sqrt{50}$ length unit.

$, CA = \sqrt{(-2+4)^2 + (5-2)^2} = \sqrt{4+9} = \sqrt{13}$ length unit.

$, \because$ BC is the greatest distance

$, BC < AB + CA$

$\therefore A , B , C$ are non-collinear points

$\because AD = \sqrt{(-2+9)^2 + (5-4)^2}$

$= \sqrt{49+1} = \sqrt{50}$ length unit.

$, CD = \sqrt{(-9+4)^2 + (4-2)^2}$

$= \sqrt{25+4} = \sqrt{29}$ length unit

$\therefore AB = CD , BC = AD$

$\therefore$ ABCD is a parallelogram

16

$AB = \sqrt{(-3-2)^2 + (0-4)^2} = \sqrt{25+16} = \sqrt{41}$ length unit.

$, BC = \sqrt{(-7+3)^2 + (5-0)^2} = \sqrt{16+25} = \sqrt{41}$ length unit.

$, CD = \sqrt{(-2+7)^2 + (9-5)^2} = \sqrt{25+16} = \sqrt{41}$ length unit.

$, DA = \sqrt{(2+2)^2 + (4-9)^2} = \sqrt{16+25} = \sqrt{41}$ length unit.

$\therefore$ ABCD is a rhombus

$, \because AC = \sqrt{(-7-2)^2 + (5-4)^2}$

$= \sqrt{81+1} = \sqrt{82}$ length unit.

$, BD = \sqrt{(-2+3)^2 + (9-0)^2} = \sqrt{1+81}$

$= \sqrt{82}$ length unit.

$\therefore AC = BD$ $\qquad \therefore$ The figure ABCD is a square.

17

$\because MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9}$

$= \sqrt{25} = 5$ length units

$, MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$ length units

and $MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$ length units

$\therefore MA = MB = MC$

$\therefore A , B$ and C lie on the circle M which its radius length is 5 length units

$\therefore$ The circumference of the circle $= 2\pi r$

$= 2 \times 3.14 \times 5$

$= 31.4$ length units

18

$\because \sqrt{(X-6)^2 + (5-1)^2} = 2\sqrt{5}$

$\therefore \sqrt{(X-6)^2 + 16} = 2\sqrt{5}$ "squaring the two sides"

$\therefore (X-6)^2 + 16 = 20$ $\qquad \therefore (X-6)^2 = 4$

$\therefore X - 6 = \pm 2$ $\qquad \therefore X - 6 = 2$

$\therefore X = 8$

or $X - 6 = -2$ $\qquad \therefore X = 4$

19

1 $\because \sqrt{(a+2)^2 + (7-3)^2} = 5$ "squaring the two sides"

$\therefore (a+2)^2 + (4)^2 = 25$ $\qquad \therefore (a+2)^2 + 16 = 25$

$\therefore (a+2)^2 = 9$ $\qquad \therefore a + 2 = \pm 3$

$\therefore a + 2 = 3$ $\qquad \therefore a = 1$

or $a + 2 = -3$ $\qquad \therefore a = -5$

2 $\because \sqrt{(3a-1-a)^2 + (-5-7)^2} = 13$ "squaring the two sides"

$\therefore (2a-1)^2 + (-12)^2 = 169$

$\therefore (2a-1)^2 + 144 = 169$

$\therefore (2a-1)^2 = 25$ $\qquad \therefore 2a - 1 = \pm 5$

$\therefore 2a - 1 = 5$ $\qquad \therefore 2a = 6$

$\therefore a = 3$ or $2a - 1 = -5$

$\therefore 2a = -4$ $\qquad \therefore a = -2$

20

$BC = \sqrt{(5-3)^2 + (1-2)^2} = \sqrt{4+1} = \sqrt{5}$ length unit.

$\therefore AB = \sqrt{5}$ length unit.

$\therefore \sqrt{(X-3)^2 + (3-2)^2} = \sqrt{5}$ "squaring the two sides"

$\therefore (X-3)^2 + (1)^2 = 5$ $\qquad \therefore X^2 - 6X + 9 + 1 = 5$

$\therefore X^2 - 6X + 5 = 0$ $\qquad \therefore (X-5)(X-1) = 0$

$\therefore X = 5$ or $X = 1$

21

$\because AB = \sqrt{(-6-9)^2 + (0-0)^2} = \sqrt{225}$

$= 15$ length units.

Let $C = (0 , y)$

$\therefore AC = \sqrt{(0-9)^2 + (y-0)^2} = \sqrt{81 + y^2}$

$, \because AB = AC$

$\therefore 15 = \sqrt{81 + y^2}$ "squaring the two sides"

$\therefore 225 = 81 + y^2$ $\qquad \therefore y^2 = 225 - 81 = 144$

$\therefore y = 12$

or $y = -12$ (refused because the point C lies on the positive part of y-axis)

$\therefore C (0 , 12)$

$\therefore CO = \sqrt{(0-0)^2 + (12-0)^2} = \sqrt{144}$

$= 12$ length units.

22

$\because$ The axis of symmetry of $\overline{CD}$ passes through the point A

$\therefore CA = DA$

$\therefore \sqrt{(6-3)^2 + (m-1)^2} = \sqrt{(6+3)^2 + (m-7)^2}$

"squaring the two sides"

$\therefore 3^2 + (m-1)^2 = 9^2 + (m-7)^2$

$\therefore 9 + m^2 - 2m + 1 = 81 + m^2 - 14m + 49$

$\therefore -2m + 14m = 81 + 49 - 9 - 1$

$\therefore 12m = 120$ $\qquad \therefore m = \frac{120}{12} = 10$

23

$\because A \in$ the X-axis

Let $A = (X , 0)$

$\because AO = AB$ $\quad \therefore X = \sqrt{(X+9)^2 + (0-15)^2}$

$\therefore X = \sqrt{X^2 + 18X + 81 + 225}$

$\therefore X^2 = X^2 + 18X + 306$ $\quad \therefore 18X = -306$

$\therefore X = -17$ $\quad \therefore AB = 17$ length units

24

$\because f(X) = X$

$\because AO = 4$ length units $\quad \therefore C$ is (X , X)

$\because OB = 6$ length units $\quad \therefore A(4 , 0)$

$\because AC = BC$ $\quad \therefore B(0 , 6)$

$\therefore \sqrt{(X-4)^2 + (X-0)^2} = \sqrt{(X-0)^2 + (X-6)^2}$

"squaring the two sides"

$\therefore X^2 - 8X + 16 + X^2 = X^2 + X^2 - 12X + 36$

$\therefore 4X = 20$ $\quad \therefore X = 5$ $\quad \therefore C(5 , 5)$

25

The point that represents Basem's house is $(1 , 9)$

The point that represents Eslam's house is $(3 , 10)$

The point that represents the school is $(10 , 2)$

The point that represents the railway station is $(4 , 0)$

1. The distance between Basem's house and the school $= \sqrt{(10-1)^2 + (2-9)^2}$
   $= \sqrt{81 + 49} = \sqrt{130}$ km.
   , the distance between Eslam's house and the school $= \sqrt{(10-3)^2 + (2-10)^2}$
   $= \sqrt{49 + 64} = \sqrt{113}$ km.
   $\therefore$ Eslam's house is nearer to the school.
2. The distance between Basem's house and the school $= \sqrt{130}$ km.
   , the distance between Basem's house and railway station $= \sqrt{(1-4)^2 + (9-0)^2}$
   $= \sqrt{9 + 81} = \sqrt{90}$ km.
   , the distance between the school and railway station $= \sqrt{(10-4)^2 + (2-0)^2} = \sqrt{36 + 4} = \sqrt{40}$ km.
   $\because$ The square of the distance between Basem's house and the school equals the sum of the squares of the distance between Basem's house and railway and the distance between the school and railway.
   $\therefore$ Basem's house , the school and railway station make a right-angled triangle at the railway station.
   $\therefore$ The way (school - railway station) is perpendicular to the way (Basem's house - railway station)

26

$AB = \sqrt{(X-4)^2 + (2+2)^2} = \sqrt{(X-4)^2 + 16}$ length unit.

$, BC = \sqrt{(X-3)^2 + (2-5)^2} = \sqrt{(X-3)^2 + 9}$ length unit.

$, CA = \sqrt{(4-3)^2 + (-2-5)^2} = \sqrt{1 + 49} = \sqrt{50}$ length unit.

$\because \Delta ABC$ is right-angled at B

$\therefore (CA)^2 = (AB)^2 + (BC)^2$

$\therefore (X-4)^2 + 16 + (X-3)^2 + 9 = 50$

$\therefore X^2 - 8X + 16 + 16 + X^2 - 6X + 9 + 9 = 50$

$\therefore 2X^2 - 14X = 0$ "dividing by 2"

$\therefore X^2 - 7X = 0$ $\quad \therefore X(X-7) = 0$

$\therefore X = 0$ or $X = 7$

At $X = 0$

$\therefore AB = \sqrt{32} = 4\sqrt{2}$ length unit

$, BC = \sqrt{18} = 3\sqrt{2}$ length unit

$\therefore$ The area of $\Delta ABC = \frac{1}{2} AB \times BC$

$= \frac{1}{2} \times 4\sqrt{2} \times 3\sqrt{2}$

$= 12$ square units.

At $X = 7$ $\quad \therefore AB = 5$ length units.

$, BC = 5$ length units.

$\therefore$ The area of $\Delta ABC = \frac{1}{2} AB \times BC = \frac{1}{2} \times 5 \times 5$

$= 12.5$ square units.

Answers of Exercise 4

1

1. The midpoint of $\overline{AB} = \left(\frac{3+7}{2} , \frac{5+1}{2}\right) = (5 , 3)$
2. The midpoint of $\overline{AB} = \left(\frac{5-1}{2} , \frac{-3+3}{2}\right) = (2 , 0)$
3. The midpoint of $\overline{AB} = \left(\frac{-5+5}{2} , \frac{4-4}{2}\right) = (0 , 0)$
4. The midpoint of $\overline{AB} = \left(\frac{0+8}{2} , \frac{4+0}{2}\right) = (4 , 2)$
5. The midpoint of $\overline{AB} = \left(\frac{2+6}{2} , \frac{4+0}{2}\right) = (4 , 2)$
6. The midpoint of $\overline{AB} = \left(\frac{7-1}{2} , \frac{-6+0}{2}\right) = (3 , -3)$

2

$\because (X , 0) = \left(\frac{1+2}{2} , \frac{-5+5}{2}\right) = \left(\frac{3}{2} , 0\right)$

$\therefore X = \frac{3}{2}$

3

$\because (5 , 3) = \left(\frac{15-5}{2} , \frac{y-2}{2}\right)$ $\quad \therefore \frac{y-2}{2} = 3$

$\therefore y - 2 = 6$ $\quad \therefore y = 8$

4

Let $B(X , y)$

$\therefore (6 , -4) = \left(\frac{5+X}{2} , \frac{-3+y}{2}\right)$

$\therefore \frac{5+X}{2} = 6$ $\quad \therefore 5 + X = 12$

$\therefore X = 7$ $\quad , \frac{-3+y}{2} = -4$

$\therefore -3 + y = -8$

$\therefore y = -5$ $\quad \therefore B(7 , -5)$

5

1. $\because (X , y) = \left(\frac{1+3}{2} , \frac{5+7}{2}\right)$
   $\therefore X = \frac{1+3}{2} = 2 , y = \frac{5+7}{2} = 6$
2. $\because (X , -3) = \left(\frac{-3+9}{2} , \frac{y+11}{2}\right)$
   $\therefore X = \frac{-3+9}{2} = 3 , \frac{y+11}{2} = -3$
   $\therefore y + 11 = -6$ $\quad \therefore y = -17$
3. $\because (-3 , y) = \left(\frac{X+9}{2} , \frac{-6-11}{2}\right)$
   $\therefore \frac{X+9}{2} = -3$ $\quad \therefore X + 9 = -6$
   $\therefore X = -15$ $\quad , y = \frac{-6-11}{2} = -8.5$
4. $\because (4 , 6) = \left(\frac{X+6}{2} , \frac{3+y}{2}\right)$
   $\therefore \frac{X+6}{2} = 4$ $\quad \therefore X + 6 = 8$ $\quad \therefore X = 2$
   $, \frac{3+y}{2} = 6$ $\quad \therefore 3 + y = 12$ $\quad \therefore y = 9$

6

1 d	2 d	3 a
4 c	5 c	6 b
7 a	8 c	9 a

7

$\because AB = BC$ $\quad \therefore B$ is the midpoint of $\overline{AC}$

$\therefore B = \left(\frac{1+5}{2} , \frac{3+1}{2}\right) = (3 , 2)$

$, \because BC = CD$ $\quad \therefore C$ is the midpoint of $\overline{BD}$

Let $D(X , y)$

$\therefore (5 , 1) = \left(\frac{3+X}{2} , \frac{2+y}{2}\right)$

$\therefore \frac{3+X}{2} = 5$ $\quad \therefore 3 + X = 10$ $\quad \therefore X = 7$

$, \frac{2+y}{2} = 1$ $\quad \therefore 2 + y = 2$ $\quad \therefore y = 0$

$\therefore D(7 , 0)$

8

Let D be the midpoint of $\overline{AB}$

$\therefore D = \left(\frac{1+9}{2} , \frac{-6+2}{2}\right) = (5 , -2)$

Let E be the midpoint of $\overline{AD}$

$\therefore E = \left(\frac{1+5}{2} , \frac{-6-2}{2}\right) = (3 , -4)$

Let X be the midpoint of $\overline{BD}$

$\therefore X = \left(\frac{9+5}{2} , \frac{2-2}{2}\right) = (7 , 0)$

9

$\because (0 , 0) = \left(\frac{X-2-2}{2} , \frac{y+2}{2}\right)$ $\quad \therefore \frac{X-4}{2} = 0$

$\therefore X - 4 = 0$ $\quad \therefore X = 4$

$, \frac{y+2}{2} = 0$ $\quad \therefore y + 2 = 0$

$\therefore y = -2$ $\quad \therefore (X , y) = (4 , -2)$

10

$\because (2a - 3 , a - b) = \left(\frac{7+3}{2} , \frac{-1+7}{2}\right) = (5 , 3)$

$\therefore 2a - 3 = 5$ $\quad \therefore 2a = 8$ $\quad \therefore a = 4$

$, a - b = 3 \therefore 4 - b = 3$ $\quad \therefore b = 1$

11

Let : $A(X , y)$

$\therefore (5 , 7) = \left(\frac{X+8}{2} , \frac{y+11}{2}\right)$

$\therefore \frac{X+8}{2} = 5$ $\quad \therefore X + 8 = 10$

$\therefore X = 2 , \frac{y+11}{2} = 7$ $\quad \therefore y + 11 = 14$

$\therefore y = 3$ $\quad \therefore A(2 , 3)$

$\therefore r = MA = \sqrt{(5-2)^2 + (7-3)^2}$

$= \sqrt{9 + 16} = 5$ length unit.

$\therefore$ The circumference of the circle $= 2\pi r$

$= 2 \times 3.14 \times 5 = 31.4$ length unit

12

$\because$ D is the midpoint of $\overline{AB}$

$\therefore D = \left(\frac{1+5}{2}, \frac{3+1}{2}\right) = (3, 2)$

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore E = \left(\frac{1+3}{2}, \frac{3+7}{2}\right) = (2, 5)$

$\therefore DE = \sqrt{(3-2)^2 + (2-5)^2} = \sqrt{10}$ length unit (1)

, $\because BC = \sqrt{(5-3)^2 + (1-7)^2}$

$= \sqrt{40} = 2\sqrt{10}$ length unit (2)

From (1) and (2)

$\therefore DE = \frac{1}{2} BC$ (Q.E.D.)

13

Let : A (X , 0) , B (0 , y)

$\therefore (3, 4) = \left(\frac{X+0}{2}, \frac{y+0}{2}\right)$

$\therefore \frac{X}{2} = 3$ $\therefore X = 6$

$\therefore A(6, 0)$ $\therefore OA = 6$ length unit

, $\frac{y}{2} = 4$ $\therefore y = 8$

$\therefore B(0, 8)$ $\therefore OB = 8$ length unit

, $AB = \sqrt{(6-0)^2 + (0-8)^2} = \sqrt{36+64} = \sqrt{100}$

$= 10$ length unit

$\therefore$ The perimeter of Δ OAB $= 6 + 8 + 10$

$= 24$ length unit

14

Let $D(X_1, 0)$ and $B(0, y_1)$

$\because$ D is the midpoint of $\overline{AB}$

$\therefore (X_1, 0) = \left(\frac{2+0}{2}, \frac{4+y_1}{2}\right)$

$\therefore X_1 = \frac{2}{2} = 1$, $\frac{4+y_1}{2} = 0$

$\therefore 4 + y_1 = 0$ $\therefore y_1 = -4$ $\therefore B(0, -4)$

Let $E(0, y_2)$ and $C(X_2, 0)$

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore (0, y_2) = \left(\frac{2+X_2}{2}, \frac{4+0}{2}\right)$

$\therefore \frac{2+X_2}{2} = 0$ $\therefore 2 + X_2 = 0$ $\therefore X_2 = -2$

, $y_2 = 2$ $\therefore C(-2, 0)$

$\therefore BC = \sqrt{(-2-0)^2 + (0+4)^2} = \sqrt{4+16} = \sqrt{20}$

$= 2\sqrt{5}$ length units.

In Δ ABC :

$\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

$\therefore DE = \frac{1}{2} BC = \frac{1}{2} \times 2\sqrt{5} = \sqrt{5}$ length unit

15

$\because \overline{AD}$ is a median in Δ ABC

$\therefore$ D is the midpoint of $\overline{BC}$

$\therefore D = \left(\frac{3-3}{2}, \frac{2+6}{2}\right) = (0, 4)$

, $\because$ M is the midpoint of $\overline{AD}$ and let A (X , y)

$\therefore (0, 6) = \left(\frac{X+0}{2}, \frac{y+4}{2}\right)$

$\therefore \frac{X}{2} = 0$ $\therefore X = 0$, $\frac{y+4}{2} = 6$

$\therefore y + 4 = 12$ $\therefore y = 8$ $\therefore A(0, 8)$

16

$\because$ The midpoint of $\overline{AC} = \left(\frac{-1+6}{2}, \frac{-1+0}{2}\right)$

$= \left(\frac{5}{2}, -\frac{1}{2}\right)$

, the midpoint of $\overline{BD} = \left(\frac{2+3}{2}, \frac{3-4}{2}\right)$

$= \left(\frac{5}{2}, -\frac{1}{2}\right)$

$\therefore$ The midpoint of $\overline{AC}$ is the same midpoint of $\overline{BD}$

$\therefore \overline{AC}$ and $\overline{BD}$ bisect each other.

17

$\because$ The midpoint of $\overline{AC} = \left(\frac{3+0}{2}, \frac{-2-7}{2}\right) = \left(\frac{3}{2}, \frac{-9}{2}\right)$

, the midpoint of $\overline{BD} = \left(\frac{-5+8}{2}, \frac{0-9}{2}\right) = \left(\frac{3}{2}, \frac{-9}{2}\right)$

$\therefore$ The midpoint of $\overline{AC}$ is the same midpoint of $\overline{BD}$

$\therefore$ The two diagonals bisect each other

$\therefore$ The points A , B , C and D are the vertices of a parallelogram.

18

1 Let E be the point of intersection of the two diagonals

$\therefore E = \left(\frac{3-1}{2}, \frac{2-2}{2}\right) = (1, 0)$

2 $AC = \sqrt{(-1-3)^2 + (-2-2)^2} = \sqrt{16+16} = \sqrt{32}$

$= 4\sqrt{2}$ length units

, $BD = \sqrt{(-2-4)^2 + (3+3)^2} = \sqrt{36+36} = \sqrt{72}$

$= 6\sqrt{2}$ length units

$\therefore$ The area of the rhombus $= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

$= 24$ square unit

19

$\because$ The midpoint of $\overline{AC} = \left(\frac{3+0}{2}, \frac{2-3}{2}\right) = \left(1\frac{1}{2}, -\frac{1}{2}\right)$

$\therefore$ The point of intersection of the two diagonals is $\left(1\frac{1}{2}, -\frac{1}{2}\right)$ (First req.)

and let D (X , y)

$\because$ The midpoint of $\overline{AC}$ = the midpoint of $\overline{BD}$

$\therefore \left(1\frac{1}{2}, -\frac{1}{2}\right) = \left(\frac{X+4}{2}, \frac{y-5}{2}\right)$ $\therefore \frac{X+4}{2} = 1\frac{1}{2}$

$\therefore X + 4 = 3$ $\therefore X = -1$

, $\frac{y-5}{2} = -\frac{1}{2}$ $\therefore y - 5 = -1$ $\therefore y = 4$

$\therefore D(-1, 4)$ (Second req.)

20

$AB = \sqrt{(2-6)^2 + (-4-0)^2} = \sqrt{16+16}$

$= \sqrt{32} = 4\sqrt{2}$ length unit

, $BC = \sqrt{(-4-2)^2 + (2+4)^2} = \sqrt{36+36} = \sqrt{72}$

$= 6\sqrt{2}$ length unit

, $CA = \sqrt{(6+4)^2 + (0-2)^2} = \sqrt{100+4} = \sqrt{104}$

$= 2\sqrt{26}$ length unit

$\because (AB)^2 + (BC)^2 = (4\sqrt{2})^2 + (6\sqrt{2})^2$

$= 32 + 72 = 104 = (CA)^2$

$\therefore \Delta$ ABC is right-angled at B (First req.)

Let E be the midpoint of $\overline{AC}$

$\therefore E = \left(\frac{6-4}{2}, \frac{0+2}{2}\right) = (1, 1)$

$\because$ In the rectangle the two diagonals bisect each other

$\therefore$ E is the midpoint of $\overline{BD}$

Let D (X , y)

$\therefore (1, 1) = \left(\frac{X+2}{2}, \frac{y-4}{2}\right)$ $\therefore \frac{X+2}{2} = 1$

$\therefore X + 2 = 2$ $\therefore X = 0$

, $\frac{y-4}{2} = 1$ $\therefore y - 4 = 2$

$\therefore y = 6$ $\therefore D(0, 6)$ (Second req.)

21

$AB = \sqrt{(3-5)^2 + (-2-3)^2} = \sqrt{4+25}$

$= \sqrt{29}$ length unit

, $BC = \sqrt{(-2-3)^2 + (-4+2)^2} = \sqrt{25+4}$

$= \sqrt{29}$ length unit

, $CA = \sqrt{(5+2)^2 + (3+4)^2} = \sqrt{49+49}$

$= \sqrt{98}$ length unit

, $\because (AB)^2 + (BC)^2 = 29 + 29 = 58$, $(AC)^2 = 98$

$\therefore (AC)^2 > (AB)^2 + (BC)^2$

$\therefore \Delta$ ABC is obtuse-angled at B (First req.)

Let E be the midpoint of $\overline{AC}$

$\therefore E = \left(\frac{5-2}{2}, \frac{3-4}{2}\right) = (1.5, -0.5)$

$\because$ In the rhombus the two diagonals bisect each other

$\therefore$ E is the midpoint of $\overline{BD}$

Let D (X , y)

$\therefore (1.5, -0.5) = \left(\frac{X+3}{2}, \frac{y-2}{2}\right)$

$\therefore \frac{X+3}{2} = 1.5$ $\therefore X + 3 = 3$ $\therefore X = 0$

, $\frac{y-2}{2} = -0.5$ $\therefore y - 2 = -1$

$\therefore y = 1$ $\therefore D(0, 1)$

, $\because AC = \sqrt{98} = 7\sqrt{2}$

, $BD = \sqrt{(3-0)^2 + (-2-1)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$\therefore$ The area of the rhombus ABCD

$= \frac{1}{2} \times 7\sqrt{2} \times 3\sqrt{2} = 21$ square unit (Second req.)

22

$AB = \sqrt{(3+3)^2 + (4-0)^2} = \sqrt{36+16} = \sqrt{52}$

$= 2\sqrt{13}$ length unit

, $BC = \sqrt{(1-3)^2 + (-6-4)^2} = \sqrt{4+100} = \sqrt{104}$

$= 2\sqrt{26}$ length unit

, $CA = \sqrt{(-3-1)^2 + (0+6)^2} = \sqrt{16+36} = \sqrt{52}$

$= 2\sqrt{13}$ length unit

$\therefore AB = AC$

$\therefore \Delta$ ABC is an isosceles triangle and its vertex is A (First req.)

Let D be the midpoint of $\overline{BC}$ (the base of Δ ABC)

$\therefore D = \left(\frac{3+1}{2}, \frac{4-6}{2}\right) = (2, -1)$

$\therefore AD = \sqrt{(2+3)^2 + (-1-0)^2} = \sqrt{25+1} = \sqrt{26}$ length unit

$\therefore$ The length of the drawn line segment perpendicular to $\overline{BC}$ from A $= \sqrt{26}$ length unit (Second req.)

23

$\because AB = \sqrt{(3-1)^2 + (1-1)^2} = \sqrt{4+0} = 2$ length units

, $BC = \sqrt{(1-3)^2 + (3-1)^2} = \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}$ length units

, $CA = \sqrt{(1-1)^2 + (1-3)^2} = \sqrt{0+4} = 2$ length units

$\therefore AB = CA$

$\therefore \Delta ABC$ is an isosceles triangle and its vertex is A (First req.)

Let the point E be the midpoint of $\overline{BC}$

$\therefore E = \left(\frac{3+1}{2}, \frac{1+3}{2}\right) = (2, 2)$

$\therefore AE = \sqrt{(2-1)^2 + (2-1)^2} = \sqrt{1+1} = \sqrt{2}$ length unit

, $\because \Delta ABC$ is an isosceles triangle and E is the midpoint of $\overline{BC}$

$\therefore \overline{AE} \perp \overline{BC}$

$\therefore$ The area of $\Delta ABC = \frac{1}{2} BC \times AE$

$= \frac{1}{2} \times 2\sqrt{2} \times \sqrt{2}$

$= 2$ square units. (Second req.)

24

$\because$ In the parallelogram the two diagonals bisect each other

$\therefore$ Let M be the point of intersection of the two diagonals

$\therefore M = \left(\frac{3-4}{2}, \frac{4-3}{2}\right) = \left(-\frac{1}{2}, \frac{1}{2}\right)$

Let $D(x_1, y_1)$

$\therefore \left(-\frac{1}{2}, \frac{1}{2}\right) = \left(\frac{x_1+2}{2}, \frac{y_1-1}{2}\right)$

$\therefore \frac{x_1+2}{2} = -\frac{1}{2}$ $\qquad \therefore x_1 + 2 = -1$

$\therefore x_1 = -3$ $\qquad , \frac{y_1-1}{2} = \frac{1}{2}$

$\therefore y_1 - 1 = 1$ $\qquad \therefore y_1 = 2$

$\therefore D = (-3, 2)$

$\because AE = 2AD$ $\qquad \therefore$ D is the midpoint of $\overline{AE}$

Let $E(x_2, y_2)$

$\therefore (-3, 2) = \left(\frac{x_2+3}{2}, \frac{y_2+4}{2}\right)$

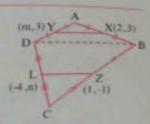
$\therefore \frac{x_2+3}{2} = -3$ $\qquad \therefore x_2 + 3 = -6$

$\therefore x_2 = -9$ $\qquad , \frac{y_2+4}{2} = 2$

$\therefore y_2 + 4 = 4$ $\qquad \therefore y_2 = 0$

$\therefore E = (-9, 0)$

25



In ΔABD:

$\because$ X, Y are the midpoints of $\overline{AB}$, $\overline{AD}$

$\therefore \overline{XY} \parallel \overline{BD}$, $XY = \frac{1}{2} BD$ (1)

, similarly in ΔCBD:

$\overline{ZL} \parallel \overline{BD}$, $ZL = \frac{1}{2} BD$ (2)

From (1), (2): $\therefore \overline{XY} \parallel \overline{ZL}$, $XY = ZL$

$\therefore$ The figure XYLZ is a parallelogram

$\therefore$ The midpoint of $\overline{XL}$ is the same midpoint of $\overline{ZY}$

$\therefore \left(\frac{2-4}{2}, \frac{3+n}{2}\right) = \left(\frac{m+1}{2}, \frac{3-1}{2}\right)$

$\therefore \frac{2-4}{2} = \frac{m+1}{2}$ $\qquad \therefore m = -3$

, $\frac{3+n}{2} = \frac{3-1}{2}$ $\qquad \therefore n = -1$

$\therefore m + n = -3 + (-1) = -4$

26

Let $E(x, y) \in \overleftrightarrow{AD}$ such that ABCE is a parallelogram

$\therefore$ The midpoint of $\overline{AC}$ = the midpoint of $\overline{BE}$

$\therefore \left(\frac{6-2}{2}, \frac{4-4}{2}\right) = \left(\frac{4+x}{2}, \frac{-2+y}{2}\right)$

$\therefore \frac{4+x}{2} = 2$ $\qquad \therefore x = 0$, $\frac{-2+y}{2} = 0$

$\therefore y = 2$ $\qquad \therefore E(0, 2)$

$\because AE = BC$ (properties of the parallelogram)

, $BC = 2AD$ $\qquad \therefore AE = 2AD$

$\therefore$ D is the midpoint of $\overline{AE}$

$\therefore D = \left(\frac{6+0}{2}, \frac{4+2}{2}\right) = (3, 3)$

Answers of Exercise 5

1

1 b	2 a	3 b	4 d
5 a	6 b	7 c	8 b
9 c	10 a	11 b	12 d
13 c	14 d	15 a	16 c
17 c	18 b	19 a	20 c
21 b	22 a	23 d	24 d

2

1 $\frac{1}{\sqrt{3}}$ $\qquad$ 2 $-\frac{1}{\sqrt{3}}$ $\qquad$ 3 $-\sqrt{3}$

3

1 zero	2 $\frac{1}{\sqrt{3}}$	3 1	4 1.5399
5 $\sqrt{3}$	6 undefined	7 17.3432	8 −1

4

1 $16° 41' 57''$ $\qquad$ 2 $20° 10' 6''$

3 $45° 41' 46''$ $\qquad$ 4 $38° 39' 35''$

5

$\because m_1 = \frac{6-2}{5-4} = 4$, $m_2 = \frac{1-5}{-1-0} = 4$ $\qquad \therefore m_1 = m_2$

$\therefore$ The two straight lines are parallel

6

$\because$ The slope of $\overleftrightarrow{AC} = \frac{-2-4}{-3+3} = \frac{-6}{\text{zero}}$

$\therefore$ The straight line $\overleftrightarrow{AC} \parallel$ y-axis ,

The slope of $\overleftrightarrow{BD} = \frac{2-2}{-3-1} = \text{zero}$

$\therefore$ The straight line $\overleftrightarrow{BD} \parallel$ x-axis

$\therefore \overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

7

$\because m_1 = \frac{3+1}{6-2} = 1$, $m_2 = \tan 45° = 1$

$\therefore m_1 = m_2$

$\therefore$ The two straight lines are parallel.

8

$\because m_1 = \frac{2\sqrt{3} - 3\sqrt{3}}{5-4} = -\sqrt{3}$, $m_2 = \tan 30° = \frac{1}{\sqrt{3}}$

$\therefore m_1 \times m_2 = -1$

$\therefore$ The two straight lines are perpendicular.

9

$\because$ The slope of $\overleftrightarrow{AD} = \frac{1-5}{2-1} = -4$

, the slope of $\overleftrightarrow{BC} = \frac{7-3}{4-(x-1)} = \frac{4}{5-x}$

$\because$ The slope of $\overleftrightarrow{AD}$ = the slope of $\overleftrightarrow{BC}$

$\therefore -4 = \frac{4}{5-x}$ $\qquad \therefore 5 - x = -1$ $\qquad \therefore x = 6$

10

$\because \Delta XYZ$ is right-angled at Y

$\therefore \overline{XY} \perp \overline{YZ}$, the slope of $\overleftrightarrow{XY} = \frac{5-2}{3-4} = -3$

$\therefore$ The slope of $\overleftrightarrow{YZ} = \frac{1}{3}$

, $\because$ the slope of $\overleftrightarrow{YZ} = \frac{a-2}{-5-4} = \frac{a-2}{-9} = \frac{1}{3}$

$\therefore a - 2 = -3$ $\qquad \therefore a = -1$

11

$\because m = \frac{7-5}{x-3}$ $\qquad \because$ m is undefined

$\therefore x - 3 = 0$ $\qquad \therefore x = 3$

12

$\because m = \frac{y-2}{-5-4}$ $\qquad$, $\because m = \text{zero}$

$\therefore y - 2 = 0$ $\qquad \therefore y = 2$

13

1 $\because m_1 = \frac{k-1}{2-3} = \frac{k-1}{-1}$, $m_2 = \tan 45° = 1$

, $\because L_1 \parallel L_2$ $\qquad \therefore m_1 = m_2$

$\therefore \frac{k-1}{-1} = 1$ $\qquad \therefore k - 1 = -1$

$\therefore k = \text{zero}$

2 $\because m_1 = \frac{k-1}{-1}$, $m_2 = 1$

, $\because L_1 \perp L_2$ $\qquad \therefore m_1 m_2 = -1$

$\therefore \frac{k-1}{-1} \times 1 = -1$ $\qquad \therefore k - 1 = 1$ $\qquad \therefore k = 2$

14

Let the measure of the positive angle be θ

$\because m = \frac{-5-3}{2-4} = 4$ $\qquad \therefore \tan\theta = 4$

$\therefore \theta = 75° 57' 50''$

15

Let the measure of the positive angle be θ

$\because m = \frac{-2-0}{2-0} = -1$

$\therefore \tan(\text{supplementary of } \theta) = 1$

$\therefore$ supplementary of $\theta = 45°$

$\therefore \theta = 180° - 45° = 135°$

16

Let the measure of the positive angle be θ

$\because$ The slope of the given straight line $= \frac{-1-5}{4+2} = -1$

$\because$ The two straight lines are perpendicular

$\therefore$ The slope of the required straight line = 1

$\therefore \tan\theta = 1$ $\qquad \therefore \theta = 45°$

17

$\because$ The slope of $\overleftrightarrow{AB} = \frac{3-1}{2-1} = \frac{2}{1} = 2$

, the slope of $\overleftrightarrow{BC} = \frac{-1-3}{0-2} = \frac{-4}{-2} = 2$

$\because$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$

$\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$

$\because$ B is a common point between the two straight lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$

$\therefore$ A , B , C are collinear points

18

Let X (0 , 1) , Y (a , 3) and Z (2 , 5)

, $\because$ the three points are collinear

$\therefore$ The slope of $\overleftrightarrow{XY}$ = the slope of $\overleftrightarrow{XZ}$

$\therefore \frac{3-1}{a-0} = \frac{5-1}{2-0}$ $\quad \therefore \frac{2}{a} = 2$ $\quad \therefore a = 1$

19

$\because$ The slope of $\overleftrightarrow{AB} = \frac{5-7}{-1-1} = 1$

, the slope of $\overleftrightarrow{BC} = \frac{2-5}{4+1} = \frac{-3}{5}$

$\because$ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

$\therefore C \notin \overleftrightarrow{AB}$

20

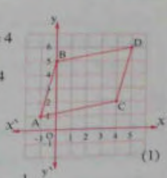
$\because$ The slope of $\overleftrightarrow{AB} = m_1 = \frac{3+1}{2+1} = \frac{4}{3}$

, the slope of $\overleftrightarrow{BC} = m_2 = \frac{0-3}{6-2} = \frac{-3}{4}$

, $\because m_1 \times m_2 = \frac{4}{3} \times \frac{-3}{4} = -1$

$\therefore \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$ $\quad \therefore \Delta$ ABC is right-angled at B

21



$\because$ The slope of $\overleftrightarrow{AB} = \frac{5-1}{0+1} = 4$

, the slope of $\overleftrightarrow{CD} = \frac{6-2}{5-4} = 4$

$\therefore$ The slope of $\overleftrightarrow{AB}$

= the slope of $\overleftrightarrow{CD}$

$\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ (1)

$\because$ The slope of $\overleftrightarrow{AC} = \frac{2-1}{4+1} = \frac{1}{5}$

, the slope of $\overleftrightarrow{BD} = \frac{6-5}{5-0} = \frac{1}{5}$

$\therefore$ The slope of $\overleftrightarrow{AC}$ = the slope of $\overleftrightarrow{BD}$

$\therefore \overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$ (2)

From (1) and (2) :

$\therefore$ The figure ABDC is a parallelogram

22

$\because$ The slope of $\overleftrightarrow{AB} = \frac{1-3}{5+1} = -\frac{1}{3}$

, the slope of $\overleftrightarrow{CD} = \frac{6-4}{0-6} = -\frac{1}{3}$

$\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ (1)

$\because$ The slope of $\overleftrightarrow{AD} = \frac{6-3}{0+1} = 3$

, the slope of $\overleftrightarrow{BC} = \frac{4-1}{6-5} = 3$

$\therefore \overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ (2)

From (1) and (2) , we deduce that :

ABCD is a parallelogram

$\because$ The slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{BC} = -\frac{1}{3} \times 3 = -1$

$\therefore \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

$\therefore$ The figure ABCD is a rectangle

23

$\because$ The slope of $\overleftrightarrow{AB} = \frac{4-3}{6-1} = \frac{1}{5}$

, the slope of $\overleftrightarrow{CD} = \frac{8-9}{2-7} = \frac{1}{5}$

$\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ (1)

$\because$ The slope of $\overleftrightarrow{AD} = \frac{8-3}{2-1} = 5$

, the slope of $\overleftrightarrow{BC} = \frac{9-4}{7-6} = 5$

$\therefore \overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ (2)

From (1) and (2) , we deduce that :

ABCD is a parallelogram.

$\because$ The slope of $\overleftrightarrow{AC} = \frac{9-3}{7-1} = 1$

, the slope of $\overleftrightarrow{BD} = \frac{8-4}{2-6} = -1$

$\therefore$ The slope of $\overleftrightarrow{AC}$ × the slope of $\overleftrightarrow{BD} = 1 \times -1 = -1$

$\therefore \overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

$\therefore$ The figure ABCD is a rhombus.

24

$\because$ The slope of $\overleftrightarrow{AB} = \frac{3+1}{2+1} = \frac{4}{3}$

, the slope of $\overleftrightarrow{CD} = \frac{-4-0}{3-6} = \frac{4}{3}$

$\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ (1)

$\because$ The slope of $\overleftrightarrow{AD} = \frac{-4+1}{3+1} = -\frac{3}{4}$

, the slope of $\overleftrightarrow{BC} = \frac{0-3}{6-2} = -\frac{3}{4}$

$\therefore \overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ (2)

From (1) and (2) , we deduce that :

ABCD is a parallelogram.

$\because$ The slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{BC} = \frac{4}{3} \times \frac{-3}{4} = -1$

$\therefore \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

$\therefore$ The figure ABCD is a rectangle.

, $\because$ the slope of $\overleftrightarrow{AC} = \frac{0+1}{6+1} = \frac{1}{7}$

, the slope of $\overleftrightarrow{BD} = \frac{-4-3}{3-2} = -7$

$\therefore$ The slope of $\overleftrightarrow{AC}$ × The slope of $\overleftrightarrow{BD}$

$= \frac{1}{7} \times -7 = -1$

$\therefore \overleftrightarrow{AC} \perp \overleftrightarrow{BD}$ $\quad \therefore$ ABCD is a square

25

$\because$ The slope of $\overleftrightarrow{AB} = \frac{2+2}{3-9} = -\frac{2}{3}$

$\because$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{CD}$

$\therefore \frac{-X+3}{X-4} = \frac{2}{-3}$ $\quad \therefore -3(-X+3) = 2(X-4)$

$\therefore 3X - 9 = 2X - 8$ $\quad \therefore X = 1$

$\therefore C = (1 , -1)$

26

$\because$ The slope of $\overleftrightarrow{AB} = \frac{0-3}{7-4} = -1$

, the slope of $\overleftrightarrow{BC} = \frac{-2-0}{1-7} = \frac{1}{3}$

$\therefore$ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

$\therefore$ A , B and C are not collinear

$\therefore$ A , B and C are vertices of a triangle (First req.)

$\because$ The slope of $\overleftrightarrow{CD} = \frac{2+2}{1-1} = \frac{4}{0}$ (undefined)

, the slope of $\overleftrightarrow{AD} = \frac{3-2}{4-1} = \frac{1}{3}$

$\therefore$ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{CD}$

, the slope of $\overleftrightarrow{BC}$ = the slope of $\overleftrightarrow{AD}$

$\therefore \overleftrightarrow{BC} \parallel \overleftrightarrow{AD}$

$\therefore$ The figure ABCD is a trapezoid (Second req.)

$AD = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{9+1} = \sqrt{10}$ length unit

$BC = \sqrt{(7-1)^2 + (0+2)^2}$

$= \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$ length unit

$\therefore AD : BC = 1 : 2$ (Third req.)

27

Let the measure of the positive angle be θ

$\therefore \sin\theta = \frac{3}{5}$

$\therefore \theta = 36° \, 52' \, 11.63''$ $\quad \therefore \tan\theta = \frac{3}{4}$

$\therefore$ The slope of the straight line $= \frac{3}{4}$

28

$\because \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{CD}$

$\therefore \frac{3-1}{3-1} = \frac{y+3X}{X-0}$ $\quad \therefore y + 3X = X$

$\therefore y + 2X = 0$ (1)

, $\because \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

$\therefore$ The slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{BC} = -1$

$\therefore \frac{3-1}{3-1} \times \frac{-3X-3}{0-3} = -1$ $\quad \therefore \frac{-3X-3}{-3} = -1$

$\therefore -3X - 3 = 3$ $\quad \therefore -3X = 6$ $\quad \therefore X = -2$

And from (1) : $y + 2 \times (-2) = 0$ $\quad \therefore y = 4$

Another solution :

$\because$ ABCD is a rectangle

$\therefore$ The two diagonals bisect each other

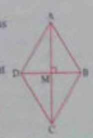
$\therefore$ The midpoint of $\overline{AC}$ = The midpoint of $\overline{BD}$

$\therefore \left(\frac{1+0}{2} , \frac{1-3X}{2}\right) = \left(\frac{3+X}{2} , \frac{3+y}{2}\right)$

$\therefore \frac{3+X}{2} = \frac{1}{2}$ $\quad \therefore 3 + X = 1$ $\quad \therefore X = -2$

, $\frac{3+y}{2} = \frac{1+6}{2}$ $\quad \therefore 3 + y = 7$ $\quad \therefore y = 4$

29



1. The two diagonals of the rhombus bisect each other

   $\therefore$ Let M be the intersection point of the diagonals

   $\therefore M = \left(\frac{3-1}{2} , \frac{2-2}{2}\right) = (1 , 0)$

   , $\because$ the slope of $\overleftrightarrow{MA} = \frac{2-0}{3-1} = \frac{2}{2} = 1$

   , the slope of $\overleftrightarrow{MB} = \frac{k-0}{4-1} = \frac{k}{3}$

   , $\because$ the two diagonals of the rhombus are perpendicular

   $\therefore \overleftrightarrow{MA} \perp \overleftrightarrow{MB}$ $\quad \therefore 1 \times \frac{k}{3} = -1$

   $\therefore k = -3$

2. Let : D (X , y) $\quad \therefore (1 , 0) = \left(\frac{4+X}{2} , \frac{-3+y}{2}\right)$

   $\therefore \frac{4+X}{2} = 1$ $\quad \therefore 4 + X = 2$

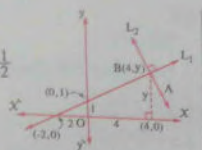
   $\therefore X = -2$, $\frac{-3+y}{2} = 0$

   $\therefore -3 + y = 0$ $\quad \therefore y = 3$ $\quad \therefore D(-2 , 3)$

   $\therefore BD = \sqrt{(4+2)^2 + (-3-3)^2}$

   $= \sqrt{36+36} = 6\sqrt{2}$ length units

30



∵ The slope of the straight line $L_1 = \frac{1-0}{0+2} = \frac{1}{2}$

Let B (4 , y)

∴ The slope of the straight line $L_1 = \frac{y-0}{4+2}$

$= \frac{y}{6}$

∴ $\frac{y}{6} = \frac{1}{2}$ ∴ y = 3 ∴ B (4 , 3)

, ∵ the straight line $L_1 \perp$ the straight line L_2

, the slope of the straight line $L_1 = \frac{1}{2}$

∴ The slope of the straight line $L_2 = -2$

, ∵ A (5 m , m) , B (4 , 3) lie on the straight line L_2

∴ $\frac{m-3}{5m-4} = -2$ ∴ $-10m + 8 = m - 3$

∴ $-11m = -11$ ∴ m = 1

Answers of Exercise 6

1

1 The slope = 5 and the intercepted part = 3 units from the negative part of y-axis

2 ∵ $2y = 4 - x$ (dividing by 2) ∴ $y = -\frac{1}{2}x + 2$

∴ The slope $= -\frac{1}{2}$ and the intercepted part = 2 units from the positive part of y-axis

3 ∵ $2x - 3y - 6 = 0$

∴ $3y = 2x - 6$ (dividing by 3)

∴ $y = \frac{2}{3}x - 2$

∴ The slope $= \frac{2}{3}$ and the intercepted part = 2 units from the negative part of y-axis

4 ∵ $\frac{y-2}{x} = \frac{1}{2}$ ∴ $y - 2 = \frac{1}{2}x$

∴ $y = \frac{1}{2}x + 2$

∴ The slope $= \frac{1}{2}$ and the intercepted part = 2 units from the positive part of the y-axis

5 ∵ $\frac{x}{2} + 3y = 6$ (multiplying by 2)

∴ $x + 6y = 12$ ∴ $6y = -x + 12$

∴ $y = -\frac{1}{6}x + 2$

∴ The slope $= -\frac{1}{6}$ and the intercepted part = 2 units from the positive part of y-axis.

6 ∵ $\frac{x}{2} + \frac{y}{3} = 1$ (multiplying by 3)

∴ $\frac{3x}{2} + y = 3$ ∴ $y = -\frac{3x}{2} + 3$

∴ The slope $= -\frac{3}{2}$ and the intercepted part = 3 units from the positive part of y-axis

2

1 $y = 2x + 7$

2 $y = -x + 3$

3 $y = 2\frac{1}{2}x - 1$

4 $y = -\frac{3}{4}x - 2\frac{1}{2}$

5 $y = -2$

3

1 ∵ The slope = tan 45° = 1 ∴ $y = x + c$

∵ The straight line passes through the point (3 , 2)

∴ 2 = 3 + c ∴ c = −1

∴ The equation is : $y = x - 1$

2 ∵ The slope of the given straight line $= \frac{2}{3}$

∴ The slope of the required straight line $= \frac{2}{3}$ and it intercepts from the negative part of y-axis 3 units

∴ The equation of the required straight line is :

$y = \frac{2}{3}x - 3$

3 ∵ The slope of the given straight line $= \frac{3}{4}$

∴ The slope of the required straight line $= -\frac{4}{3}$ and it intercepts from the positive part of y-axis 6 units

∴ The equation of the required straight line is

$y = -\frac{4}{3}x + 6$

4 ∵ The slope of the given straight line $= \frac{7-1}{2+2} = \frac{3}{2}$

∴ The slope of the required straight line $= -\frac{2}{3}$ and intercepts from the positive part of y-axis 5 units

∴ The equation of the required straight line is

$y = -\frac{2}{3}x + 5$

5 ∵ The straight line passes through the two points (4 , 0) , (0 , 9)

∴ The slope of the straight line $= \frac{9-0}{0-4} = -\frac{9}{4}$ and the intercepted part = 9 units from the positive part of y-axis

∴ The equation of the straight line is :

$y = -\frac{9}{4}x + 9$

6 ∵ The slope = 2 ∴ $y = 2x + c$

, ∵ the straight line passes through the point (2 , −1)

∴ $-1 = 2 \times 2 + c$ ∴ c = −5

∴ $y = 2x - 5$

7 ∵ The slope of the given straight line $= \frac{1}{2}$

∴ The slope of the required straight line = −2

∴ The equation of the required straight line is :

$y = -2x + c$

∵ The straight line passes through the point (−2 , 3)

∴ $3 = -2 \times (-2) + c$ ∴ c = −1

∴ The equation of the required straight line is :

$y = -2x - 1$

8 ∵ The slope of the given straight line $= -\frac{1}{2}$

∴ The slope of the required straight line $= -\frac{1}{2}$

∴ The equation of the required straight line is

$y = -\frac{1}{2}x + c$

∵ The straight line passes through the point (3 , −5)

∴ $-5 = -\frac{1}{2} \times 3 + c$ ∴ $c = -3\frac{1}{2}$

∴ The equation of the required straight line is

$y = -\frac{1}{2}x - 3\frac{1}{2}$

9 ∵ The slope of the given straight line $= \frac{1-5}{-2-1} = \frac{4}{3}$

∴ The slope of the required straight line $= \frac{4}{3}$

∴ The equation of the required straight line is :

$y = \frac{4}{3}x + c$

, ∵ The straight line passes through the point (3 , −1)

∴ $-1 = \frac{4}{3} \times 3 + c$ ∴ c = −5

∴ The equation of the required straight line is :

$y = \frac{4}{3}x - 5$

10 ∵ The slope of $\overleftrightarrow{AB} = \frac{-4+3}{5-2} = -\frac{1}{3}$

∴ The slope of the required straight line = 3

∴ The equation of the required straight line is :

$y = 3x + c$

∵ The straight line passes through the point (1 , 2)

∴ $2 = 3 \times 1 + c$ ∴ c = −1

∴ The equation of the required straight line is :

$y = 3x - 1$

11 ∵ The slope of the given straight line = tan 45° = 1

∴ The slope of the required straight line = −1

∴ The equation of the required straight line is :

$y = -x + c$

∵ The straight line passes through the point (2 , −2)

∴ $-2 = -2 + c$ ∴ c = 0

∴ The equation of the required straight line is :

$y = -x$

12 ∵ The slope of the straight line $= \frac{1+1}{1-2} = -2$

∴ The equation of the straight line is $y = -2x + c$

∵ The straight line passes through the point (1 , 1)

∴ $1 = -2 \times 1 + c$ ∴ c = 3

∴ The equation of the straight line is

$y = -2x + 3$

13 ∵ The slope of the straight line $= \frac{-1-2}{-2-4} = \frac{1}{2}$

∴ The equation of the straight line is $y = \frac{1}{2}x + c$

∵ The straight line passes through the point (4 , 2)

∴ $2 = \frac{1}{2} \times 4 + c$ ∴ c = zero

∴ The equation of the straight line is $y = \frac{1}{2}x$

∴ The intercepted part of y-axis = zero

∴ The straight line passes through the origin point

14 ∵ $\frac{y-1}{x} = \frac{1}{3}$ ∴ $y - 1 = \frac{1}{3}x$

∴ $y = \frac{1}{3}x + 1$

∴ The slope of the given straight line $= \frac{1}{3}$

∴ The slope of the required straight line $= \frac{1}{3}$

∴ The equation of the required straight line is

$y = \frac{1}{3}x - 3$

15 ∵ The slope of $\overleftrightarrow{AB} = \frac{1-6}{2+3} = -1$

∴ The slope of the required straight line = 1

∴ The equation of the required straight line is

$y = x + c$

∵ The required straight line passes through the point A (−3 , 6) ∴ 6 = −3 + c ∴ c = 9

∴ The equation of the required straight line is

$y = x + 9$

16 ∵ The slope of $\overleftrightarrow{AB} = \frac{5-3}{3-1} = 1$

∴ The slope of the required straight line = −1

∴ The equation of the required straight line is

$y = -x + c$

, ∵ the midpoint of $\overline{AB} = \left(\frac{1+3}{2} , \frac{3+5}{2}\right) = (2 , 4)$

, ∵ the required straight line passes through the midpoint of $\overline{AB}$

$\therefore 4 = -2 + c \qquad \therefore c = 6$

∴ The equation of the required straight line is $y = -x + 6$

17 $\because 2y = 4x - 5 \qquad \therefore y = 2x - \frac{5}{2} \qquad \therefore m = 2$

∴ The slope of the required straight line $= 2$

∴ The equation of the required straight line is $y = 2x + c$

∵ The midpoint of $\overline{AB} = \left(\frac{4-2}{2}, \frac{8+4}{2}\right) = (1, 6)$

∴ $(1, 6)$ satisfies its equation

$\therefore 6 = 2 \times 1 + c \qquad \therefore c = 4$

∴ The equation of the required straight line is $y = 2x + 4$

18 ∵ The slope of the given straight line $= 2$

∴ The slope of the required straight line $= -\frac{1}{2}$

∴ The equation of the required straight line is $y = -\frac{1}{2}x + c$

∵ The midpoint of $\overline{AB} = \left(\frac{3-1}{2}, \frac{6+4}{2}\right) = (1, 5)$

, the required straight line passes through the midpoint of $\overline{AB}$

$\therefore 5 = -\frac{1}{2} \times 1 + c \qquad \therefore c = 5\frac{1}{2}$

∴ The equation of the required straight line is $y = -\frac{1}{2}x + 5\frac{1}{2}$

19 ∵ The required straight line intercepts from the positive part of x-axis 4 units

∴ The required straight line passes through the point $(4, 0)$

∴ The slope of the required straight line $= \frac{0-3}{4-2} = \frac{-3}{2}$

∴ The equation of the required straight line is : $y = \frac{-3}{2}x + c$

, ∵ the required straight line passes through the point $(2, 3)$

$\therefore 3 = \frac{-3}{2} \times 2 + c \qquad \therefore c = 6$

∴ The equation of the required straight line is : $y = \frac{-3}{2}x + 6$

4

1 d	2 d	3 c	4 b
5 d	6 d	7 a	8 d
9 c	10 d	11 a	12 a
13 a	14 c	15 d	16 a
17 c	18 c	19 d	20 b
21 a	22 b	23 c	24 c

5

∵ The slope of $\overleftrightarrow{AB} = \frac{2-1}{1-3} = \frac{-1}{2}$

, the slope of the other straight line $= \frac{-2}{4} = \frac{-1}{2}$

∴ The slope of $\overleftrightarrow{AB}$ = the slope of the other straight line

∴ The two straight lines are parallel.

6 ∵ The slope of the straight line :

$\sqrt{3}x + y = 5$ is : $\frac{-\sqrt{3}}{1} = -\sqrt{3}$

, the slope of the other straight line $= \tan 30° = \frac{1}{\sqrt{3}}$

, $\because -\sqrt{3} \times \frac{1}{\sqrt{3}} = -1$

∴ The two straight lines are perpendicular.

7 $y = 2$, $x = -3$

8

∵ The slope of the straight line $= \frac{-3}{-2} = \frac{3}{2}$

, the slope of the straight line $= \tan\theta$

$\therefore \tan\theta = \frac{3}{2} \qquad \therefore \theta \simeq 56° \, 18' \, 36''$

Put $x = 0$

$\therefore 3 \times 0 - 2y + 6 = 0 \quad \therefore -2y = -6 \quad \therefore y = 3$

∴ The intersection point with y-axis is $(0, 3)$

9

1 Let $A(x, 0)$

$\therefore$ At $y = 0 \qquad \therefore 2x - 3 \times 0 - 6 = 0$

$\therefore 2x = 6 \qquad \therefore x = 3$

∴ The straight line cuts the x-axis at the point $A(3, 0)$

Let $B(0, y)$

$\therefore$ At $x = 0 \qquad \therefore 2 \times 0 - 3y - 6 = 0$

$\therefore -3y = 6 \qquad \therefore y = -2$

∴ The straight line cuts the y-axis at the point $B(0, -2)$

2 Let D be the midpoint of $\overline{AB}$

$\therefore D = \left(\frac{3+0}{2}, \frac{0-2}{2}\right) = \left(\frac{3}{2}, -1\right)$

, ∵ the straight line is parallel to the y-axis

∴ Its slope is undefined

, ∵ the straight line passes through the point $D\left(\frac{3}{2}, -1\right)$

∴ The equation of the straight line is : $x = \frac{3}{2}$

10

$\because m_1 = \frac{1-(-1)}{5-2} = \frac{2}{3}$, $m_2 = \frac{-a}{3}$

, ∵ the two straight lines are parallel

$\therefore m_1 = m_2 \qquad \therefore \frac{2}{3} = \frac{-a}{3}$

$\therefore a = -2$

11

$\because m_1 = \frac{-3-2}{6-5} = -5 \qquad \therefore m_2 = \frac{1}{5}$

$\because m_2 = \frac{a}{5} \qquad \therefore \frac{a}{5} = \frac{1}{5} \qquad \therefore a = 1$

12

∵ The slope of the straight line $L = \frac{4}{3}$

∴ The slope of $\overleftrightarrow{AB} = \frac{4}{3}$

∵ The slope of $\overleftrightarrow{AB} = \frac{y+3}{5-2} \qquad \therefore \frac{y+3}{3} = \frac{4}{3}$

$\therefore y + 3 = 4 \qquad \therefore y = 1$

13

$\because m_1 = \frac{-\text{Coefficient of } x}{\text{Coefficient of } y} = 2k - 1$

, $m_2 = \tan 45° = 1$

, ∵ the two straight lines are parallel

$\therefore m_1 = m_2 \qquad \therefore 2k - 1 = 1$

$\therefore 2k = 2 \qquad \therefore k = 1$

14

∵ The slope of $\overleftrightarrow{XY} = \frac{6+2}{-5-3} = -1$

∴ The slope of the axis of symmetry of $\overline{XY} = 1$

∴ The equation of the axis of symmetry of $\overline{XY}$ is : $y = x + c$

, ∵ the midpoint of $\overline{XY} = \left(\frac{3-5}{2}, \frac{-2+6}{2}\right) = (-1, 2)$

∴ $(-1, 2)$ satisfies the equation : $y = x + c$

$\therefore 2 = -1 + c \qquad \therefore c = 3$

∴ The equation of the axis of symmetry of $\overline{XY}$ is : $y = x + 3$

15

Let D be the midpoint of $\overline{BC}$

$\therefore D = \left(\frac{3+1}{2}, \frac{7-3}{2}\right) = (2, 2)$

∴ The slope of $\overleftrightarrow{AD} = \frac{-6-2}{5-2} = -\frac{8}{3}$

∴ The equation of $\overleftrightarrow{AD}$ is $y = -\frac{8}{3}x + c$

$\because D \in \overleftrightarrow{AD}$

∴ $(2, 2)$ satisfies its equation

$\therefore 2 = -\frac{8}{3} \times 2 + c \qquad \therefore c = \frac{22}{3}$

∴ The equation of $\overleftrightarrow{AD}$ is $y = -\frac{8}{3}x + \frac{22}{3}$

16

∵ The slope of $\overleftrightarrow{BC} = \frac{1+1}{-2-5} = -\frac{2}{7}$

∴ The slope of the perpendicular straight line to it $= \frac{7}{2}$

∴ The equation of the perpendicular to $\overleftrightarrow{BC}$ is $y = \frac{7}{2}x + c$

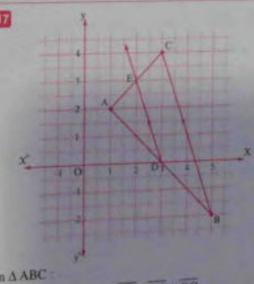
∵ $A \in$ the perpendicular to $\overleftrightarrow{BC}$

∴ $(0, 6)$ satisfies the equation

$\therefore 6 = \frac{7}{2} \times 0 + c \qquad \therefore c = 6$

∴ The equation of the perpendicular to $\overleftrightarrow{BC}$ from the point A is $y = \frac{7}{2}x + 6$

17



In ΔABC :

∵ D is the midpoint of $\overline{AB}$, $\overline{DE} \parallel \overline{BC}$

∴ E is the midpoint of $\overline{AC}$, $DE = \frac{1}{2} BC$

$\therefore DE = \frac{1}{2}\sqrt{(5-3)^2+(-2-4)^2}$

$= \frac{1}{2}\sqrt{4+36} = \frac{1}{2}\sqrt{40}$

$= \frac{1}{2} \times 2\sqrt{10} = \sqrt{10}$ length unit

$\because$ The slope of $\overleftrightarrow{BC} = \frac{4+2}{3-5} = -3$

$\therefore$ The slope of $\overleftrightarrow{DE} = -3$

$\therefore$ The equation of $\overleftrightarrow{DE}$ is $y = -3x + c$

$\because$ D is the midpoint of $\overline{AB} = \left(\frac{1+5}{2}, \frac{2-2}{2}\right) = (3, 0)$

$\therefore (3, 0)$ satisfies its equation

$\therefore 0 = -3 \times 3 + c \qquad \therefore c = 9$

$\therefore$ The equation of $\overleftrightarrow{DE}$ is $y = -3x + 9$

18

$\because$ The slope of $\overleftrightarrow{AC} = \frac{6-4}{-1-5} = -\frac{2}{6} = -\frac{1}{3}$

, $\because$ the two diagonals of the square are perpendicular

$\therefore$ The slope of $\overleftrightarrow{BD} = 3$

$\therefore$ The equation of $\overleftrightarrow{BD}$ is : $y = 3x + c$

$\because$ The midpoint of $\overline{AC} = \left(\frac{5-1}{2}, \frac{6+4}{2}\right) = (2, 5)$

$\therefore (2, 5)$ satisfies the equation of $\overleftrightarrow{BD}$

$\therefore 5 = 2 \times 3 + c \qquad \therefore c = -1$

$\therefore$ The equation of $\overleftrightarrow{BD}$ is : $y = 3x - 1$

19

$\because$ The slope of $\overleftrightarrow{AC} = \frac{3-0}{1-6} = -\frac{3}{5}$

, $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

$\therefore$ The slope of $\overleftrightarrow{BD} = \frac{5}{3}$

$\therefore$ The equation of $\overleftrightarrow{BD}$ is $y = \frac{5}{3}x + c$

$\because$ The two diagonals of the rhombus bisect each other

$\therefore$ The midpoint of $\overline{AC} = \left(\frac{1+6}{2}, \frac{3+0}{2}\right) = (3.5, 1.5)$

$\therefore (3.5, 1.5)$ satisfies the equation of $\overleftrightarrow{BD}$

$\therefore 1.5 = \frac{5}{3} \times 3.5 + c \qquad \therefore c = -4\frac{1}{3}$

$\therefore$ The equation of $\overleftrightarrow{BD}$ is $y = \frac{5}{3}x - 4\frac{1}{3}$

20

$\because$ The slope of $\overleftrightarrow{AB} = \frac{-3-3}{-1-2} = 2$

$\therefore$ The equation of $\overleftrightarrow{AB}$ is : $y = 2x + c$

, $\because \overleftrightarrow{AB}$ passes through the point $(2, 3)$

$\therefore 3 = 2 \times 2 + c$

$\therefore c = -1$

$\therefore$ The equation of $\overleftrightarrow{AB}$ is : $y = 2x - 1$

by substituting in the equation of $\overleftrightarrow{AB}$ by $x = 2k + 1$

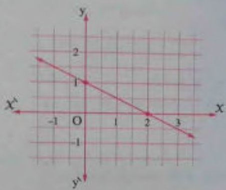
$\therefore y = 2(2k+1) - 1 = 4k + 2 - 1 = 4k + 1$

$\therefore$ The point $C(2k+1, 4k+1)$ satisfies the equation of $\overleftrightarrow{AB}$

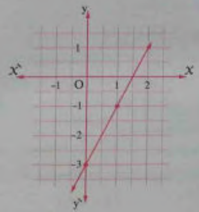
$\therefore C \in \overleftrightarrow{AB}$

21

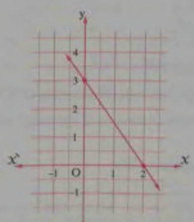
1



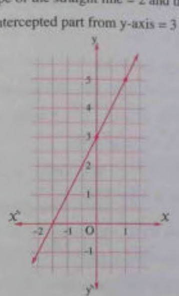
2



3



22 The slope of the straight line = 2 and the length of the intercepted part from y-axis = 3 units



23

1. $\because$ The straight line passes through the two points $(0, 3)$ and $(4, 5)$

   $\therefore$ The slope $= \frac{5-3}{4-0} = \frac{1}{2}$

2. 3 units from the positive part of y-axis

3. The equation is $y = \frac{1}{2}x + 3$

4. 6 units from the negative part of x-axis

5. The area of the triangle $= \frac{1}{2} \times 3 \times 6 = 9$ square units

24

1. $\because$ The slope of the straight line $= \frac{3-1}{2-1} = 2$

   $\therefore$ The equation of the straight line is $y = 2x + c$

   $\because$ The point $(1, 1) \in$ the straight line

   $\therefore 1 = 2 \times 1 + c \qquad \therefore c = -1$

   $\therefore$ The equation of the straight line is $y = 2x - 1$

2. One unit of the negative part of y-axis

3. $\because$ The point $(3, a)$ satisfies the equation

   $\therefore a = 2 \times 3 - 1 = 5$

25

$\because \overleftrightarrow{AB}$ cuts two equal parts of the two axes

$\therefore OA = OB$

$\therefore$ In ΔAOB which is right-angled at O

$m(\angle ABO) = m(\angle BAO) = 45^\circ$

$\therefore \overleftrightarrow{AB}$ makes with the positive direction of the x-axis an angle of measure 135°

$\therefore$ The slope of $\overleftrightarrow{AB} = \tan 135^\circ = -1$

$\therefore k =$ the slope of $\overleftrightarrow{AB} = -1$

$\therefore y = -x + c$

, $\because (2, 3) \in \overleftrightarrow{AB}$

$\therefore 3 = -2 + c \qquad \therefore c = 5$ (First req.)

$\therefore OA = OB = 5$ length units

$\therefore$ The area of $\Delta ABO = \frac{1}{2} \times 5 \times 5 = 12.5$ square units (Second req.)

26

$\because \Delta ABO$ is equilateral

, C is the midpoint of $\overline{AB}$

$\therefore \overline{OC} \perp \overline{AB}$

$\therefore m(\angle BOC) = 30^\circ$

$\therefore \tan(\angle BOC) = \tan 30^\circ = \frac{1}{\sqrt{3}}$

$\therefore$ The equation of $\overleftrightarrow{OC}$ is : $y = \frac{1}{\sqrt{3}}x + c$

, $\because O \in \overleftrightarrow{OC} \quad \therefore c = 0$

$\therefore$ The equation of $\overleftrightarrow{OC}$ is : $y = \frac{1}{\sqrt{3}}x$

27

Let : $D(x, y)$

, $\because$ the midpoint of $\overline{AB}$ = The midpoint of $\overline{OD}$

, $\because$ the midpoint of $\overline{AB} = \left(\frac{6+2}{2}, \frac{6+2}{2}\right) = (4, 4)$

, $\because$ the midpoint of $\overline{OD} = \left(\frac{0+x}{2}, \frac{0+y}{2}\right)$

$\therefore (4, 4) = \left(\frac{x}{2}, \frac{y}{2}\right) \qquad \therefore \frac{x}{2} = 4 \qquad \therefore x = 8$

, $\frac{y}{2} = 4 \qquad \therefore y = 8$

$\therefore D(8, 8)$ (First req.)

$\because \overleftrightarrow{OD}$ passes through the origin point

$\therefore$ Its equation is : $y = mx$ (m is the slope)

, $\because$ the slope of $\overleftrightarrow{OD} = \frac{8-0}{8-0} = 1$

$\therefore$ The equation of $\overleftrightarrow{OD}$ is : $y = x$ (Second req.)

, $\because$ the slope of $\overleftrightarrow{OD} = \tan(\angle DOC)$

$\therefore \tan(\angle DOC) = 1 \qquad \therefore m(\angle DOC) = 45^\circ$

$\therefore m(\angle DOE) = 180^\circ - 45^\circ = 135^\circ$ (Third req.)

28

Let the point $A(0, y)$

$\because$ The straight line L_1 passes through the point $A(0, y)$

$\therefore 2 \times 0 - y + 2 = 0$ $\quad \therefore y = 2$ $\quad \therefore A(0, 2)$

$, \because m_1 = \frac{-2}{-1} = 2$ $, L_1 \perp L_2$

$\therefore m_1 m_2 = -1$ $\quad \therefore 2 \times m_2 = -1$

$\therefore m_2 = \frac{-1}{2}$

$\therefore$ The equation of L_2 is : $y = \frac{-1}{2} X + c$

$, \because$ the straight line L_2 passes through the point $A(0, 2)$

$\therefore 2 = \frac{-1}{2} \times 0 + c$ $\quad \therefore c = 2$

$\therefore$ The equation of L_2 is : $y = \frac{-1}{2} X + 2$

29

1 **First :** $\because \tan(\angle ABO) = \frac{4}{3}$

$\therefore m(\angle ABO) = 53° \ 7' \ 48''$

From ΔABO :

$m(\angle BAO) = 180° - (90° + 53° \ 7' \ 48'')$

$= 36° \ 52' \ 12''$

Second : Let the point $(X, 0)$

$, \because \tan(\angle ABO) = \frac{4}{3}$

$\therefore \frac{8}{X} = \frac{4}{3}$ $\quad \therefore 4X = 24$

$\therefore X = 6$ $\quad \therefore B(6, 0)$

2 **First :** The slope of $\overleftrightarrow{AB} = \frac{0-8}{6-0} = \frac{-8}{6} = \frac{-4}{3}$

Second : $\because$ The required straight line is perpendicular to $\overleftrightarrow{AB}$

$\because$ The slope of $\overleftrightarrow{AB} = -\frac{4}{3}$

$\therefore$ The slope of the required straight line is $= \frac{3}{4}$

$\therefore$ The equation of the required straight line is :

$y = \frac{3}{4} X + c$

$, \because$ the straight line passes through the point $O(0, 0)$

$\therefore 0 = \frac{3}{4} \times 0 + c$ $\quad \therefore c = 0$

$\therefore$ The equation of the required straight line is :

$y = \frac{3}{4} X$

30

1 $O = (0, 0)$

$, \because A = (X, 0), B = (0, y), C = (4, 3)$

where C is the midpoint of $\overline{AB}$

$\therefore \frac{X+0}{2} = 4$ $\quad \therefore X = 8$

$, \frac{y+0}{2} = 3$ $\quad \therefore y = 6$

$\therefore A(8, 0), B(0, 6)$

2 $OA = 8$ length units $, OB = 6$ length units

$, CA = \sqrt{(8-4)^2 + (0-3)^2} = 5$ length units

$, CB = CA = 5$ length units

$, CO = \sqrt{4^2 + 3^2} = 5$ length units

3 The slope of $\overleftrightarrow{AB} = \frac{6-0}{0-8} = -\frac{3}{4}$

$,$ the slope of $\overleftrightarrow{OC} = \frac{0-3}{0-4} = \frac{3}{4}$

$,$ the slope of $\overleftrightarrow{OA} =$ zero

$,$ the slope of $\overleftrightarrow{OB}$ is undefined

4 The equation of $\overleftrightarrow{AB}$ is : $y = -\frac{3}{4} X + 6$

$,$ the equation of $\overleftrightarrow{CO}$ is : $y = \frac{3}{4} X$

31

Let $C(X, 0)$

$, \because$ the slope of $\overleftrightarrow{AB} = \frac{3+3}{2+4} = 1$

$, \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

$\therefore$ The slope of $\overleftrightarrow{BC} = \frac{0-3}{X-2} = -1$

$\therefore X - 2 = 3$ $\quad \therefore X = 5$

$\therefore C(5, 0)$ (First req.)

$, \because$ the slope of $\overleftrightarrow{AC} = \frac{0+3}{5+4} = \frac{1}{3}$

$\therefore$ The equation of $\overleftrightarrow{AC}$ is : $y = \frac{1}{3} X + c$

$, \because \overleftrightarrow{AC}$ passes through the point $C(5, 0)$

$\therefore 0 = \frac{1}{3} \times 5 + c$ $\quad \therefore c = \frac{-5}{3}$

$\therefore$ The equation of $\overleftrightarrow{AC}$ is : $y = \frac{1}{3} X - \frac{5}{3}$ (Second req.)

32

$\because$ O is the midpoint of $\overline{AB}$

$\therefore (0, 0) = \left(\frac{8+N}{2}, \frac{H-6}{2}\right)$

$\therefore \frac{8+N}{2} = 0$ $\quad \therefore 8 + N = 0$ $\quad \therefore N = -8$

$, \frac{H-6}{2} = 0$ $\quad \therefore H - 6 = 0$ $\quad \therefore H = 6$

$\therefore H + N = 6 - 8 = -2$ (First req.)

$, \because A(8, 6), B(-8, -6)$

$\therefore AB = \sqrt{(8+8)^2 + (6+6)^2} = 20$ length units

$, \because \Delta ABC$ is right-angled at C $, \overline{CO}$ is a median

$\therefore CO = \frac{1}{2} AB = \frac{1}{2} \times 20 = 10$ length units

$\therefore C(0, 10)$ $\quad \therefore$ the slope of $\overleftrightarrow{AC} = \frac{10-6}{0-8} = \frac{-1}{2}$

$\therefore$ The equation of $\overleftrightarrow{AC}$ is : $y = \frac{-1}{2} X + 10$

(Second req.)

33

1 The slope of $\overleftrightarrow{AC} = \frac{-1}{-1} = 1$

$,$ from the equation of $\overleftrightarrow{AC}$: $y = X - 3$

$\therefore OH = 3$ length unit.

2 $\because$ The slope of $\overleftrightarrow{BC} = \sqrt{3}$

$\therefore \tan(\angle CBD) = \sqrt{3}$ $\quad \therefore m(\angle CBD) = 60°$

$, \because$ the slope of $\overleftrightarrow{AC} = 1$

$\therefore \tan(\angle CAD) = 1$ $\quad \therefore m(\angle CAD) = 45°$

3 $\because \angle CBD$ is an exterior angle of ΔABC

$\therefore m(\angle CBD) = m(\angle ACB) + m(\angle CAD)$

$\therefore m(\angle ACB) = 60° - 45° = 15°$

34

$\because C(5, 2)$

$\therefore OB = 5$ length units $, BC = 2$ length units

$, \because ABCD$ is a square

$\therefore AB = BC = AD = 2$ length units

$\therefore OA = OB - BA = 5 - 2 = 3$ length units

$\therefore$ The slope of the straight line $L = \tan(\angle AOD)$

$= \frac{AD}{OA} = \frac{2}{3}$

$\therefore$ The equation of the straight line L is :

$y = \frac{2}{3} X + c$

$, \because$ the straight line L passes through the origin point

$\therefore c = 0$

$\therefore$ The equation of the straight line L is : $y = \frac{2}{3} X$

35

Let the length of $\overline{OA} = \ell$ length unit

$\because ABCD$ is a square $\quad \therefore AB = BC$

$, \because OA = AB$

$\therefore OA = AB = BC = \ell$ length unit

In ΔOBC : $\because \overline{BC} \perp \overline{BO}$ (properties of the square)

$\therefore \tan(\angle BOC) = \frac{BC}{BO} = \frac{\ell}{2\ell} = \frac{1}{2}$

$\therefore$ The slope of $\overleftrightarrow{OC} = \tan(\angle BOC) = \frac{1}{2}$

$\therefore$ The equation $\overleftrightarrow{OC}$ is : $y = \frac{1}{2} X + c$

$, \because \overleftrightarrow{OC}$ passes through the origin point

$\therefore c = 0$

$\therefore$ The equation of $\overleftrightarrow{OC}$ is : $y = \frac{1}{2} X$

36

$\because$ The straight line L_1 passes through the origin point

$\therefore$ Its equation is : $y = m X$

$, \because$ the slope of the straight line $L_1 = \tan 45° = 1$

$\therefore$ The equation of the straight line L_1 is : $y = X$

(First req.)

$, \because$ the straight line L_1 // the straight line L_2

$\therefore$ The slope of the straight line $L_1 =$ the slope of the straight line $L_2 = 1$

$\therefore$ The equation of the straight line L_2 is : $y = X + c$

$, \because$ the straight line L_2 passes through the point $A(1, 5)$

$\therefore 5 = 1 + c$ $\quad \therefore c = 4$

$\therefore$ The equation of the straight line L_2 is :

$y = X + 4$ (Second req.)

Let $B(X, y)$

$\because$ B belongs to the straight line L_1 $\quad \therefore y = X$

$, \overleftrightarrow{AB} \perp L_1$ $\quad \therefore$ The slope of $\overleftrightarrow{AB} = -1$

$\therefore \frac{y-5}{X-1} = -1$ $\quad \therefore y - 5 = -X + 1$

$, \because X = y$ $\quad \therefore y - 5 = -y + 1$

$\therefore 2y = 6$ $\quad \therefore y = 3$

$\therefore X = 3$ $\quad \therefore B(3, 3)$

$\therefore$ The length of $\overline{AB} = \sqrt{(1-3)^2 + (5-3)^2}$

$= 2\sqrt{2}$ length unit (Third req.)

37

1 2 m.

2 The velocity of the particle = The slope of the straight line passing through the two points $(0, 2), (4, 4)$

$= \frac{4-2}{4-0} = \frac{1}{2}$

$\therefore$ The velocity $= \frac{1}{2}$ m./sec.

3 $d = \frac{1}{2}t + 2$

4 2 metre

5 7 seconds

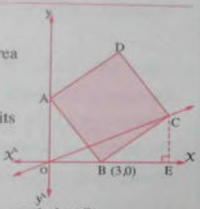
38

1 90 km. 2 2.5 hours

3 The velocity of the car = The slope of the straight line which passes through the two points (0.5 , 30) , (2 , 120) $= \frac{120 - 30}{2 - 0.5} = 60$ km./hr.

4 $d = 60\,t$

39



$\because$ ABCD is a square its area equals 25 square units

$\therefore AB = BC = 5$ length units

, $\because B(3 , 0)$

$\therefore OB = 3$ length units

$\therefore$ In Δ AOB which is right-angled at O

$\therefore AO = 4$ length units (Pythagoras)

, $\because \Delta AOB \equiv \Delta BEC$ (prove by yourself)

$\therefore EC = OB = 3$ length units

, $EB = AO = 4$ length units

$\therefore OE = 7$ length units $\therefore$ The point C (7 , 3)

, $\because \overleftrightarrow{CO}$ passes througth the origin point

$\therefore$ The equation of $\overleftrightarrow{CO}$ is :

$y = m\,x$ (where m is the slope)

, $\because$ the slope of $\overleftrightarrow{CO} = \frac{0-3}{0-7} = \frac{3}{7}$

$\therefore$ The equation of $\overleftrightarrow{CO}$ is : $y = \frac{3}{7}x$

i.e. $7y = 3x$ (The req.)

40

Let : $OB = OA = x$

, $\because \Delta$ AOB is right-angled at O

$\therefore x^2 + x^2 = (2\sqrt{2})^2$ (Pythagoras)

$\therefore 2x^2 = 8$ $\therefore x^2 = 4$

$\therefore x = 2$ $\therefore OB = OA = 2$

$\therefore$ The point B (0 , 2)

Let the equation of $\overleftrightarrow{AB}$ by : $y = m\,x + n$

, $\because$ the slope of $\overleftrightarrow{AB} = \tan(\angle BAO) = \frac{BO}{AO} = 1$

, $n = 2$

$\therefore$ The equation of $\overleftrightarrow{AB}$ is : $y = x + 2$

, $\because C = (1 , k) \in \overleftrightarrow{AB}$ $\therefore k = 1 + 2$

$\therefore k = 3$ $\therefore C(1 , 3)$

, $\because \overleftrightarrow{CD} \perp \overleftrightarrow{AB}$, the slope of $\overleftrightarrow{AB} = 1$

$\therefore$ The slope of $\overleftrightarrow{CD} = -1$

$\therefore$ The equation of $\overleftrightarrow{CD}$ is : $y = -x + \ell$

, $\because (1 , 3) \in \overleftrightarrow{CD}$ $\therefore 3 = -1 + \ell$

$\therefore \ell = 4$

$\therefore$ The equation of $\overleftrightarrow{CD}$ is : $y = -x + 4$ (The req.)

Answers of accumulative basic skills

1 d	2 a	3 c	4 b
5 b	6 a	7 a	8 a
9 d	10 d	11 b	12 a
13 d	14 a	15 a	16 d
17 c	18 c	19 d	20 c
21 c	22 c	23 b	24 d
25 c	26 c	27 d	28 c
29 c	30 d	31 b	32 d
33 a	34 c	35 c	36 b
37 d	38 b	39 c	

Guide Answers

Of The Notebook

(Algebra and Statistics)

Answers of accumulative tests on algebra & statistics

Accumulative test 1

1 1 b 2 b 3 b
4 b 5 a 6 c

2 1 $X \times Y = \{(2 , 3) , (2 , 4) , (2 , 5)\}$
2 $n(Y^2) = 9$ 3 $X^2 = \{(2 , 2)\}$

3 $x + 2y = 11$

Accumulative test 2

1 1 b 2 d 3 d
4 a 5 a 6 c

2 1 $(X \cap Y) \times Z = \{(2 , 7) , (2 , 2) , (3 , 7) , (3 , 2)\}$
2 $(X - Y) \times Z = \{(1 , 7) , (1 , 2) , (4 , 7) , (4 , 2)\}$

3 $R = \{(\frac{1}{2} , 2) , (1 , 1) , (-\frac{1}{2} , -2) , (-1 , -1)\}$
, represent by yourself , R isn't a function.
Show by yourself.

Accumulative test 3

1 1 b 2 d 3 b
4 c 5 d 6 d

2 1 The domain $= \{3 , 5 , 7\}$
2 $f(x) = 3x$

3 $b = 1$

Accumulative test 4

1 1 b 2 c 3 b
4 b 5 c 6 a

2 Graph by yourself.
1 The equation of the axis of symmetry is : $x = 3$
2 The minimum value of the function $= 0$

3 1 $(X \cap Y) \times Z = \varnothing$
2 $X \times (Y - Z) = \{(3 , 4)\}$
3 $n(X^2) = 1$

Accumulative test 5

1 1 b 2 b 3 c
4 d 5 d 6 a

2 1 $f(3) + 3g(\sqrt[3]{2}) = 12$
2 Prove by yourself.

3 The number is : 1

Accumulative test 6

1 1 c 2 b 3 b
4 c 5 b 6 d

2 Prove by yourself.

3 $\frac{28}{3}$

Accumulative test 7

1 1 d 2 c 3 c
4 a 5 c 6 d

2 Prove by yourself.

3 Prove by yourself.

Accumulative test 8

1 1 b 2 c 3 b
4 c 5 a 6 d

2 1 $y = 2x$ 2 $x = \frac{1}{4}$

3 Prove by yourself.

Accumulative test 9

1 1 b 2 a 3 b
4 d 5 c 6 b

2 The mean = 23 , the standard deviation $\simeq 4.24$

3 Prove by yourself.

Answers of important questions on algebra & statistics

Unit one

First Answers of multiple choice questions

1 (d)	2 (d)	3 (b)	4 (a)	5 (b)
6 (d)	7 (d)	8 (c)	9 (b)	10 (d)
11 (a)	12 (b)	13 (a)	14 (d)	15 (d)
16 (a)	17 (a)	18 (a)	19 (b)	20 (d)
21 (d)	22 (b)	23 (a)	24 (a)	25 (c)
26 (b)	27 (c)	28 (a)	29 (a)	30 (c)

Second Answers of essay questions

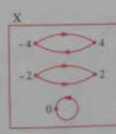
1
1 $X \times (Y \cap Z) = \{3 , 4\} \times \{5\} = \{(3 , 5) , (4 , 5)\}$
2 $Y^2 = \{4 , 5\} \times \{4 , 5\} = \{(4 , 4) , (4 , 5) , (5 , 4) , (5 , 5)\}$
3 $n(X^2) = 2^2 = 4$

2
1 $(X \cap Y) \times Z = \{1\} \times \{3 , 4\} = \{(1 , 3) , (1 , 4)\}$
2 $(X \cup Y) \times (Z - Y) = \{1 , 2 , 3\} \times \{4\} = \{(1 , 4) , (2 , 4) , (3 , 4)\}$

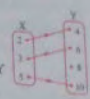
3
1 $Y \times X = \{(1 , 1) , (5 , 1) , (3 , 1) , (1 , 4) , (5 , 4) , (3 , 4)\}$
2 $X = \{1 , 4\}$
$, X^2 = \{1 , 4\} \times \{1 , 4\} = \{(1 , 1) , (1 , 4) , (4 , 1) , (4 , 4)\}$

4
1 $(Y \cap X) \times Y = \{3 , 5\} \times \{3 , 5 , 6\} = \{(3 , 3) , (3 , 5) , (3 , 6) , (5 , 3) , (5 , 5) , (5 , 6)\}$
2 $n(Y^2) = 3^2 = 9$

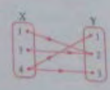
5
$R = \{(-4 , 4) , (-2 , 2) , (0 , 0) , (2 , -2) , (4 , -4)\}$
, R is a function on X because each element in X has a unique image in X



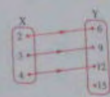
6
$R = \{(2 , 4) , (3 , 6) , (5 , 10)\}$
, R is a function because every element in X has only one image in Y
, the range $= \{4 , 6 , 10\}$



7
1 $R = \{(1 , 2) , (3 , 2) , (4 , 1) , (4 , 3)\}$
2 $\because 2a\,R\,3 \quad \therefore 2a = 4$
$\therefore a = 2$

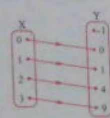


8
$R = \{(2 , 6) , (3 , 9) , (4 , 12)\}$

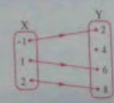


, R is a function because every element in X has only one image in Y

9
$R = \{(0 , 0) , (1 , 1) , (2 , 4) , (3 , 9)\}$
, R is a function because every element in X has only one image in Y



10
$R = \{(-1 , 2) , (1 , 6) , (2 , 8)\}$
, R is a function because every element in X has only one image in Y



11
$X = \{0 , 1 , 2\}$
$, R = \{(0 , 0) , (1 , 2) , (2 , 1)\}$
, R is a function

12
1 The range $= \{3 , 5\}$

1 ∵ R is a function

∴ Each element in X appears as the first projection once

∴ a = 3 , b = 5 or a = 5 , b = 3

∴ a + b = 3 + 5 = 5 + 3 = 8

2

13

∵ $f(2) = 2 \times 2^2 - 5 \times 2 + 2 = 0$

, $f\left(\frac{1}{2}\right) = 2 \times \left(\frac{1}{2}\right)^2 - 5 \times \frac{1}{2} + 2 = 0$

∴ $f(2) = f\left(\frac{1}{2}\right)$

14

1 $f(\sqrt{2}) + 3\,g(\sqrt{2})$

$= (\sqrt{2})^2 - 3 \times \sqrt{2} + 3(\sqrt{2} - 3)$

$= 2 - 3\sqrt{2} + 3\sqrt{2} - 9 = -7$

2 ∵ $f(3) = 3^2 - 3 \times 3 = 0$, $g(3) = 3 - 3 = 0$

∴ $f(3) = g(3) = 0$

15

∵ $f(\sqrt{2}) + g(2) = 5$ ∴ a + 2 + 1 = 5

∴ a + 3 = 5 ∴ a = 2

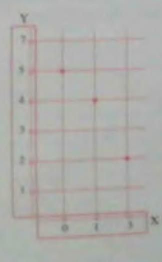
16

1 ∵ $f(0) = 5 - 0 = 5$

, $f(1) = 4$, $f(3) = 2$

∴ The range of f = {5 , 4 , 2}

2



17

1 The domain = {1 , 2 , 3 , 4 , 5}

2 The range = {3 , 5 , 7 , 9 , 11}

3 The rule of the function $f : f(X) = 2X + 1$

18

∵ The straight line intersects the X-axis at (2 , b)

∴ b = 0

, ∵ (2 , 0) belongs to the straight line

∴ 4 × 2 − a = 0 ∴ 8 − a = 0

∴ a = 8

19

∵ The straight line which represents the function f cuts the X-axis at (3 , 0)

∴ 0 = 3 a − 3 ∴ 3 a = 3

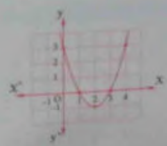
∴ a = 1 ∴ $f(X) = X - 3$

∴ $f(5) = 5 - 3 = 2$

20

$f(X) = X^2 - 4X + 3$

X	0	1	2	3	4
f (X)	3	0	−1	0	3



From the graph :

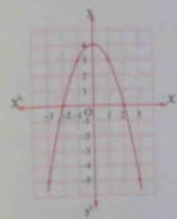
1 The minimum value = −1

2 The equation of axis of symmetry is : X = 2

21

$f(X) = 4 - X^2$

X	−3	−2	−1	0	1	2	3
f (X)	−5	0	3	4	3	0	−5



From the graph :

- The vertex of the curve is : (0 , 4)
- The maximum value = 4
- The equation of the symmetry axis is : X = 0

22

∵ $3f(2) + 3\ell(X) = 6$ ∴ $f(2) + \ell(X) = 2$

∴ $a + 2^2 + c = 2$ ∴ a + 4 + c = 2

∴ a + c = −2

∴ $2f(0) + 2\ell(7) = 2[f(0) + \ell(7)] = 2[a + 0^2 + c]$

$= 2(a + c) = 2 \times -2 = -4$

23

∵ $\overleftrightarrow{AB}$ represents the function $f : f(X) = 3$

, the point A ∈ y-axis ∴ A (0 , 3)

∴ OA = 3 length units.

, ∵ the area of Δ AOB = $\frac{1}{2}$ AB × OA

∴ 6 = $\frac{1}{2}$ AB × 3 ∴ $\frac{1}{2}$ AB = 2

∴ AB = 4 length units. ∴ B (−4 , 3)

, ∵ O (0 , 0) satisfies the function r (X)

∴ 0 = n × 0 + k ∴ k = 0

∴ r (X) = n X

, ∵ B (−4 , 3) satisfies the function r (X)

∴ 3 = −4 n ∴ n = $\frac{-3}{4}$

24

1 Let A (X , 0)

, ∵ A (X , 0) belongs to the straight line of the function f

∴ 4 − 2 X = 0 ∴ −2 X = −4

∴ X = 2 ∴ A (2 , 0)

Let B (0 , y)

, ∵ B (0 , y) belongs to the straight line of the function f

∴ y = 4 − 2 × 0 ∴ y = 4

∴ B (0 , 4)

2 The area of Δ AOB = $\frac{1}{2}$ OA × OB

= $\frac{1}{2}$ × 2 × 4 = 4 square units

25

1 ∵ The curve of the function intersects the X-axis at the two points A and C

∴ $0 = 9 - X^2$ ∴ $X^2 = 9$

∴ X = 3 or X = −3

∴ A = (3 , 0) , C = (−3 , 0)

2 ∵ AC = 6 length units.

, at X = 0 ∴ $f(X) = 9$

∴ B = (0 , 9)

∴ OB = 9 length units.

∴ The area of Δ ABC = $\frac{1}{2}$ AC × OB = $\frac{1}{2}$ × 6 × 9

= 27 square units.

26

1 ∵ A (−2 , m) satisfies the function f

∴ $m = (-2)^2 = 4$

, ∵ A (−2 , 4) satisfies the function g

∴ 4 = k − (−2) ∴ k = 2

2 Let B (n , 0)

, ∵ B (n , 0) satisfies the function g :

g (X) = 2 − X

∴ 0 = 2 − n ∴ n = 2

∴ B (2 , 0) ∴ OB = 2 length units

∴ The area of Δ AOB = $\frac{1}{2}$ × 2 × 4 = 4 square units

Unit two

First Answers of multiple choice questions

1 (b)	2 (a)	3 (d)	4 (a)	5 (c)
6 (c)	7 (c)	8 (d)	9 (c)	10 (d)
11 (d)	12 (c)	13 (b)	14 (a)	15 [illegible]
16 (d)	17 (d)	18 (d)	19 (d)	20 [illegible]
21 (c)	22 (c)			

Second Answers of essay questions

1

$\because \frac{x-2y}{x+3y}=\frac{3}{5}$ $\therefore 5x-10y=3x+9y$

$\therefore 5x-3x=9y+10y$ $\therefore 2x=19y$

$\therefore \frac{x}{y}=\frac{19}{2}$

2

$\because \frac{x}{y}=\frac{2}{3}$ $\therefore x=2m , y=3m$

$\therefore \frac{3x+2y}{6y-x}=\frac{6m+6m}{18m-2m}=\frac{12m}{16m}=\frac{3}{4}$

3

$\because \frac{a}{b}=\frac{c}{d}=m$ $\therefore a=bm , c=dm$

$\therefore \frac{a+2c}{b+2d}=\frac{bm+2dm}{b+2d}=\frac{m(b+2d)}{b+2d}=m$ (1)

$, \frac{c-a}{d-b}=\frac{dm-bm}{d-b}=\frac{m(d-b)}{d-b}=m$ (2)

From (1) and (2) : $\therefore \frac{a+2c}{b+2d}=\frac{c-a}{d-b}$

4

$\because \frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$

$\therefore c=dm , b=dm^2 , a=dm^3$

$\therefore \frac{a}{b+d}=\frac{dm^3}{dm^2+d}=\frac{dm^3}{d(m^2+1)}=\frac{m^3}{m^2+1}$ (1)

$, \frac{c^3}{c^2d+d^3}=\frac{d^3m^3}{d^3m^2+d^3}=\frac{d^3m^3}{d^3(m^2+1)}=\frac{m^3}{m^2+1}$ (2)

From (1) and (2) : $\therefore \frac{a}{b+d}=\frac{c^3}{c^2d+d^3}$

5

Let the number be x

$\therefore \frac{3+x}{5+x}=\frac{8+x}{12+x}$

$\therefore 40+13x+x^2=36+15x+x^2$

$\therefore 40-36=15x-13x$

$\therefore 4=2x$ $\therefore x=2$

$\therefore$ The required number = 2

6

$\because \frac{a}{b-a}=\frac{c}{d-c}$ $\therefore ad-ac=bc-ac$

$\therefore ad=bc$ $\therefore \frac{a}{b}=\frac{c}{d}$

$\therefore a , b , c , d$ are proportional quantities

7

$\because \frac{x}{4}=\frac{y}{5}=\frac{z}{3}=m$

$\therefore x=4m , y=5m , z=3m$

$\therefore$ L.H.S. $=\frac{x-y+z}{x+y-z}=\frac{4m-5m+3m}{4m+5m-3m}=\frac{2m}{6m}$

$=\frac{1}{3}=$ R.H.S.

8

$\because \frac{x}{3}=\frac{y}{4}=\frac{z}{5}=m$ (where $m>0$)

$\therefore x=3m , y=4m , z=5m$

$\therefore \sqrt{x^2+y^2}=\sqrt{9m^2+16m^2}=\sqrt{25m^2}=5m$ (1)

$, 2x+y-z=6m+4m-5m=5m$ (2)

From (1) and (2) :

$\therefore \sqrt{x^2+y^2}=2x+y-z$

9

$\because a:b:c=1:2:3$

$\therefore a=m , b=2m , c=3m$

$, \because b+c=25$ $\therefore 2m+3m=25$

$\therefore 5m=25$ $\therefore m=5$

$\therefore a=5 , b=2\times5=10 , c=3\times5=15$

10

$\because \frac{a}{2}=\frac{b}{3}=\frac{c}{4}=\frac{2a-2b+5c}{3x}$

, multiplying the two terms of the 1st ratio by 2 , the 2nd ratio by (– 2) and the 3rd ratio by 5 and adding the antecedents and consequents of the three ratios.

$\therefore \frac{2a-2b+5c}{4-6+20}=$ one of the given ratios.

$\therefore \frac{2a-2b+5c}{18}=\frac{2a-2b+5c}{3x}$

$\therefore 3x=18$ $\therefore x=6$

11

Let the number be x

$\therefore \frac{5+x}{11+x}=\frac{4}{7}$ $\therefore 35+7x=44+4x$

$\therefore 7x-4x=44-35$ $\therefore 3x=9$

$\therefore x=3$ $\therefore$ The number is : 3

12

Let the number be x $\therefore \frac{7+x^2}{11+x^2}=\frac{4}{5}$

$\therefore 35+5x^2=44+4x^2$

$\therefore 5x^2-4x^2=44-35$ $\therefore x^2=9$

$\therefore x=3$ or $x=-3$ (refused)

$\therefore$ The number is : 3

13

Let the two numbers be a and b

$\therefore \frac{a}{b}=\frac{2}{3}$ $\therefore a=2m , b=3m$

$\therefore \frac{2m+7}{3m-12}=\frac{5}{3}$ $\therefore 6m+21=15m-60$

$\therefore 21+60=15m-6m$ $\therefore 81=9m$

$\therefore m=9$

$\therefore$ The two numbers are 18 and 27

14

$\because \frac{a}{2x-y}=\frac{b}{2y-x}$

, multiplying the two terms of the 1st ratio by 2 and adding the antecedents and the consequents of the two ratios

$\therefore \frac{2a+b}{4x-2y+2y-x}=\frac{2a+b}{3x}$

= one of the given ratios (1)

, multiplying the two terms of the 2nd ratio by 2 and adding the antecedents and the consequents of the two ratios

$\therefore \frac{a+2b}{2x-y+4y-2x}=\frac{a+2b}{3y}$

= one of the given ratios (2)

From (1) and (2) : $\therefore \frac{2a+b}{3x}=\frac{a+2b}{3y}$

$\therefore \frac{2a+b}{x}=\frac{a+2b}{y}$

15

$\because \frac{x+y}{7}=\frac{y+z}{5}=\frac{z+x}{8}$

, adding the antecedents and consequents of the three ratios.

$\therefore \frac{x+y+y+z+z+x}{7+5+8}=\frac{2(x+y+z)}{20}=\frac{x+y+z}{10}$

= one of the given ratios. (1)

, multiplying the two terms of the 2nd ratio by (– 1) and adding the antecedents and consequents of the 1st and 2nd ratios.

$\therefore \frac{x+y-y-z}{7-5}=\frac{x-z}{2}=$ one of the given ratios. (2)

From (1) and (2) : $\therefore \frac{x+y+z}{10}=\frac{x-z}{2}$

$\therefore \frac{x+y+z}{x-z}=\frac{10}{2}=5$

16

$\because \frac{x}{2a+b}=\frac{y}{2b-c}=\frac{z}{2c-a}$

, multiplying the two terms of the 1st ratio by 2 and adding the antecedents and consequents of the 1st and the 2nd ratios.

$\therefore \frac{2x+y}{4a+2b+2b-c}=\frac{2x+y}{4a+4b-c}$

= one of the given ratios. (1)

, multiplying the two terms of the 1st ratio by 2 and the 2nd by 2 and adding the antecedents and consequents of the three ratios

$\therefore \frac{2x+2y+z}{4a+2b+4b-2c+2c-a}=\frac{2x+2y+z}{3a+6b}$

= one of the given ratios. (2)

From (1) and (2) :

$\therefore \frac{2x+y}{4a+4b-c}=\frac{2x+2y+z}{3a+6b}$

17

$\because \frac{x+y}{3}=\frac{y+z}{8}=\frac{z+x}{6}$

, adding the antecedents and consequents of the three ratios.

$\therefore \frac{x+y+y+z+z+x}{3+8+6}=\frac{2(x+y+z)}{17}$

= one of the given ratios. (1)

, multiplying the two terms of the 2nd ratio by 2 and adding the antecedents and consequents of the three ratios.

$\therefore \frac{x+y+2y+2z+z+x}{3+16+6}=\frac{2x+3y+3z}{25}$

= one of the given ratios.(2)

From (1) and (2) : $\therefore \frac{2(x+y+z)}{17}=\frac{2x+3y+3z}{25}$

$\therefore \frac{x+y+z}{2x+3y+3z}=\frac{17}{2\times25}=\frac{17}{50}$

18

$\because \frac{x+y}{5}=\frac{y+z}{8}=\frac{z+x}{7}$

, multiplying the two terms of the 2nd ratio by (– 1) and adding the antecedents and consequents of the three ratios.

$\therefore \frac{x+y-y-z+z+x}{5-8+7}=\frac{2x}{4}=\frac{x}{2}$

= one of the given ratios (1)

, multiplying the two terms of the 3rd ratio by (– 1) and adding the antecedents and consequents of the three ratios

$\therefore \frac{[illegible]}{[illegible]} = \frac{2z}{z} = \frac{?}{?}$

= one of the given ratios (2)

, multiplying the two terms of the 1^{st} ratio by (−1) and adding the antecedents and consequents of the three ratios

$\therefore \frac{-X - y + y + z + z + X}{-5 + 8 + 7} = \frac{2z}{10} = \frac{z}{5}$

= one of the given ratios (3)

From (1), (2) and (3) : $\therefore \frac{X}{2} = \frac{y}{3} = \frac{z}{5}$

19

$\because$ b is the middle proportional between a and c

$\therefore b^2 = ac$

$\therefore$ L.H.S. $= \frac{a^2 + b^2}{b^2 + c^2} = \frac{a^2 + ac}{ac + c^2} = \frac{a(a + c)}{c(a + c)} = \frac{a}{c}$ = R.H.S.

20

1 $\because y \propto X$ $\therefore y = m X$

$\therefore 20 = 4 m$ $\therefore m = 5$

$\therefore y = 5 X$

2 When y = 40 $\therefore 40 = 5 X$

$\therefore X = 8$

21

1 $\because y \propto \frac{1}{X}$ $\therefore X y = m$

$\therefore 3 \times 2 = m$ $\therefore m = 6$

$\therefore X y = 6$

2 When X = 1.5 $\therefore 1.5 y = 6$

$\therefore y = 4$

22

$\because \frac{21 X - y}{7 X - z} = \frac{7}{z}$ $\therefore 21 X z - y z = 7 X y - y z$

$\therefore 21 X z = 7 X y$ $\therefore 3 z = y$

$\therefore y \propto z$

23

$\because X = z + 8$, $z \propto \frac{1}{y}$ $\therefore z = \frac{m}{y}$

$\therefore 2 = \frac{m}{3}$ $\therefore m = 6$

$\therefore z = \frac{6}{y}$ $\therefore X = \frac{6}{y} + 8$

When X = 3 $\therefore 3 = \frac{6}{y} + 8$

$\therefore \frac{6}{y} = -5$ $\therefore y = -\frac{6}{5}$

24

$\because y = 1 + b$, $b \propto \frac{1}{X^2}$ $\therefore b = \frac{m}{X^2}$

$\therefore y = 1 + \frac{m}{X^2}$ $\therefore 5 = 1 + \frac{m}{2^2}$

$\therefore 4 = \frac{m}{4}$ $\therefore m = 16$

$\therefore y = 1 + \frac{16}{X^2}$

At X = 4 $\therefore y = 1 + \frac{16}{4^2} = 1 + 1 = 2$

25

$\because X^4 y^2 - 14 X^2 y + 49 = 0$

$\therefore (X^2 y - 7)^2 = 0$

$\therefore X^2 y - 7 = 0$ $\therefore X^2 y = 7$

$\therefore y \propto \frac{1}{X^2}$

26

1 The variation is inverse.

2 $\because y \propto \frac{1}{X}$ $\therefore X y = m$ $\therefore m = 12$

3 At X = 3 $\therefore 3 y = 12$ $\therefore y = 4$

4 At $y = 2\frac{2}{5}$ $\therefore (2\frac{2}{5}) X = 12$

$\therefore \frac{12}{5} X = 12$ $\therefore X = 5$

Unit three

First Answers of multiple choice questions

1 (c) **2** (c) **3** (a) **4** (b) **5** (b)
6 (d) **7** (c) **8** (b) **9** (c) **10** (b)
11 (d) **12** (c) **13** (c) **14** (a) **15** (a)
16 (b)

Second Answers of essay questions

1

Form the table by yourself

, then the mean $(\bar{X}) = 16$, $\sigma \simeq 3.29$

2

Form the tables by yourself

, then $\sigma \simeq 1.73$

3

Form the table by yourself

, then the mean $(\bar{X}) = 11.6$, $\sigma \simeq 5.66$

Answers of model examinations of the school book of algebra & statistics

Model 1

1

1 b 2 c 3 b 4 a 5 c 6 b

2

[a] 1 $Y = \{2, 5, 7\}$

2 $Y \times X = \{(2, 2), (5, 2), (7, 2)\}$

[b] Let $\frac{a}{b} = \frac{c}{d} = m$, where $m > 0$

$\therefore a = b m$, $c = d m$

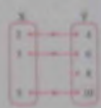
$\therefore$ L.H.S. $= \frac{a}{b - a} = \frac{b m}{b - b m} = \frac{b m}{b(1 - m)} = \frac{m}{1 - m}$ (1)

, R.H.S. $= \frac{c}{d - c} = \frac{d m}{d - d m} = \frac{d m}{d(1 - m)} = \frac{m}{1 - m}$ (2)

From (1), (2) : $\therefore \frac{a}{b - a} = \frac{c}{d - c}$

3

[a] 1 $R = \{(2, 4), (3, 6), (5, 10)\}$



2 R is a function because every element of X has only one image in Y

[b] Let the number be X

$\therefore \frac{X + 7}{X + 11} = \frac{2}{3}$

$\therefore 3 X + 21 = 2 X + 22$ $\therefore X = 1$

$\therefore$ The required number is 1

4

[a] 1 The range $= \{3, 1, 5\}$

2 a + b = 8

[b] 1 $\because y \propto \frac{1}{X}$ $\therefore X y = m$

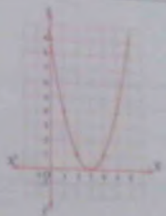
$\therefore m = 2 \times 3 = 6$ $\therefore X y = 6$

2 At X = 1.5 $\therefore y = \frac{6}{1.5} = 4$

5

[a] $f(X) = (X - 3)^2$

X	0	1	2	3	4	5	6
f(X)	9	4	1	0	1	4	9



From the graph :

The vertex of the curve is (3, 0)

, the minimum value = 0

, the equation of the axis of symmetry is : X = 3

[b] Form the table by yourself

, then the arithmetic mean = 7

, the standard deviation ≃ 1.41

Model 2

1

1 a 2 c 3 d 4 b 5 c 6 a

2

[a] 1 $n(X \times Z) = 2$

2 $(Y \cap X) \times Z = \{2\} \times \{3\} = \{(2, 3)\}$

[b] $\because$ b is the middle proportional between a and c

$\therefore \frac{a}{b} = \frac{b}{c} = m$ $\therefore b = c m$, $a = c m^2$

$\therefore$ L.H.S. $= \frac{a - b}{a - c} = \frac{c m^2 - c m}{c m^2 - c} = \frac{c m(m - 1)}{c(m^2 - 1)}$

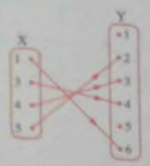
$= \frac{c m(m - 1)}{c(m - 1)(m + 1)} = \frac{m}{m + 1}$ (1)

, R.H.S. $= \frac{b}{b + c} = \frac{c m}{c m + c} = \frac{c m}{c(m + 1)} = \frac{m}{m + 1}$ (2)

From (1), (2) $\therefore \frac{a - b}{a - c} = \frac{b}{b + c}$

3

[a] ① $R = \{(1, 6), (3, 4), (4, 3), (5, 2)\}$



② R is a function because every element of X has only one image in Y

[b] $\because 5a = 3b \qquad \therefore \frac{a}{b} = \frac{3}{5}$

$\therefore a = 3m, b = 5m$

$\therefore \frac{7a + 9b}{4a + 2b} = \frac{7 \times 3m + 9 \times 5m}{4 \times 3m + 2 \times 5m} = \frac{66m}{22m} = 3$

4

[a] $\because f(X) = 4X + b, f(3) = 15$

$\therefore 4 \times 3 + b = 15 \qquad \therefore b = 3$

[b] ① $\because y \propto X \qquad \therefore y = mX$

$\therefore 6 = m \times 3 \qquad \therefore m = 2$

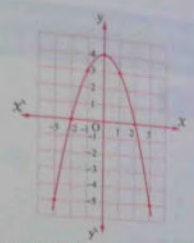
$\therefore y = 2X$

② At $X = 5 \qquad \therefore y = 2 \times 5 = 10$

5

[a] $f(X) = 4 - X^2$

X	-3	-2	-1	0	1	2	3
$f(X)$	-5	0	3	4	3	0	-5



From the graph : The vertex of the curve is $(0, 4)$

, the maximum value $= 4$

, the equation of the axis of symmetry is : $X = 0$

[b] Form the tables by yourself

, then the mean $= 2.26$

, the standard deviation $\simeq 1.06$

Model for the merge students

1

① the first ② the third ③ 30

④ X ⑤ 9 ⑥ 9

2

① a ② a ③ d

④ b ⑤ c ⑥ c

3

① ✔ ② ✘ ③ ✘

④ ✔ ⑤ ✔ ⑥ ✔

4

① 1 ② 6 ③ 8

④ 10 ⑤ ± 6 ⑥ 2

Answers of governorates' examinations of algebra & statistics

1 Cairo

1

① d ② d ③ a ④ b ⑤ a ⑥ b

2

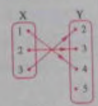
[a] Let the number be : X

$\therefore \frac{5 + X}{11 + X} = \frac{4}{7} \qquad \therefore 35 + 7X = 44 + 4X$

$\therefore 7X - 4X = 44 - 35 \qquad \therefore 3X = 9$

$\therefore X = 3 \qquad \therefore$ The number is : 3

[b] ① $R = \{(1, 4), (2, 3), (3, 2)\}$



② R is a function because every element in X has only one image in Y

3

[a] Let the fourth proportional be : X

$\therefore \frac{3}{5} = \frac{6}{X} \qquad \therefore X = \frac{6 \times 5}{3} = 10$

[b] ① $Y = \{1, 4, 5\}$

② $Y \times X = \{(1, 2), (4, 2), (5, 2)\}$

③ $n(Y^2) = 9$

4

[a] ① $\because y \propto \frac{1}{X} \qquad \therefore Xy = m \qquad \therefore 3 \times 4 = m$

$\therefore m = 12 \qquad \therefore Xy = 12$

② When $X = 6 \qquad \therefore 6y = 12 \qquad \therefore y = 2$

[b] $\because \frac{X}{2} = \frac{y}{3} = \frac{z}{5} = m$

$\therefore X = 2m, y = 3m, z = 5m$

$\therefore \frac{2X + y}{7} = \frac{4m + 3m}{7} = \frac{7m}{7} = m \qquad (1)$

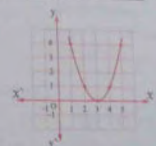
$, \frac{2y + z}{11} = \frac{6m + 5m}{11} = \frac{11m}{11} = m \qquad (2)$

From (1) and (2) : $\therefore \frac{2X + y}{7} = \frac{2y + z}{11}$

5

[a] $f(X) = (X - 3)^2$

X	1	2	3	4	5
$f(X)$	4	1	0	1	4



From the graph :

① The equation of the axis of symmetry is : $X = 3$

② The minimum value $= 0$

[b] Form the table by yourself , then $\sigma \simeq 1.41$

2 Giza

1

① a ② c ③ d ④ a ⑤ d ⑥ c

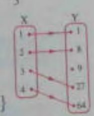
2

[a] $\because \frac{X}{3} = \frac{y}{4} = \frac{c}{5} = m$

$\therefore X = 3m, y = 4m, c = 5m$

$\therefore \frac{2X + 3y}{7c - 2y} = \frac{6m + 12m}{35m - 8m} = \frac{18m}{27m} = \frac{2}{3}$

[b] ① $R = \{(1, 1), (2, 8), (3, 27), (4, 64)\}$



② R is a function

, the range $= \{1, 8, 27, 64\}$

3

[a] ① $\because y \propto X \qquad \therefore y = mX \qquad \therefore 6 = 2m$

$\therefore m = 3 \qquad \therefore y = 3X$

② When $X = 5 \qquad \therefore y = 3 \times 5 = 15$

[b] $\because \frac{a}{b} = \frac{b}{c} = m \qquad \therefore b = cm, a = cm^2$

$\therefore \frac{a - b}{a - c} = \frac{cm^2 - cm}{cm^2 - c} = \frac{cm(m - 1)}{c(m^2 - 1)}$

$= \frac{cm(m - 1)}{c(m - 1)(m + 1)} = \frac{m}{m + 1} \qquad (1)$

$, \frac{b}{b+c} = \frac{c\,m}{c\,m+c} = \frac{c\,m}{c\,(m+1)} = \frac{m}{m+1}$ (2)

From (1) and (2) : $\therefore \frac{a-b}{a-c} = \frac{b}{b+c}$

4

[a] $\because (2X-1 , X+y) = (5 , 8)$

$\therefore 2X-1=5$ $\therefore 2X=6$ $\therefore X=3$

$\therefore 3+y=8$ $\therefore y=5$

[b] $\because \frac{X-2y}{X+3y} = \frac{3}{5}$

$\therefore 5X-10y=3X+9y$

$\therefore 5X-3X=9y+10y$ $\therefore 2X=19y$

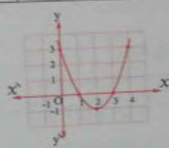
$\therefore \frac{X}{y} = \frac{19}{2}$

5

[a] Form the table by yourself , then the mean $(\overline{X})=5$, $\sigma \simeq 2.24$

[b] $f(X)=X^2-4X+3$

X	0	1	2	3	4
$f(X)$	3	0	−1	0	3



From the graph :

1 The minimum value = −1

2 The equation of the axis of symmetry is : $X=2$

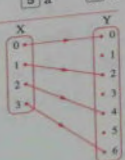
3 Alexandria

1

1 b 2 b 3 d 4 b 5 a 6 d

2

[a] 1 $R=\{(0,0),(1,2),(2,4),(3,6)\}$



2 R is a function because every element in X has only one image in Y

[b] $\because \frac{X}{3}=\frac{y}{4}=\frac{z}{5}=m$

$\therefore X=3m , y=4m , z=5m$

$\therefore \text{L.H.S.} = \frac{2y-z}{3X-2y+z} = \frac{8m-5m}{9m-8m+5m}$

$= \frac{3m}{6m} = \frac{1}{2} = \text{R.H.S.}$

3

[a] $f(\sqrt{2})+3g(\sqrt{2}) = (\sqrt{2})^2 - 3\sqrt{2} + 3(\sqrt{2}-3)$

$= 2-3\sqrt{2}+3\sqrt{2}-9=-7$

[b] Let the number be : X

$\therefore \frac{5+X^2}{11+X^2}=\frac{3}{5}$ $\therefore 25+5X^2=33+3X^2$

$\therefore 5X^2-3X^2=33-25$

$\therefore 2X^2=8$ $\therefore X^2=4$

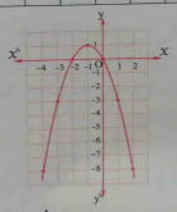
$\therefore X=2$ or $X=-2$ (refused)

$\therefore$ The number is : 2

4

[a] $f(X)=-X^2-2X$

X	−4	−3	−2	−1	0	1	2
$f(X)$	−8	−3	0	1	0	−3	−8



From the graph :

1 The vertex of the curve is : $(-1 , 1)$

2 The equation of the axis of symmetry is : $X=-1$

3 The maximum value = 1

[b] $\because \frac{a}{b}=\frac{b}{c}=\frac{c}{d}=m$

$\therefore c=dm$, $b=dm^2$, $a=dm^3$

$\therefore \frac{a^2-3c^2}{b^2-3d^2} = \frac{d^2m^6-3d^2m^2}{d^2m^4-3d^2} = \frac{d^2m^2(m^4-3)}{d^2(m^4-3)}$

$=m^2$ (1)

$, \frac{b}{d} = \frac{d\,m^2}{d} = m^2$ (2)

From (1) and (2) : $\therefore \frac{a^2-3c^2}{b^2-3d^2}=\frac{b}{d}$

5

[a] 1 $\because y \propto \frac{1}{X}$ $\therefore Xy=m$ $\therefore 2\times3=m$

$\therefore m=6$ $\therefore Xy=6$

2 When $X=1.5$ $\therefore 1.5y=6$

$\therefore y=4$

[b] Form the table by yourself , then $\sigma \simeq 3.286$

4 El-Kalyoubia

1

1 b 2 a 3 c 4 c 5 d 6 c

2

[a] 1 $X\times Y=\{(2,-1),(2,5),(-1,-1),(-1,5)\}$

2 $(X-Y)\times Z=\{2\}\times\{2,3\}$

$=\{(2,2),(2,3)\}$

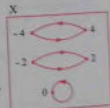
[b] 1 $\because y\propto X$ $\therefore y=mX$ $\therefore 5=15m$

$\therefore m=\frac{1}{3}$ $\therefore y=\frac{1}{3}X$

2 When $X=30$ $\therefore y=\frac{1}{3}\times30=10$

3

[a] $R=\{(-4,4),(-2,2),(0,0),(2,-2),(4,-4)\}$



, R is a function because every element in X has only one image in X

[b] $\because \frac{a}{b}=\frac{c}{d}=m$ $\therefore a=bm , c=dm$

$\therefore \text{L.H.S} = \frac{a+b}{c+d} = \frac{bm+b}{dm+d} = \frac{b(m+1)}{d(m+1)} = \frac{b}{d} = \text{R.H.S.}$

4

[a] Let the number be : X

$\therefore \frac{7+X}{11+X}=\frac{4}{5}$ $\therefore 35+5X=44+4X$

$\therefore 5X-4X=44-35$ $\therefore X=9$

$\therefore$ The number is : 9

[b] $\because \frac{2}{a}=\frac{a}{b}=\frac{b}{54}=m$

$\therefore b=54m , a=54m^2 , 2=54m^3$

$\therefore m^3=\frac{2}{54}=\frac{1}{27}$ $\therefore m=\frac{1}{3}$

$\therefore a=54\times\left(\frac{1}{3}\right)^2=6 , b=54\times\frac{1}{3}=18$

$\therefore a+b=6+18=24$

5

[a] $f(X)=X^2+2X-3$

X	−4	−3	−2	−1	0	1	2
$f(X)$	5	0	−3	−4	−3	0	5

From the graph :

1 The minimum value = −4

2 The equation of the axis of symmetry is : $X=-1$

[b] Form the table by yourself , then the mean $(\overline{X})=16$, $\sigma \simeq 3.29$

5 El-Sharkia

1

1 b 2 c 3 d 4 b 5 c 6 b

2

[a] $\because \frac{a}{3}=\frac{b}{2}=\frac{c}{5}=m$

$\therefore a=3m , b=2m , c=5m$

$\therefore \text{L.H.S.} = \frac{a-2b+3c}{2a+b+c} = \frac{3m-4m+15m}{6m+2m+5m}$

$=\frac{14m}{13m}=\frac{14}{13}=\text{R.H.S.}$

[b] 1 $X=\{1,2,3\} , Y=\{2,3\}$

2 $(X\cap Y)\times Y=\{2,3\}\times\{2,3\}$

$=\{(2,2),(2,3),(3,2),(3,3)\}$

3

[a] $\because a \propto X$ $\therefore a = m X$ $\therefore 4 = 2 m$

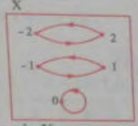
$\therefore m = 2$ $\therefore a = 2 X$

$\therefore y = 2 X + 2$

At $X = 1$ $\therefore y = 2 \times 1 + 2 = 4$

[b] $X = \{-2 , -1 , 0 , 1 , 2\}$

$, R = \{(-2 , 2) , (-1 , 1) , (0 , 0) , (1 , -1) , (2 , -2)\}$



R is a function because every element in X has only one image in X

4

[a] $\because \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = d m \quad , b = d m^2 \quad , a = d m^3$

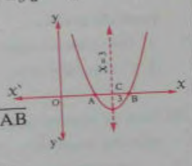
$\therefore \frac{a^2 - 3 c^2}{b^2 - 3 d^2} = \frac{d^2 m^6 - 3 d^2 m^2}{d^2 m^4 - 3 d^2} = \frac{d^2 m^2 (m^4 - 3)}{d^2 (m^4 - 3)} = m^2$ (1)

$, \frac{b}{d} = \frac{d m^2}{d} = m^2$ (2)

From (1) and (2) : $\therefore \frac{a^2 - 3 c^2}{b^2 - 3 d^2} = \frac{b}{d}$

[b] $\because$ The equation of the axis of symmetry is :

$X = \frac{-b}{2 a} = \frac{6}{2} = 3$



$, \because$ The axis of symmetry bisects $\overline{AB}$

$\therefore$ CA = 1 unit

$\therefore$ AO = 3 − 1 = 2 units

$\therefore A (2 , 0)$

$, \because A (2 , 0)$ satisfies the function

$\therefore 0 = 2^2 - 6 \times 2 + m$ $\therefore 0 = 4 - 12 + m$

$\therefore m = 8$

$\therefore f (X) = X^2 - 6 X + 8$

$\therefore$ The minimum value $= f \left(\frac{-b}{2 a}\right) = f (3) = 3^2 - 6 \times 3 + 8 = -1$

5

[a] Let the number be : X

$\therefore \frac{3 + X}{5 + X} = \frac{8 + X}{12 + X}$

$\therefore (3 + X)(12 + X) = (8 + X)(5 + X)$

$\therefore 36 + 15 X + X^2 = 40 + 13 X + X^2$

$\therefore 15 X - 13 X = 40 - 36$

$\therefore 2 X = 4$ $\therefore X = 2$

$\therefore$ The number is : 2

[b] Form the table by yourself

, then the mean $(\overline{X}) = 16$, $\sigma \simeq 3.29$

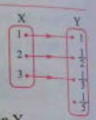
6 El-Gharbia

1

1 c 2 b 3 b 4 a 5 a 6 d

2

[a] $R = \left\{(1 , 1) , \left(2 , \frac{1}{2}\right) , \left(3 , \frac{1}{3}\right)\right\}$



R is a function because every element in X has only one image in Y

[b] $\because$ b is the middle proportional between a and c

$\therefore b^2 = a c$

$\therefore L.H.S. = \frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{a^2}{a c} + \frac{a c}{c^2} = \frac{a}{c} + \frac{a}{c} = \frac{2 a}{c} = R.H.S$

3

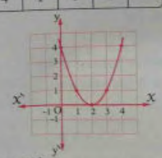
[a] $\because y \propto \frac{1}{X}$ $\therefore X y = m$ $\therefore 3 \times 10 = m$

$\therefore m = 30$ $\therefore X y = 30$

When $X = 5$ $\therefore 5 y = 30$ $\therefore y = 6$

[b] $f (X) = (X - 2)^2$

X	0	1	2	3	4
$f (X)$	4	1	0	1	4



From the graph :

1 The vertex of the curve is : (2 , 0)

2 The equation of the axis of symmetry is : $X = 2$

4

[a] Let the number be : X

$\therefore \frac{3 + X}{7 + X} = \frac{1}{2}$ $\therefore 6 + 2 X = 7 + X$

$\therefore 2 X - X = 7 - 6$ $\therefore X = 1$

$\therefore$ The number is : 1

[b] 1 $X = \{1\}$, $Y = \{1 , 3 , 5\}$

2 $Y \times X = \{(1 , 1) , (3 , 1) , (5 , 1)\}$

3 $X^2 = \{(1 , 1)\}$

5

[a] $\because 5 a = 3 b$ $\therefore \frac{a}{b} = \frac{3}{5}$

$\therefore a = 3 m \quad , b = 5 m$

$\therefore \frac{7 a + 9 b}{4 a + 2 b} = \frac{21 m + 45 m}{12 m + 10 m} = \frac{66 m}{22 m} = 3$

[b] Form the table by yourself

, then the mean $(\overline{X}) = 8$, $\sigma \simeq 2.8$

7 El-Dakahlia

1

[a] 1 a 2 a 3 d

[b] 1 $(X \cap Y) \times Z = \{1\} \times \{3 , 4\} = \{(1 , 3) , (1 , 4)\}$

2 $(X \cup Y) \times (Z - Y) = \{1 , 2 , 3\} \times \{4\} = \{(1 , 4) , (2 , 4) , (3 , 4)\}$

2

[a] 1 c 2 a 3 d

[b] $\because$ b is the middle proportional between a and c

$\therefore b^2 = a c$

$\therefore L.H.S = \frac{a^2 + b^2}{b^2 + c^2} = \frac{a^2 + a c}{a c + c^2} = \frac{a (a + c)}{c (a + c)} = \frac{a}{c} = R.H.S$

3

[a] Form the table by yourself , then $\sigma \simeq 1.41$

[b] $R = \{(-1 , 1) , (0 , 0) , (1 , 1)\}$

R is a function because every element in X has only one image in X

, the range $= \{1 , 0\}$

4

[a] $\because \frac{X + y}{9} = \frac{y + z}{7}$

multiplying the two terms of the 2nd ratio by − 1 and adding the antecedents and consequents of the two ratios

$\therefore \frac{X + y - y - z}{9 - 7} = \frac{X - z}{2}$

= one of the given ratios (1)

, adding the antecedents and consequents of the two ratios

$\therefore \frac{X + y + y + z}{9 + 7} = \frac{X + 2 y + z}{16}$

= one of the given ratios (2)

From (1) and (2) : $\therefore \frac{X - z}{2} = \frac{X + 2 y + z}{16}$

$\therefore \frac{X - z}{X + 2 y + z} = \frac{2}{16} = \frac{1}{8}$

[b] $\because d \propto t$ $\therefore \frac{d_1}{d_2} = \frac{t_1}{t_2}$

$\therefore \frac{150}{d_2} = \frac{6}{10}$ $\therefore d_2 = \frac{150 \times 10}{6} = 250$ km.

5

[a] $\because X^4 y^2 - 14 X^2 y + 49 = 0$

$\therefore (X^2 y - 7)^2 = 0$ $\therefore X^2 y - 7 = 0$

$\therefore X^2 y = 7$ $\therefore y \propto \frac{1}{X^2}$

[b] 1 $\because A (-2 , m)$ satisfies $f (X)$

$\therefore m = (-2)^2 = 4$

$, \because A (-2 , 4)$ satisfies $g (X)$

$\therefore 4 = k - (-2)$ $\therefore 4 = k + 2$

$\therefore k = 2$

2 $\because g (X) = 2 - X$, $B (X , 0)$ satisfies $g (X)$

$\therefore 0 = 2 - X$ $\therefore X = 2$

$\therefore$ BO = 2 length units

$\therefore$ The area of $\Delta AOB = \frac{1}{2} \times 2 \times 4$ = 4 square units.

8 Kafr El-Sheikh

1

1 c 2 a 3 b 4 b 5 c 6 d

2

[a] 1 $R = \{(1 , 6) , (3 , 4) , (4 , 3) , (5 , 2)\}$

2 R is a function because every element in X has only one image in Y

[b] $\because \frac{a}{b} = \frac{3}{5}$ $\therefore a = 3 m , b = 5 m$

$\therefore \frac{4 a + 2 b}{7 a + 9 b} = \frac{12 m + 10 m}{21 m + 45 m} = \frac{22 m}{66 m} = \frac{1}{3}$

3

[a] $\because \frac{a}{b} = \frac{c}{d} = m$ $\therefore a = bm , c = dm$

$\therefore \frac{a-b}{c-d} = \frac{bm-b}{dm-d} = \frac{b(m-1)}{d(m-1)} = \frac{b}{d}$ (1)

$, \frac{a}{c} = \frac{bm}{dm} = \frac{b}{d}$ (2)

From (1) and (2) : $\therefore \frac{a-b}{c-d} = \frac{a}{c}$

[b] 1 $\because y \propto X$ $\therefore y = mX$

$\therefore 10 = 5m$ $\therefore m = 2$ $\therefore y = 2X$

2 When $X = 3$ $\therefore y = 2 \times 3 = 6$

4

[a] Form the table by yourself

, then the mean $(\overline{X}) = 8 , \sigma = 4$

[b] 1 $\because f(1) = 7$ $\therefore 7 = 2 \times 1 + c$ $\therefore c = 5$

$\therefore f(X) = 2X + 5$

2 $f(2) = 2 \times 2 + 5 = 9$

5

[a] $\because$ b is the middle proportional between a and c

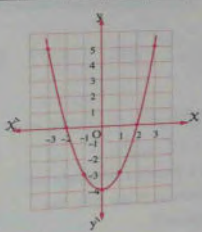
$\therefore b^2 = ac$

$\therefore$ L.H.S $= \frac{b^2 + c^2}{a^2 + b^2} = \frac{ac + c^2}{a^2 + ac} = \frac{c(a+c)}{a(a+c)}$

$= \frac{c}{a} =$ R.H.S.

[b] $f(X) = X^2 - 4$

X	-3	-2	-1	0	1	2	3
$f(X)$	5	0	-3	-4	-3	0	5



From the graph :

The vertex of the curve is : $(0 , -4)$

9 El-Beheira

1

1 b 2 c 3 c 4 c 5 a 6 d

2

[a] Let the number be : X

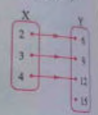
$\therefore \frac{5 + X^2}{7 + X^2} = \frac{7}{8}$ $\therefore 40 + 8X^2 = 49 + 7X^2$

$\therefore 8X^2 - 7X^2 = 49 - 40$

$\therefore X^2 = 9$ $\therefore X = 3$ or $X = -3$ (refused)

$\therefore$ The number is : 3

[b] $R = \{(2 , 6)$
$, (3 , 9)$
$, (4 , 12)\}$



R is a function because every element in X has only one image in Y

3

[a] $\because y \propto X$ $\therefore y = mX$

$\therefore 6 = 3m$ $\therefore m = 2$ $\therefore y = 2X$

When $X = 5$ $\therefore y = 2 \times 5 = 10$

[b] $\because \frac{X}{2} = \frac{y}{5} = \frac{z}{7} = m$

$\therefore X = 2m , y = 5m , z = 7m$

$\therefore$ L.H.S. $= \frac{5y - 3z}{2z - 3X} = \frac{25m - 21m}{14m - 6m} = \frac{4m}{8m}$

$= \frac{1}{2} =$ R.H.S.

4

[a] 1 $X \times (Y \cap Z) = \{3 , 4\} \times \{5\}$

$= \{(3 , 5) , (4 , 5)\}$

2 $(X - Y) \times Z = \{3\} \times \{5 , 6 , 7\}$

$= \{(3 , 5) , (3 , 6) , (3 , 7)\}$

3 $n(Z^2) = 3 \times 3 = 9$

[b] $\because$ b is the middle proportional between a and c

$\therefore b^2 = ac$

$\therefore$ L.H.S. $= \frac{a^2 + b^2}{b^2 + c^2} = \frac{a^2 + ac}{ac + c^2} = \frac{a(a+c)}{c(a+c)}$

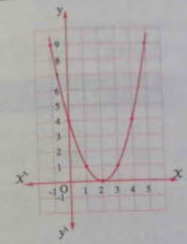
$= \frac{a}{c} =$ R.H.S.

5

[a] Form the table by yourself , then $\sigma \simeq 9.32$

[b] $f(X) = (X - 2)^2$

X	-1	0	1	2	3	4	5
$f(X)$	9	4	1	0	1	4	9



From the graph :

1 The vertex of the curve is : $(2 , 0)$

2 The minimum value $= 0$

, the equation of the axis of symmetry is : $X = 2$

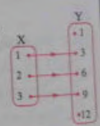
10 El-Menia

1

1 b 2 c 3 d 4 d 5 b 6 b

2

[a] 1 $R = \{(1 , 3) , (2 , 6) , (3 , 9)\}$



2 R is a function because every element in X has only one image in Y

[b] 1 $\because y \propto X$ $\therefore y = mX$

$\therefore 14 = 42m$ $\therefore m = \frac{1}{3}$ $\therefore y = \frac{1}{3}X$

2 When $X = 60$ $\therefore y = \frac{1}{3} \times 60 = 20$

3

[a] $\because (2 , b)$ is the intersection point of the straight line with the X-axis

$\therefore b = 0$

$\therefore (2 , 0)$ satisfies the function

$\therefore 0 = 4 \times 2 + a$ $\therefore a = -8$

[b] Form the table by yourself , then $\sigma \simeq 1.41$

4

[a] $\because \frac{a}{b} = \frac{b}{c} = m$ $\therefore b = cm , a = cm^2$

$\therefore \left(\frac{b-c}{a-b}\right)^2 = \left(\frac{cm - c}{cm^2 - cm}\right)^2 = \left(\frac{c(m-1)}{cm(m-1)}\right)^2$

$= \frac{1}{m^2}$ (1)

$, \frac{c}{a} = \frac{c}{cm^2} = \frac{1}{m^2}$ (2)

From (1) and (2) :

$\therefore \left(\frac{b-c}{a-b}\right)^2 = \frac{c}{a}$

[b] $\because X^4 y^2 - 14 X^2 y + 49 = 0$

$\therefore (X^2 y - 7)^2 = 0$ $\therefore X^2 y - 7 = 0$

$\therefore X^2 y = 7$ $\therefore y \propto \frac{1}{X^2}$

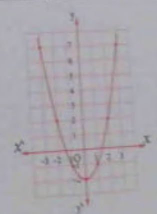
5

[a] $\because \frac{a}{b} = \frac{3}{5}$ $\therefore a = 3m , b = 5m$

$\therefore \frac{20a - 7b}{15a + b} = \frac{60m - 35m}{45m + 5m} = \frac{25m}{50m} = \frac{1}{2}$

[b] $f(X) = X^2 - 2$

X	-3	-2	-1	0	1	2	3
$f(X)$	7	2	-1	-2	-1	2	7



From the graph :

- The vertex of the curve is : $(0 , -2)$
- The minimum value $= -2$

11 Souhag

1

1 c 2 a 3 d 4 b 5 a 6 c

2

[a] 1 $(X \cap Y) \times Z = \varnothing \times \{6 , 5\} = \varnothing$

2 $X \times (Y - Z) = \{3\} \times \{4\} = \{(3 , 4)\}$

3 $n(X^2) = 1 \times 1 = 1$

[b] $\because \frac{x - 3y}{x + 2y} = \frac{2}{3}$ $\quad \therefore 3x - 9y = 2x + 4y$

$\therefore 3x - 2x = 4y + 9y$ $\quad \therefore x = 13y$

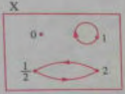
$\therefore \frac{x}{y} = 13$

3

[a] $\because \frac{a}{b} = \frac{c}{d} = m$ $\quad \therefore a = bm , c = dm$

$\therefore \text{L.H.S.} = \frac{3a - 2c}{5a + 3c} = \frac{3bm - 2dm}{5bm + 3dm} = \frac{m(3b - 2d)}{m(5b + 3d)} = \frac{3b - 2d}{5b + 3d} = \text{R.H.S.}$

[b] 1 $R = \{(1 , 1) , (2 , \frac{1}{2}) , (\frac{1}{2} , 2)\}$



2 R is not a function because $0 \in X$ has no image in X

4

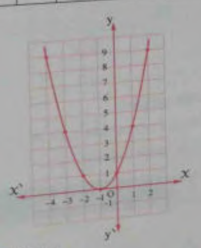
[a] $\because \frac{a}{2} = \frac{2}{4} = \frac{4}{b}$ $\quad \therefore 4a = 4$ $\quad \therefore a = 1$

$, 2b = 16$ $\quad \therefore b = 8$

$\therefore a + b = 1 + 8 = 9$

[b] $f(x) = (x + 1)^2$

x	-4	-3	-2	-1	0	1	2
$f(x)$	9	4	1	0	1	4	9



From the graph :

1 The vertex of the curve is : $(-1 , 0)$

2 The minimum value $= 0$

3 The equation of the axis of symmetry is : $x = -1$

5

[a] 1 $\because y \propto x$ $\quad \therefore y = mx$

$\therefore 20 = 4m$ $\quad \therefore m = 5$ $\quad \therefore y = 5x$

2 When $y = 40$ $\quad \therefore 40 = 5x$ $\quad \therefore x = 8$

[b] Form the tables by yourself

, then $\sigma \simeq 1.73$ years

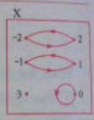
12 Qena

1

1 c 2 d 3 c 4 a 5 b 6 a

2

[a] $R = \{(-2 , 2) , (-1 , 1) , (0 , 0) , (1 , -1) , (2 , -2)\}$



R is not a function

[b] $\because \frac{a}{b} = \frac{b}{c} = m$ $\quad \therefore b = cm \quad , \quad a = cm^2$

$\therefore \frac{a^3 + b^3}{b^3 + c^3} = \frac{c^3 m^6 + c^3 m^3}{c^3 m^3 + c^3} = \frac{c^3 m^3 (m^3 + 1)}{c^3 (m^3 + 1)} = m^3$ (1)

$, \frac{a^2}{bc} = \frac{c^2 m^4}{cm \times c} = m^3$ (2)

From (1) and (2) : $\therefore \frac{a^3 + b^3}{b^3 + c^3} = \frac{a^2}{bc}$

3

[a] $\because$ The straight line intersects the x-axis at $(1 , a - 3)$ $\quad \therefore a - 3 = 0$ $\quad \therefore a = 3$

$, \because (1 , 0)$ satisfies the function

$\therefore 0 = 2 \times 1 - b$ $\quad \therefore b = 2$

[b] $\because \frac{a}{b} = \frac{c}{d} = m$ $\quad \therefore a = bm , c = dm$

$\therefore \frac{a + b}{b} = \frac{bm + b}{b} = \frac{b(m + 1)}{b} = m + 1$ (1)

$, \frac{c + d}{d} = \frac{dm + d}{d} = \frac{d(m + 1)}{d} = m + 1$ (2)

From (1) and (2) : $\therefore \frac{a + b}{b} = \frac{c + d}{d}$

4

[a] $\because y \propto \frac{1}{x}$ $\quad \therefore xy = m$

$\therefore 2 \times 3 = m$ $\quad \therefore m = 6$ $\quad \therefore xy = 6$

When $x = 1.5$

$\therefore 1.5y = 6$ $\quad \therefore y = 4$

[b] Form the table by yourself

, then the mean $(\overline{X}) = 5 , \sigma \simeq 1.41$

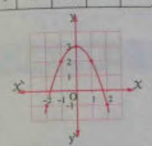
5

[a] $\because \frac{x}{y} = \frac{2}{3}$ $\quad \therefore x = 2m , y = 3m$

$\therefore \frac{3x + 2y}{6y - x} = \frac{6m + 6m}{18m - 2m} = \frac{12m}{16m} = \frac{3}{4}$

[b] $f(x) = 3 - x^2$

x	-2	-1	0	1	2
$f(x)$	-1	2	3	2	-1



From the graph :

- The vertex of the curve is : $(0 , 3)$
- The maximum value $= 3$
- The equation of the axis of symmetry is : $x = 0$

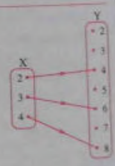
13 Aswan

1

1 a 2 d 3 c 4 d 5 d 6 b

2

[a] 1 $R = \{(2 , 4) , (3 , 6) , (4 , 8)\}$



2 R is a function because every element in X has only one image in Y

, the range $= \{4 , 6 , 8\}$

[b] 1 $\because y \propto \frac{1}{x}$ $\quad \therefore xy = m$

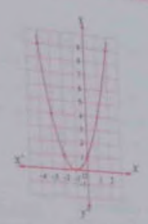
$\therefore 2 \times 6 = m$ $\quad \therefore m = 12$ $\quad \therefore xy = 12$

2 When $x = 3$ $\quad \therefore 3y = 12$ $\quad \therefore y = 4$

3

[a] $f(x) = x^2 + 2x + 1$

x	-4	-3	-2	-1	0	1	2
$f(x)$	9	4	1	0	1	4	9



From the graph :

- The vertex of the curve is : $(-1 , 0)$
- The minimum value $= 0$
- The equation of the axis of symmetry is : $x = -1$

[b] $\because$ b is the middle proportional between a and c

$\therefore b^2 = ac$

$\therefore \text{L.H.S.} = \frac{a^2 + b^2}{b^2 + c^2} = \frac{a^2 + ac}{ac + c^2} = \frac{a(a + c)}{c(a + c)} = \frac{a}{c} = \text{R.H.S.}$

4

[a] $\because f(3) = 3^2 - 3 \times 3 = 9 - 9 = 0$

$, g(3) = 3 - 3 = 0$

$\therefore f(3) = g(3)$

[b] $\because \frac{a}{b} = \frac{3}{5} = m$ $\quad \therefore a = 3m , b = 5m$

$\therefore \frac{7a + 9b}{4a + 2b} = \frac{21m + 45m}{12m + 10m} = \frac{66m}{22m} = 3$

5

[a] 1 $n(Y^2) = 2 \times 2 = 4$

2 $X \times (Y \cap Z) = \{1\} \times \{2\} = \{(1 , 2)\}$

[b] Form the table by yourself

, then the mean $(\overline{X}) = 16$

$, \sigma \simeq 3.29$

14 South Sinai

1

1 a 2 b 3 d 4 a 5 b 6 d

2

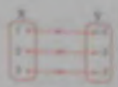
[a] $(X - Y) \times Z = \{-1\} \times \{4, 6, 8\}$
$= \{(-1, 4), (-1, 6), (-1, 8)\}$

[b] $\because y^2 - 10Xy + 25X^2 = 0$ $\therefore (y - 5X)^2 = 0$
$\therefore y - 5X = 0$ $\therefore y = 5X$
$\therefore y \propto X$

3

[a] Let the number be : X
$\therefore \frac{7 + X^2}{11 + X^2} = \frac{4}{5}$ $\therefore 35 + 5X^2 = 44 + 4X^2$
$\therefore 5X^2 - 4X^2 = 44 - 35$
$\therefore X^2 = 9$ $\therefore X = 3$ or $X = -3$
$\therefore$ The number is : 3 or -3

[b] $R = \{(1, -1), (2, -2), (3, -3)\}$



R is a function because every element in X has only one image in Y

4

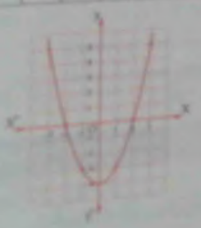
[a] $\because y \propto \frac{1}{X}$ $\therefore Xy = m$
$\therefore 3 \times 2 = m$ $\therefore m = 6$ $\therefore Xy = 6$
When $y = 6$ $\therefore 6X = 6$ $\therefore X = 1$

[b] Form the tables by yourself , then $\sigma \simeq 1.31$

5

[a] $f(X) = X^2 - 4$

X	-3	-2	-1	0	1	2	3
$f(X)$	5	0	-3	-4	-3	0	5



From the graph :

- The vertex of the curve is : $(0, -4)$
- The equation of the axis of symmetry is : $X = 0$

[b] $\because \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$
$\therefore c = dm$, $b = dm^2$, $a = dm^3$
$\therefore \frac{a^2 - 3c^2}{b^2 - 3d^2} = \frac{d^2m^6 - 3d^2m^2}{d^2m^4 - 3d^2}$
$= \frac{d^2m^2(m^4 - 3)}{d^2(m^4 - 3)} = m^2$ (1)
, $\frac{b}{d} = \frac{dm^2}{d} = m^2$ (2)
From (1) and (2) : $\therefore \frac{a^2 - 3c^2}{b^2 - 3d^2} = \frac{b}{d}$

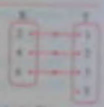
15 Matrouh

1

1 c 2 c 3 b 4 c 5 d 6 b

2

[a] 1 $R = \{(2, 1), (4, 2), (6, 3)\}$



2 R is a function because every element in X has only one image in Y
, the range $= \{1, 2, 3\}$

[b] $\because \frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - 2b + 5c}{3X}$
, multiplying the two terms of the 1st ratio by (2) and the 2nd by (-2) and the 3rd by (5) and adding the antecedents and consequents of the three ratios.
$\therefore \frac{2a - 2b + 5c}{4 - 6 + 20}$ = one of the given ratios
$\therefore \frac{2a - 2b + 5c}{18} = \frac{2a - 2b + 5c}{3X}$
$\therefore 3X = 18$ $\therefore X = 6$

3

[a] 1 $X = \{1, 4, 5\}$
2 $Y \times X = \{(2, 1), (2, 4), (2, 5)\}$
3 $n(X^2) = 3 \times 3 = 9$

[b] $\because$ b is the middle proportional between a and c
$\therefore b^2 = ac$
$\therefore$ L.H.S. $= \frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{a^2}{ac} + \frac{ac}{c^2} = \frac{a}{c} + \frac{a}{c}$
$= \frac{2a}{c} =$ R.H.S.

4

[a] Form the table by yourself
, then the mean $(\bar{X}) = 10$, $\sigma \simeq 2.83$

[b] 1 $\because y \propto \frac{1}{X}$ $\therefore Xy = m$
$\therefore 2 \times 3 = m$ $\therefore m = 6$ $\therefore Xy = 6$
2 When $X = 6$ $\therefore 6y = 6$ $\therefore y = 1$

5

[a] $\because (a - 3, 7) = (2, b^3 - 1)$
$\therefore a - 3 = 2$ $\therefore a = 5$
, $b^3 - 1 = 7$ $\therefore b^3 = 8$ $\therefore b = \sqrt[3]{8} = 2$
$\therefore \frac{a + 2b}{2a - b} = \frac{5 + 2 \times 2}{2 \times 5 - 2} = \frac{9}{8}$

[b] $f(X) = 1 - X^2$

X	-2	-1	0	1	2
$f(X)$	-3	0	1	0	-3

From the graph :

1 The vertex of the curve is : $(0, 1)$
2 The maximum value = 1
3 The equation of the symmetry axis is : $X = 0$

Answers of examinations on Port Said specifications of algebra & statistics

Exam 1 Port Said 2023

First Answers of multiple choice questions

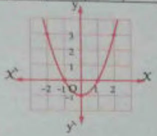
1 (c) 2 (d) 3 (d) 4 (b) 5 (c)
6 (c) 7 (d) 8 (b) 9 (a) 10 (a)
11 (d) 12 (a) 13 (b) 14 (d) 15 (c)
16 (c) 17 (a) 18 (c) 19 (a) 20 (b)
21 (a)

Second Answers of essay questions

22

$f(X) = X^2 - 1$

X	-2	-1	0	1	2
$f(X)$	3	0	-1	0	3



From the graph :

1 The minimum value $= -1$

2 The equation of the symmetry axis is : $X = 0$

23

$\because y \propto \frac{1}{X}$ $\quad\therefore \frac{y_1}{y_2} = \frac{X_2}{X_1}$

$\therefore \frac{3}{y_2} = \frac{6}{4}$ $\quad\therefore y_2 = \frac{3 \times 4}{6} = 2$

24

Form the table by yourself , then $\sigma \simeq 2.83$

Exam 2 Port Said 2024

First Answers of multiple choice questions

1 (d) 2 (c) 3 (b) 4 (d) 5 (c)
6 (b) 7 (d) 8 (c) 9 (d) 10 (c)
11 (a) 12 (b) 13 (b) 14 (a) 15 (b)
16 (d) 17 (d) 18 (d) 19 (c) 20 (b)
21 (c)

Second Answers of essay questions

22

1 The vertex of the curve is : $(2, 4)$

2 The equation of the symmetry axis is : $X = 2$

3 The maximum value $= 4$

23

$\because \frac{a}{b} = \frac{c}{d} = m$ $\quad\therefore a = bm , c = dm$

$\therefore \frac{a + 2c}{b + 2d} = \frac{bm + 2dm}{b + 2d} = \frac{m(b + 2d)}{b + 2d} = m$ (1)

$, \frac{c - a}{d - b} = \frac{dm - bm}{d - b} = \frac{m(d - b)}{d - b} = m$ (2)

From (1) and (2) : $\therefore \frac{a + 2c}{b + 2d} = \frac{c - a}{d - b}$

24

Form the table by yourself , then the mean $(\overline{X}) = 7$, $\sigma \simeq 1.41$

Exam 3

First Answers of multiple choice questions

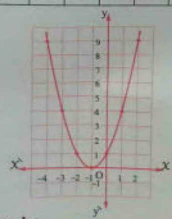
1 (c) 2 (b) 3 (a) 4 (c) 5 (b)
6 (a) 7 (d) 8 (b) 9 (a) 10 (d)
11 (c) 12 (c) 13 (a) 14 (b) 15 (d)
16 (a) 17 (b) 18 (d) 19 (a) 20 (b)
21 (c)

Second Answers of essay questions

22

$f(X) = X^2 + 2X + 1$

X	-4	-3	-2	-1	0	1	2
$f(X)$	9	4	1	0	1	4	9



From the graph :

• The vertex of the curve is : $(-1, 0)$

• The minimum value $= 0$

23

$\because \frac{X}{3} = \frac{y}{4} = \frac{z}{5} = m$

$\therefore X = 3m , y = 4m , z = 5m$

$\therefore \frac{2y - z}{3X - 2y + z} = \frac{8m - 5m}{9m - 8m + 5m} = \frac{3m}{6m} = \frac{1}{2}$

24

Form the table by yourself , then the mean $(\overline{X}) = 8$, $\sigma = 4$

Exam 4

First Answers of multiple choice questions

1 (b) 2 (d) 3 (b) 4 (d) 5 (c)
6 (c) 7 (b) 8 (a) 9 (c) 10 (b)
11 (c) 12 (a) 13 (b) 14 (d) 15 (b)
16 (c) 17 (d) 18 (d) 19 (a) 20 (c)
21 (d)

Second Answers of essay questions

22

$\because$ b is the middle proportional between a and c

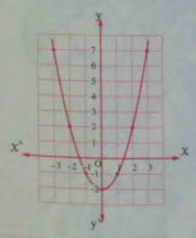
$\therefore b^2 = ac$

$\therefore \text{L.H.S.} = \frac{a^2 + b^2}{b^2 + c^2} = \frac{a^2 + ac}{ac + c^2} = \frac{a(a + c)}{c(a + c)} = \frac{a}{c} = \text{R.H.S.}$

23

$f(X) = X^2 - 2$

X	-3	-2	-1	0	1	2	3
$f(X)$	7	2	-1	-2	-1	2	7



From the graph :

1 The vertex of the curve is : $(0, -2)$

2 The equation of the axis of symmetry is : $X = 0$

24

Form the table by yourself , then the mean $(\overline{X}) = 63$, $\sigma \simeq 7.07$

Exam 5

First Answers of multiple choice questions

1 (c) 2 (c) 3 (b) 4 (c) 5 (d)
6 (a) 7 (c) 8 (b) 9 (a) 10 (b)
11 (d) 12 (c) 13 (b) 14 (d) 15 (b)
16 (a) 17 (a) 18 (c) 19 (b) 20 (d)
21 (b)

Second Answers of essay questions

22

$\because \frac{a}{b} = \frac{c}{d} = m$ $\quad\therefore a = bm , c = dm$

$\therefore \frac{a - b}{a} = \frac{bm - b}{bm} = \frac{b(m - 1)}{bm} = \frac{m - 1}{m}$ (1)

$, \frac{c - d}{c} = \frac{dm - d}{dm} = \frac{d(m - 1)}{dm} = \frac{m - 1}{m}$ (2)

From (1) and (2) : $\therefore \frac{a - b}{a} = \frac{c - d}{c}$

23

Form the table by yourself , then $\sigma \simeq 1.41$

24

$f(X) = (X - 2)^2$

X	0	1	2	3	4
$f(X)$	4	1	0	1	4

From the graph :

1 The vertex of the curve is : $(2, 0)$

2 The equation of the axis of symmetry is : $X = 2$

3 The minimum value $= 0$

Guide Answers

Of The Notebook

(Trigonometry and Geometry)

Answers of accumulative tests on trigonometry & geometry

Accumulative test 1

1 1 b　　2 b　　3 c

4 a　　5 c　　6 d

2 1 $\tan(\angle ACB) + \tan(\angle ACD) = \frac{56}{15}$

2 $\sin(\angle B)\cos(\angle CAD) + \cos(\angle B)\sin(\angle CAD) = \frac{56}{65}$

3 1 4 cm.　　2 $\frac{1}{4}$

Accumulative test 2

1 1 b　　2 b　　3 c

4 c　　5 b　　6 a

2 1 Prove by yourself.

2 $22^\circ\ 37'$

3 $X = \frac{1}{16}$

Accumulative test 3

1 1 a　　2 c　　3 b

4 c　　5 c　　6 c

2 Prove by yourself.

3 $m(\angle X) = 45^\circ$

Accumulative test 4

1 1 d　　2 d　　3 b

4 a　　5 a　　6 a

2 DC = 5 cm. , $\cos(\angle BCD) = \frac{4}{5}$

3 The perimeter of the triangle AOB = 24 length units.

Accumulative test 5

1 1 c　　2 c　　3 d

4 d　　5 a　　6 b

2 Prove by yourself.

3 1 M $(-1, 3)$

2 D $(-4, 7)$

Accumulative test 6

1 1 b　　2 b　　3 c

4 b　　5 a　　6 c

2 $y = -9X + 15$

3 1 $\cos A \cos C - \sin A \sin C = 0$

2 $m(\angle C) = 36^\circ\ 52'\ 12''$

Answers of important questions on trigonometry & geometry

Unit four

First Answers of multiple choice questions

1 (a)	2 (b)	3 (a)	4 (c)	5 (b)
6 (a)	7 (b)	8 (d)	9 (c)	10 (c)
11 (b)	12 (a)	13 (b)	14 (c)	15 (d)
16 (d)	17 (c)	18 (c)	19 (b)	

Second Answers of essay questions

1

Let the measures of the two angles be $3X$ and $5X$

$\therefore 3X + 5X = 180°$ $\qquad \therefore 8X = 180°$

$\therefore X = \frac{180°}{8} = 22.5°$

$\therefore$ The measure of the first angle $= 3 \times 22.5° = 67.5° = 67° \; 30'$

, the measure of the second angle $= 5 \times 22.5° = 112.5° = 112° \; 30'$

2

Let the measures of the interior angles of the triangle be $3X$, $4X$, $7X$

$\therefore 3X + 4X + 7X = 180°$ $\qquad \therefore 14X = 180°$

$\therefore X = \frac{180°}{14}$

$\therefore$ The measure of the first angle

$= 3 \times \frac{180°}{14} \simeq 38° \; 34' \; 17''$

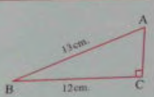
, the measure of the second angle

$= 4 \times \frac{180°}{14} \simeq 51° \; 25' \; 43''$

, the measure of the third angle $= 7 \times \frac{180°}{14} = 90°$

3

In ΔABC :



$\because m(\angle C) = 90°$

$\therefore (AC)^2 = (13)^2 - (12)^2 = 25$

$\therefore AC = 5$ cm.

$\therefore$ L.H.S. $= \sin A \cos B + \cos A \sin B$

$= \frac{12}{13} \times \frac{12}{13} + \frac{5}{13} \times \frac{5}{13} = 1 =$ R.H.S.

4

$\sin 45° \cos 45° + \sin 30° \cos 60° - \cos^2 30°$

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{2} - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{2} + \frac{1}{4} - \frac{3}{4} = 0$

5

$\because \tan^2 60° - \tan^2 45° = (\sqrt{3})^2 - (1)^2 = 2$ (1)

, $4 \sin 30° = 4 \times \frac{1}{2} = 2$ (2)

From (1) and (2) :

$\therefore \tan^2 60° - \tan^2 45° = 4 \sin 30°$

6

$\because \sin^3 30° = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ (1)

, $9 \cos^3 60° - \tan^2 45° = 9 \times \left(\frac{1}{2}\right)^3 - (1)^2$

$= \frac{9}{8} - 1 = \frac{1}{8}$ (2)

From (1) and (2) :

$\therefore \sin^3 30° = 9 \cos^3 60° - \tan^2 45°$

7

$\because X \sin 30° = \sin 30° \cos 60° + \cos 30° \sin 60°$

$\therefore X \times \frac{1}{2} = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$\therefore \frac{1}{2} X = \frac{1}{4} + \frac{3}{4}$

$\therefore \frac{1}{2} X = 1$ $\qquad \therefore X = 2$

8

$\because X \sin 30° \cos^2 45° = \sin^2 60°$

$\therefore X \times \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$

$\therefore X \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{4}$ $\qquad \therefore \frac{1}{4} X = \frac{3}{4}$

$\therefore X = 3$

9

$\because \tan X = 4 \sin 30° \cos 60°$ $\qquad \therefore \tan X = 4 \times \frac{1}{2} \times \frac{1}{2}$

$\therefore \tan X = 1$ $\qquad \therefore X = 45°$

10

$\because \sin E = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

$\therefore m(\angle E) = 30°$

11

In ΔABC :

$\because m(\angle A) = 90°$ $\qquad \therefore (BC)^2 = (20)^2 + (15)^2 = 625$

$\therefore BC = 25$ cm.

$\therefore \cos C \cos B - \sin C \sin B = \frac{15}{25} \times \frac{20}{25} - \frac{20}{25} \times \frac{15}{25}$

$= \frac{300}{625} - \frac{300}{625} = 0$

12

In ΔADC :

$\because m(\angle D) = 90°$

$\therefore (AD)^2 = (17)^2 - (15)^2 = 64$

$\therefore AD = 8$ cm.

$\therefore 3 \tan C + \sin B = 3 \times \frac{8}{15} + \frac{8}{10} = \frac{12}{5}$

13

In ΔABC :

$\because m(\angle A) = 90°$

, $\overline{AD} \perp \overline{BC}$

$\therefore (AD)^2 = 9 \times 16 = 144$

$\therefore AD = 12$ cm.

$\therefore \tan B \tan C = \frac{12}{9} \times \frac{12}{16} = 1$

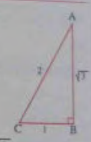
14

$\because 2AB = \sqrt{3}\, AC$ $\qquad \therefore \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

Let $AB = \sqrt{3}$ length units

, $AC = 2$ length units

$\therefore BC = 1$ length unit



$\therefore \sin C = \frac{\sqrt{3}}{2}$, $\cos C = \frac{1}{2}$, $\tan C = \sqrt{3}$

15

1 In ΔABC :

$\because m(\angle C) = 90°$

$\therefore (AB)^2 = \ell^2 + \ell^2 = 2\ell^2$ $\qquad \therefore AB = \sqrt{2}\,\ell$

$\therefore AC : BC : AB = \ell : \ell : \sqrt{2}\,\ell = 1 : 1 : \sqrt{2}$

2 $\tan B = \tan 45° = 1$, $\sin A = \sin 45° = \frac{1}{\sqrt{2}}$

16

$\because m(\angle B) = 90°$

$\therefore \frac{\sin A}{\cos C} = \frac{\frac{BC}{AC}}{\frac{BC}{AC}} = 1$

, $\because \tan E = \frac{\sin A}{\cos C} = 1$

$\therefore m(\angle E) = 45°$

17

1 $\because \tan C = \frac{AB}{BC}$ $\qquad \therefore \frac{3}{4} = \frac{6}{BC}$

$\therefore BC = 8$ cm.

, $\because (AC)^2 = (6)^2 + (8)^2 = 100$

$\therefore AC = 10$ cm.

2 $\sin A + \cos A = \frac{8}{10} + \frac{6}{10} = \frac{14}{10} = \frac{7}{5}$

18

$\because 2 \cos X - \sqrt{3} = 0$ $\qquad \therefore 2 \cos X = \sqrt{3}$

$\therefore \cos X = \frac{\sqrt{3}}{2}$ $\qquad \therefore X = 30°$

$\therefore \tan 2X = \tan 60° = \sqrt{3}$

19

Draw $\overline{AF} \perp \overline{BC}$

, $\overline{DE} \perp \overline{BC}$

$\because \overline{AD} \parallel \overline{BC}$

$\therefore$ AFED is a rectangle , FE = 4 cm.

$\therefore BF + CE = 8$ cm.

, $\because BF = CE = 4$ cm. ($\Delta ABF \equiv \Delta DCE$)

$\therefore$ From ΔABF which is right-angled at F

$(AF)^2 = (5)^2 - (4)^2 = 9$

$\therefore AF = 3$ cm.

, $\because$ AFED is a rectangle $\qquad \therefore DE = AF = 3$ cm.

$\therefore \frac{\tan B \cos C}{\cos^2 C + \sin^2 C} = \frac{\frac{3}{4} \times \frac{4}{5}}{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = \frac{3}{5}$

20

Draw $\overline{DF} \perp \overline{BC}$

$\therefore \overline{AD} \parallel \overline{BC}$, $\overline{AB} \perp \overline{BC}$

, $\overline{DF} \perp \overline{BC}$

$\therefore$ ABFD is a rectangle

$\therefore BF = AD = 6$ cm.

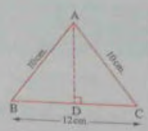
$\therefore FC = 4$ cm. , $DF = AB = 3$ cm.

$\therefore$ From ΔDFC which is right-angled at F :

$(DC)^2 = 3^2 + 4^2 = 25$ $\qquad \therefore DC = 5$ cm.

$\therefore \cos(\angle DCB) - \tan(\angle ACB) = \frac{4}{5} - \frac{3}{10} = \frac{1}{2}$

21



Draw $\overline{AD} \perp \overline{BC}$ to cut it at D

$\because AB = AC$, $\overline{AD} \perp \overline{BC}$

$\therefore BD = DC = 6$ cm.

In Δ ABD : $\because \cos B = \frac{6}{10}$

$\therefore m(\angle B) \simeq 53° \ 7' \ 48''$

, $\because m(\angle ADB) = 90°$

$\therefore (AD)^2 = (10)^2 - (6)^2 = 64$ $\quad \therefore AD = 8$ cm.

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 12 \times 8 = 48 \text{ cm}^2$.

22

1 In Δ ABC : $\because m(\angle B) = 90°$

$\therefore (AC)^2 = (5)^2 + (12)^2 = 169$

$\therefore AC = 13$ cm.

2 $\because$ ABCD is a rectangle

$\therefore AB = CD = 5$ cm. , $BC = AD = 12$ cm.

$\therefore 5 \tan(\angle ACD) - 13 \sin(\angle DAC)$

$= 5 \times \frac{12}{5} - 13 \times \frac{5}{13} = 12 - 5 = 7$

Unit five

First Answers of multiple choice questions

1 (d)	2 (c)	3 (a)	4 (c)	5 (c)
6 (b)	7 (c)	8 (c)	9 (c)	10 (b)
11 (a)	12 (a)	13 (b)	14 (d)	15 (c)
16 (d)	17 (d)	18 (c)	19 (d)	20 (d)
21 (c)	22 (b)	23 (b)	24 (a)	25 (b)
26 (d)	27 (c)	28 (a)	29 (d)	30 (c)
31 (b)	32 (b)	33 (d)	34 (a)	35 (a)

Second Answers of essay questions

1

$\because$ The slope of $\overleftrightarrow{AB} = \frac{5+1}{6+3} = \frac{2}{3}$

, the slope of $\overleftrightarrow{BC} = \frac{3-5}{3-6} = \frac{2}{3}$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ $\quad \therefore \overleftrightarrow{AB} \mathbin{/\mkern-3mu/} \overleftrightarrow{BC}$

, $\because$ B is a common point between the two straight lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$

$\therefore$ A , B , C are collinear points.

2

$\because AB = \sqrt{(-2-3)^2 + (4+1)^2} = \sqrt{25+25} = \sqrt{50}$

$= 5\sqrt{2}$ length unit

, $BC = \sqrt{(3-4)^2 + (-1-5)^2} = \sqrt{1+36}$

$= \sqrt{37}$ length unit

, $AC = \sqrt{(-2-4)^2 + (4-5)^2} = \sqrt{36+1}$

$= \sqrt{37}$ length unit

$\therefore BC = AC$

$\therefore \Delta$ ABC is an isosceles triangle.

3

$\because \sqrt{(X-6)^2 + (5-1)^2} = 2\sqrt{5}$ (squaring both sides)

$\therefore (X-6)^2 + (4)^2 = 20$ $\quad \therefore (X-6)^2 = 4$

$\therefore X - 6 = \pm 2$ $\quad \therefore X - 6 = -2$ $\quad \therefore X = 4$

$\therefore$ or $X - 6 = 2$ $\quad \therefore X = 8$

4

$\because MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9}$

$= \sqrt{25} = 5$ length units

, $MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$ length units

, $MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$ length units

$\therefore MA = MB = MC$

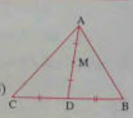
$\therefore$ A , B and C are located on the circle M whose radius length is 5 length units

$\therefore$ The circumference of the circle $= 2\pi r$

$= 2 \times 5 \times \pi$

$= 10\pi$ length units

5



$\because \overline{AD}$ is a median in Δ ABC

$\therefore$ D is the midpoint of $\overline{BC}$

$\therefore D = \left(\frac{-2+0}{2}, \frac{4+6}{2}\right) = (-1, 5)$

Let A (X, y)

, $\because$ M is the midpoint of $\overline{AD}$

$\therefore (-3, -2) = \left(\frac{X-1}{2}, \frac{y+5}{2}\right)$

$\therefore \frac{X-1}{2} = -3$ $\quad \therefore X - 1 = -6$ $\quad \therefore X = -5$

, $\frac{y+5}{2} = -2$ $\quad \therefore y + 5 = -4$ $\quad \therefore y = -9$

$\therefore A(-5, -9)$

6

$\because AB = \sqrt{(-1-1)^2 + (-2-4)^2} = \sqrt{4+36}$

$= \sqrt{40}$ length unit.

, $BC = \sqrt{(2+1)^2 + (-3+2)^2} = \sqrt{9+1}$

$= \sqrt{10}$ length unit.

, $CA = \sqrt{(1-2)^2 + (4+3)^2} = \sqrt{1+49}$

$= \sqrt{50}$ length unit

$\therefore (CA)^2 = (AB)^2 + (BC)^2$

$\therefore \Delta$ ABC is right-angled at B

$\therefore$ Its area $= \frac{1}{2}$ AB $\times$ BC $= \frac{1}{2}\sqrt{40} \times \sqrt{10}$

$= 10$ square units.

7

$\because$ The slope of $\overleftrightarrow{AB} = \frac{-2-3}{6-5} = -5$

, the slope of $\overleftrightarrow{CD} = \frac{4+1}{0-1} = -5$

$\therefore \overleftrightarrow{AB} \mathbin{/\mkern-3mu/} \overleftrightarrow{CD}$ (1)

, $\because$ the slope of $\overleftrightarrow{AD} = \frac{4-3}{0-5} = \frac{-1}{5}$

, the slope of $\overleftrightarrow{BC} = \frac{-1+2}{1-6} = \frac{-1}{5}$

$\therefore \overleftrightarrow{AD} \mathbin{/\mkern-3mu/} \overleftrightarrow{BC}$ (2)

From (1) and (2) : $\therefore$ ABCD is a parallelogram.

, $\because$ the slope of $\overleftrightarrow{AC} = \frac{-1-3}{1-5} = 1$

, the slope of $\overleftrightarrow{BD} = \frac{4+2}{0-6} = -1$

$\therefore$ The slope of $\overleftrightarrow{AC}$ $\times$ the slope of $\overleftrightarrow{BD} = 1 \times -1 = -1$

$\therefore \overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

$\therefore$ ABCD is a rhombus.

8

Let B (X, y)

, $\because$ C is the midpoint of $\overline{AB}$

$\therefore (6, -4) = \left(\frac{5+X}{2}, \frac{-3+y}{2}\right)$

$\therefore \frac{5+X}{2} = 6$ $\quad \therefore 5 + X = 12$ $\quad \therefore X = 7$

, $\frac{-3+y}{2} = -4$ $\quad \therefore -3 + y = -8$ $\quad \therefore y = -5$

$\therefore B(7, -5)$

9

$\because$ C is the midpoint of $\overline{AB}$

$\therefore (3, 1) = \left(\frac{1+X}{2}, \frac{y+3}{2}\right)$

$\therefore \frac{1+X}{2} = 3$ $\quad \therefore 1 + X = 6$ $\quad \therefore X = 5$

, $\frac{y+3}{2} = 1$ $\quad \therefore y + 3 = 2$ $\quad \therefore y = -1$

$\therefore (X, y) = (5, -1)$

10

$\because AB = \sqrt{(3-0)^2 + (3-3)^2} = \sqrt{9+0} = 3$ length unit.

, $BC = \sqrt{(0-0)^2 + (3-0)^2} = \sqrt{0+9} = 3$ length unit.

, $CD = \sqrt{(0-3)^2 + (0-0)^2} = \sqrt{9+0} = 3$ length unit

, $DA = \sqrt{(3-3)^2 + (0-3)^2} = \sqrt{0+9} = 3$ length unit.

$\therefore AB = BC = CD = DA$ $\quad \therefore$ ABCD is a rhombus

, $\because AC = \sqrt{(3-0)^2 + (3-0)^2} = \sqrt{9+9} = \sqrt{18}$

$= 3\sqrt{2}$ length unit

, $BD = \sqrt{(0-3)^2 + (3-0)^2} = \sqrt{9+9} = \sqrt{18}$

$= 3\sqrt{2}$ length unit.

$\therefore AC = BD$

$\therefore$ ABCD is a square , the length of its diagonal $= 3\sqrt{2}$ length unit ,

its area $= 3 \times 3 = 9$ square unit.

11

$\because$ ABCD is a rhombus $\quad \therefore AB = BC$

$\therefore \sqrt{(6-5)^2 + (-2-3)^2} = \sqrt{(1-6)^2 + (m+2)^2}$

(squaring both sides)

$\therefore (6-5)^2 + (-2-3)^2 = (1-6)^2 + (m+2)^2$

$\therefore (m+2)^2 + 25 = 1 + 25$ $\quad \therefore (m+2)^2 = 1$

$\therefore m + 2 = \pm 1$

$\therefore m + 2 = 1$ $\quad \therefore m = -1$

or $m + 2 = -1$ $\quad \therefore m = -3$

12

$\because$ B is the midpoint of $\overline{AC}$

$\therefore (X, 3) = \left(\frac{3+5}{2}, \frac{y+2}{2}\right)$ $\quad \therefore X = \frac{3+5}{2} = 4$

, $\frac{y+2}{2} = 3$ $\quad \therefore y + 2 = 6$

$\therefore y = 4$ $\quad \therefore X + y = 4 + 4 = 8$

13

1 Let A (X, y)

$\therefore (5, 7) = \left(\frac{X+8}{2}, \frac{y+11}{2}\right)$

$\therefore \frac{X+8}{2} = 5$ $\quad \therefore X + 8 = 10$ $\quad \therefore X = 2$

, $\frac{y+11}{2}=7$ $\therefore y+11=14$

$\therefore y=3$ $\therefore A(2, 3)$

2 $\because r = MA = \sqrt{(5-2)^2+(7-3)^2}$

$= \sqrt{9+16} = 5$ length unit.

$\therefore$ The circumference of the circle

$= 2\pi r = 2 \times 3.14 \times 5 = 31.4$ length unit

14

$\because AB = \sqrt{(3+3)^2+(4-0)^2} = \sqrt{36+16} = \sqrt{52}$

$= 2\sqrt{13}$ length unit

, $BC = \sqrt{(1-3)^2+(-6-4)^2} = \sqrt{4+100} = \sqrt{104}$

$= 2\sqrt{26}$ length unit

, $CA = \sqrt{(-3-1)^2+(0+6)^2} = \sqrt{16+36} = \sqrt{52}$

$= 2\sqrt{13}$ length unit

$\therefore AB = AC$

$\therefore \Delta ABC$ is an isosceles triangle and its vertex is A

Let D be the midpoint of $\overline{BC}$ (the base of ΔABC)

$\therefore D = \left(\frac{3+1}{2}, \frac{4-6}{2}\right) = (2, -1)$

$\therefore AD = \sqrt{(-3-2)^2+(0+1)^2} = \sqrt{25+1}$

$= \sqrt{26}$ length unit

$\therefore$ The length of the drawn line segment perpendicular to $\overline{BC}$ from A equals $\sqrt{26}$ length unit

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 2\sqrt{26} \times \sqrt{26}$

$= 26$ square units

15

1 $\because$ The midpoint of $\overline{AC} = \left(\frac{3+5}{2}, \frac{3-1}{2}\right) = (4, 1)$

$\therefore$ The point of intersection of the diagonals $= (4, 1)$

2 Let $D(x, y)$

, $\because$ the midpoint of $\overline{AC}$ = the midpoint of $\overline{BD}$

$\therefore (4, 1) = \left(\frac{2+x}{2}, \frac{-2+y}{2}\right)$

$\therefore \frac{2+x}{2} = 4$ $\therefore 2+x=8$ $\therefore x=6$

, $\frac{-2+y}{2} = 1$ $\therefore -2+y=2$ $\therefore y=4$

$\therefore D(6, 4)$

16

$\because m_1 = \frac{5+2}{4+3} = 1$, $m_2 = \tan 45° = 1$

$\therefore m_1 = m_2$ $\therefore L_1 \parallel L_2$

17

$\because m_1 = \frac{k-1}{2-3} = \frac{k-1}{-1}$, $m_2 = \tan 45° = 1$

, $\because L_1 \perp L_2$ $\therefore m_1 m_2 = -1$

$\therefore \frac{k-1}{-1} \times 1 = -1$ $\therefore k-1=1$

$\therefore k=2$

18

$\because$ The slope of the straight line $= \tan 45° = 1$

$\therefore$ The equation of the straight line is : $y = x + c$

, $\because$ the straight line passes through the point $(3, 2)$

$\therefore 2 = 3 + c$ $\therefore c = -1$

$\therefore$ The equation of the straight line is : $y = x - 1$

19

$\because$ The slope of the given straight line $= \frac{-1}{2}$

$\therefore$ The slope of the required straight line $= \frac{-1}{2}$

$\therefore$ The equation of the required straight line is :

$y = \frac{-1}{2}x + c$

, $\because (3, -5)$ satisfies the equation

$\therefore -5 = \frac{-1}{2} \times 3 + c$ $\therefore c = -\frac{7}{2}$

$\therefore$ The equation is : $y = \frac{-1}{2}x - \frac{7}{2}$

20

$\because$ The slope $= \tan\theta = 2$, and it intercepts 7 units from the positive part of the y-axis

$\therefore$ The equation is : $y = 2x + 7$

21

$\because$ The slope of the given straight line $= \frac{-4+3}{5-2} = \frac{-1}{3}$

$\therefore$ The slope of the required straight line $= 3$

$\therefore$ The equation of the required straight line is :

$y = 3x + c$

, $\because (1, 2)$ satisfies the equation

$\therefore 2 = 3 \times 1 + c$ $\therefore c = -1$

$\therefore$ The equation is : $y = 3x - 1$

22

$\because$ The straight line passes through the two points $(4, 0)$, $(0, 9)$

$\therefore$ The slope of the straight line $= \frac{9-0}{0-4} = -\frac{9}{4}$ and the intercepted part $= 9$ units from the positive part of y-axis

$\therefore$ The equation of the straight line is : $y = -\frac{9}{4}x + 9$

23

$\because$ The slope of the straight line $= \frac{-1-2}{-2-4} = \frac{1}{2}$

$\therefore$ The equation of the straight line is : $y = \frac{1}{2}x + c$

, $\because (4, 2)$ satisfies the equation.

$\therefore 2 = \frac{1}{2} \times 4 + c$ $\therefore c = 0$

$\therefore$ The equation of the straight line is : $y = \frac{1}{2}x$

, $\because c = 0$

$\therefore$ The straight line passes through the origin point.

24

$\because$ The slope of $\overleftrightarrow{AB} = \frac{3+1}{5-3} = 2$

$\therefore$ The slope of the axis of symmetry of $\overline{AB} = \frac{-1}{2}$

$\therefore$ The equation of the axis of symmetry of $\overline{AB}$ is :

$y = \frac{-1}{2}x + c$

, $\because$ the midpoint of $\overline{AB} = \left(\frac{3+5}{2}, \frac{-1+3}{2}\right) = (4, 1)$

$\therefore (4, 1)$ satisfies the equation : $y = \frac{-1}{2}x + c$

$\therefore 1 = \frac{-1}{2} \times 4 + c$ $\therefore c = 3$

$\therefore$ The equation of the axis of symmetry of $\overline{AB}$ is :

$y = \frac{-1}{2}x + 3$

25

$\because \frac{y-1}{x} = \frac{1}{3}$ $\therefore y - 1 = \frac{1}{3}x$

$\therefore y = \frac{1}{3}x + 1$

$\therefore$ The slope of the given straight line $= \frac{1}{3}$

$\therefore$ The slope of the required straight line $= \frac{1}{3}$

, $\because$ the straight line intercepts a negative part of y-axis of 4 length units.

$\therefore$ The equation of the required straight line is

$y = \frac{1}{3}x - 4$

26

$\because$ The slope of $\overleftrightarrow{BC} = \frac{3+7}{1-3} = -5$

$\therefore$ The slope of the required straight line $= -5$

$\therefore$ The equation of the required straight line is :

$y = -5x + c$

, $\because A(5, 1)$ satisfies the equation of the required straight line

$\therefore 1 = -5 \times 5 + c$ $\therefore c = 26$

$\therefore$ The equation of the required straight line is :

$y = -5x + 26$

27

$\because \frac{x}{3} + \frac{y}{2} = 1$ (multiplying by 2)

$\therefore \frac{2x}{3} + y = 2$ $\therefore y = \frac{-2}{3}x + 2$

$\therefore$ The slope $= \frac{-2}{3}$

and the intercepted part $= 2$ units from the positive part of y-axis.

28

$\because$ The slope of the given straight line $= \frac{-3}{-4} = \frac{3}{4}$

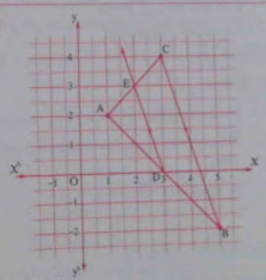
$\therefore$ The slope of the required straight line $= \frac{-4}{3}$

and it intercepts from the positive part of y-axis 4 units.

$\therefore$ The equation of the required straight line is :

$y = \frac{-4}{3}x + 4$

29



1 In ΔABC :

$\because$ D is the midpoint of $\overline{AB}$, $\overline{DE} \parallel \overline{BC}$

$\therefore$ E is the midpoint of $\overline{AC}$, $DE = \frac{1}{2}BC$

$\therefore DE = \frac{1}{2}\sqrt{(5-3)^2+(-2-4)^2}$

$= \frac{1}{2}\sqrt{4+36} = \frac{1}{2}\sqrt{40}$

$= \frac{1}{2} \times 2\sqrt{10} = \sqrt{10}$ length unit.

2 $\because$ The slope of $\overleftrightarrow{BC} = \frac{4+2}{3-5} = -3$

$\therefore$ The slope of $\overleftrightarrow{DE} = -3$

$\therefore$ The equation of $\overleftrightarrow{DE}$ is : $y = -3x + c$

, $\because$ D (the midpoint of $\overline{AB}$) $= \left(\frac{1+5}{2}, \frac{2-2}{2}\right)$

$= (3, 0)$

$\therefore (3, 0)$ satisfies its equation.

[b] $\because AB = \sqrt{(3-1)^2 + (3-5)^2} = \sqrt{4+4}$
$= 2\sqrt{2}$ length units

, $BC = \sqrt{(1-1)^2 + (5-3)^2} = \sqrt{4} = 2$ length units

, $AC = \sqrt{(3-1)^2 + (3-3)^2} = \sqrt{4} = 2$ length units

$\therefore BC = AC$ $\therefore \Delta ABC$ is isosceles.

3

[a] $\because$ The slope of the straight line $= \frac{-3-3}{-1-1} = 3$

$\therefore$ The equation of the straight line is : $y = 3X + c$

, $\because (1, 3)$ satisfies the equation.

$\therefore 3 = 3 \times 1 + c$ $\therefore c = 0$

$\therefore$ The equation of the straight line is : $y = 3X$

, $\because c = 0$

$\therefore$ The straight line passes through the origin point.

[b] $\because (3, 1) = \left(\frac{1+X}{2}, \frac{y+3}{2}\right)$

$\therefore \frac{1+X}{2} = 3$ $\therefore 1 + X = 6$

$\therefore X = 5$, $\frac{y+3}{2} = 1$

$\therefore y + 3 = 2$ $\therefore y = -1$

$\therefore (X, y) = (5, -1)$

4

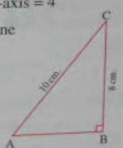
[a] $\because$ The straight line passes through the two points $(1, 0)$ and $(0, 4)$

$\therefore$ The slope $= \frac{4-0}{0-1} = -4$

$\therefore$ The equation of the straight line is :
$y = -4X + c$

, $\because$ the intercepted part from y-axis $= 4$

$\therefore$ The equation of the straight line is : $y = -4X + 4$



[b] $\because m(\angle B) = 90^\circ$

$\therefore (AB)^2 = (10)^2 - (8)^2 = 36$

$\therefore AB = 6$ cm.

$\therefore \sin^2 A + 1 = \left(\frac{8}{10}\right)^2 + 1 = \frac{41}{25}$ (1)

, $2\cos^2 C + \cos^2 A = 2 \times \left(\frac{8}{10}\right)^2 + \left(\frac{6}{10}\right)^2 = \frac{41}{25}$ (2)

From (1) and (2) :

$\therefore \sin^2 A + 1 = 2\cos^2 C + \cos^2 A$

5

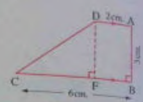
[a] $\because m_1 = \frac{4-3}{2+1} = \frac{1}{3}$

, $m_2 = \frac{1}{3}$ $\therefore m_1 = m_2$ $\therefore L_1 \parallel L_2$

[b] **Const :** Draw $\overline{DF} \perp \overline{BC}$

Proof : $\because \overline{AD} \parallel \overline{BC}$, $\overline{AB} \perp \overline{BC}$

, $\overline{DF} \perp \overline{BC}$



$\therefore$ ABFD is a rectangle

$\therefore BF = AD = 2$ cm.

, $AB = DF = 3$ cm.

$\therefore FC = 6 - 2 = 4$ cm.

From Δ DFC which is right-angled at F

$\therefore (DC)^2 = (3)^2 + (4)^2 = 25$

$\therefore DC = 5$ cm.

$\therefore \cos(\angle BCD) = \frac{4}{5}$

Model for the merge students

1

1 ✔	2 ✔	3 ✘
4 ✘	5 ✘	6 ✔

2

1 b	2 c	3 d
4 c	5 a	6 c

3

1 0	2 1	3 10
4 2	5 -3	6 $\frac{\sqrt{3}}{2}$

4

1 $\frac{1}{2}$	2 $\frac{3}{5}$	3 3
4 2	5 5 length units	6 $(-5, 2)$

Answers of governorates' examinations of trigonometry & geometry

1 Cairo

1

1 d 2 d 3 b 4 c 5 b 6 a

2

[a] $\because 2X \times 1 = \sqrt{3} \times \frac{\sqrt{3}}{2} = \frac{3}{2}$

$\therefore X = \frac{3}{4}$

[b] $\because$ The slope $= 3$

$\therefore$ The equation is : $y = 3X + c$

, $\because (1, 5)$ satisfies the equation

$\therefore 5 = 3 \times 1 + c$ $\therefore c = 2$

$\therefore$ The equation is : $y = 3X + 2$

3

[a] $\because \cos^2 60^\circ = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ (1)

, $\tan 45^\circ - \sin^2 60^\circ = 1 - \left(\frac{\sqrt{3}}{2}\right)^2 = 1 - \frac{3}{4} = \frac{1}{4}$ (2)

From (1) and (2) :

$\therefore \cos^2 60^\circ = \tan 45^\circ - \sin^2 60^\circ$

[b] 1 $\because$ The diagonals of the parallelogram bisect each other

$\therefore$ M is the midpoint of $\overline{AC}$

$\therefore M = \left(\frac{3-5}{2}, \frac{4+2}{2}\right) = (-1, 3)$

2 Let D (X, y)

$\therefore (-1, 3) = \left(\frac{2+X}{2}, \frac{-1+y}{2}\right)$

$\therefore \frac{2+X}{2} = -1$ $\therefore 2 + X = -2$ $\therefore X = -4$

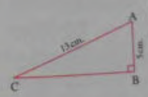
, $\frac{-1+y}{2} = 3$

$\therefore -1 + y = 6$ $\therefore y = 7$

$\therefore D(-4, 7)$

4

[a] In Δ ABC :



$\because m(\angle B) = 90^\circ$

$\therefore (BC)^2 = (13)^2 - (5)^2$
$= 144$

$\therefore BC = 12$ cm.

$\therefore \sin^2 C + \cos^2 C = \left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2 = \frac{169}{169} = 1$

[b] $\because m_1 = \frac{3-2}{1-3} = \frac{-1}{2}$, $m_2 = 2$

$\therefore m_1 \times m_2 = \frac{-1}{2} \times 2 = -1$

$\therefore$ The two straight lines are perpendicular.

5

[a] $\because r = MA = \sqrt{(-1-2)^2 + (3-7)^2} = \sqrt{9+16}$
$= 5$ length units

$\therefore d = 2r = 2 \times 5 = 10$ length units

[b] 1 The equation is : $y = 3X + 6$

2 At $y = 0$ $\therefore 0 = 3X + 6$

$\therefore 3X = -6$ $\therefore X = -2$

$\therefore$ The intersection point of the straight line with the X-axis is $(-2, 0)$

2 Giza

1

1 d 2 c 3 b 4 c 5 a 6 b

2

[a] $\because 2\sin X = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$

$\therefore \sin X = \frac{1}{2}$ $\therefore X = 30^\circ$

[b] $\because$ The slope of the given straight line $= -2$

$\therefore$ The slope of the required straight line $= -2$

$\therefore$ Its equation is : $y = -2X + c$

, $\because (2, -5)$ satisfies the equation

$\therefore -5 = -2 \times 2 + c$ $\therefore c = -1$

$\therefore$ The equation is : $y = -2X - 1$

3

[a] In Δ ABC :

$\because m(\angle B) = 90^\circ$

$\therefore (AB)^2 = (5)^2 - (4)^2 = 9$

$\therefore AB = 3$ cm.

$\therefore \sin A \cos C + \cos A \sin C$
$= \frac{4}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{3}{5} = \frac{16}{25} + \frac{9}{25} = 1$

[b] Let B (X, y)

$\therefore (3, 4) = \left(\frac{1+X}{2}, \frac{2+y}{2}\right)$

$\therefore \frac{1+X}{2} = 3$ $\therefore 1 + X = 6$ $\therefore X = 5$

$, \frac{2+y}{2} = 4$ $\qquad \therefore 2 + y = 8$

$\therefore y = 6$ $\qquad \therefore B(5, 6)$

4

[a] $\because 2X - 3y + 6 = 0$ $\qquad \therefore 3y = 2X + 6$

$\therefore y = \frac{2}{3} X + 2$

$\therefore$ The slope $= \frac{2}{3}$ and the intercepted part $= 2$ units from the positive part of the y-axis

[b] $\because \sqrt{(X-6)^2 + (5-1)^2} = 2\sqrt{5}$ (squaring both sides)

$\therefore (X-6)^2 + 16 = 20$ $\qquad \therefore (X-6)^2 = 4$

$\therefore X - 6 = \pm 2$ $\qquad \therefore X - 6 = 2$

$\therefore X = 8$

or $X - 6 = -2$ $\qquad \therefore X = 4$

5

[a] $\because AB = \sqrt{(3+2)^2 + (-1-4)^2} = \sqrt{25+25}$

$= \sqrt{50}$ length units.

$, BC = \sqrt{(4-3)^2 + (5+1)^2} = \sqrt{1+36}$

$= \sqrt{37}$ length units.

$, AC = \sqrt{(4+2)^2 + (5-4)^2} = \sqrt{36+1}$

$= \sqrt{37}$ length units.

$\therefore BC = AC$

$\therefore \Delta ABC$ is isosceles

[b] 1 $\because OA = 3$ units $\qquad \therefore A(3, 0)$

$, \because OB = 4$ units $\qquad \therefore B(0, 4)$

$\therefore$ The midpoint of $\overline{AB} = \left(\frac{3+0}{2}, \frac{0+4}{2}\right) = \left(\frac{3}{2}, 2\right)$

2 $\because$ The slope of $\overleftrightarrow{AB} = \frac{4-0}{0-3} = \frac{-4}{3}$ $, OB = 4$ units

$\therefore$ The equation of $\overleftrightarrow{AB}$ is : $y = \frac{-4}{3} X + 4$

3 Alexandria

1

1 b 2 c 3 a 4 d 5 c 6 d

2

[a] $\because \sin^3 30° = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ (1)

$, 9\cos^3 60° - \tan^2 45° = 9 \times \left(\frac{1}{2}\right)^3 - (1)^2$

$= \frac{9}{8} - 1 = \frac{1}{8}$ (2)

From (1) and (2) :

$\therefore \sin^3 30° = 9\cos^3 60° - \tan^2 45°$

[b] $\because AB = \sqrt{(3+3)^2 + (4-0)^2} = \sqrt{36+16}$

$= \sqrt{52}$ length units

$, BC = \sqrt{(1-3)^2 + (-6-4)^2} = \sqrt{4+100}$

$= \sqrt{104}$ length units

$, CA = \sqrt{(-3-1)^2 + (0+6)^2} = \sqrt{16+36}$

$= \sqrt{52}$ length units

$\therefore AB = AC$

$\therefore \Delta ABC$ is an isosceles triangle.

$, \because \left(\sqrt{104}\right)^2 = \left(\sqrt{52}\right)^2 + \left(\sqrt{52}\right)^2$

i.e. $(BC)^2 = (AB)^2 + (AC)^2$

$\therefore \Delta ABC$ is right-angled at A

$\therefore$ Its area $= \frac{1}{2} \times AB \times AC = \frac{1}{2} \times \sqrt{52} \times \sqrt{52}$

$= 26$ square units.

3

[a] $\because 3\sin X \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$

$\therefore \frac{3}{2} \sin X = \frac{3}{4}$ $\qquad \therefore \sin X = \frac{1}{2}$

$\therefore X = 30°$

[b] $\because \frac{X}{2} + \frac{y}{3} = 1$ (multiplying by 3)

$\therefore \frac{3}{2} X + y = 3$ $\qquad \therefore y = \frac{-3}{2} X + 3$

$\therefore$ The slope $= \frac{-3}{2}$ and the straight line intercepts 3 units from the positive part of the y-axis.

4

[a] $\because \overleftrightarrow{CD}$ // the X-axis

$\therefore$ The slope of $\overleftrightarrow{CD} = 0$ $\qquad \therefore \frac{y-2}{-5-4} = 0$

$\therefore y - 2 = 0$ $\qquad \therefore y = 2$

[b] 1 $\because$ ABCD is a rectangle

$\therefore m(\angle B) = 90°$

$\therefore (AC)^2 = (5)^2 + (12)^2 = 169$

$\therefore AC = 13$ cm.

2 $\because CD = AB = 5$ cm. $, AD = BC = 12$ cm.

$, m(\angle D) = 90°$

$\therefore 5\tan(\angle ACD) - 13\sin(\angle DAC)$

$= 5 \times \frac{12}{5} - 13 \times \frac{5}{13} = 12 - 5 = 7$

5

[a] $\because m_1 = a, m_2 = \frac{-3-2}{6-5} = -5$

$, \because L_1 \perp L_2$ $\qquad \therefore m_1 \times m_2 = -1$

$\therefore a \times -5 = -1$ $\qquad \therefore a = \frac{1}{5}$

[b] $\because \overline{AD}$ is a median of ΔABC

$\therefore$ D is the midpoint of $\overline{BC}$

$\therefore D = \left(\frac{-2-4}{2}, \frac{3-3}{2}\right) = (-3, 0)$

$\therefore$ The slope of $\overleftrightarrow{AD} = \frac{0-2}{-3-1} = \frac{1}{2}$

$\therefore$ The equation of $\overleftrightarrow{AD}$ is : $y = \frac{1}{2} X + c$

$, \because A(1, 2)$ satisfies the equation of $\overleftrightarrow{AD}$

$\therefore 2 = \frac{1}{2} \times 1 + c$ $\qquad \therefore c = \frac{3}{2}$

$\therefore$ The equation is : $y = \frac{1}{2} X + \frac{3}{2}$

4 El-Kalyoubia

1

1 d 2 c 3 d 4 a 5 d 6 b

2

[a] $\because (3, 1) = \left(\frac{1+X}{2}, \frac{y+3}{2}\right)$

$\therefore \frac{1+X}{2} = 3$ $\qquad \therefore 1 + X = 6$ $\qquad \therefore X = 5$

$, \frac{y+3}{2} = 1$ $\qquad \therefore y + 3 = 2$ $\qquad \therefore y = -1$

$\therefore (X, y) = (5, -1)$

[b] $\because X \times \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$

$\therefore \frac{1}{4} X = \frac{3}{4}$ $\qquad \therefore X = 3$

3

[a] $\because AB = \sqrt{(-3-2)^2 + (0-4)^2} = \sqrt{25+16}$

$= \sqrt{41}$ length units.

$, BC = \sqrt{(-7+3)^2 + (5-0)^2} = \sqrt{16+25}$

$= \sqrt{41}$ length units.

$, CD = \sqrt{(-2+7)^2 + (9-5)^2} = \sqrt{25+16}$

$= \sqrt{41}$ length units.

$, DA = \sqrt{(2+2)^2 + (4-9)^2} = \sqrt{16+25}$

$= \sqrt{41}$ length units.

$, AC = \sqrt{(-7-2)^2 + (5-4)^2} = \sqrt{81+1}$

$= \sqrt{82}$ length units.

$, BD = \sqrt{(-2+3)^2 + (9-0)^2} = \sqrt{1+81}$

$= \sqrt{82}$ length units

$\therefore AB = BC = CD = DA, AC = BD$

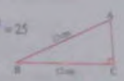
$\therefore$ The figure ABCD is a square.

[b] 1 In ΔABC :

$\because m(\angle C) = 90°$

$\therefore (AC)^2 = (13)^2 - (12)^2 = 25$

$\therefore AC = 5$ cm.



2 $1 + \tan^2 A = 1 + \left(\frac{12}{5}\right)^2$

$= \frac{169}{25}$

4

[a] Let $X(0, 1), Y(a, 3), Z(2, 5)$

$, \because$ The three points are collinear

$\therefore$ The slope of $\overleftrightarrow{XY}$ = the slope of $\overleftrightarrow{XZ}$

$\therefore \frac{3-1}{a-0} = \frac{5-1}{2-0}$ $\qquad \therefore \frac{2}{a} = 2$ $\qquad \therefore a = 1$

[b] $\because m_1 = \frac{2\sqrt{3} - 3\sqrt{3}}{5-4} = -\sqrt{3}$ $, m_2 = \tan 30° = \frac{1}{\sqrt{3}}$

$\therefore m_1 \times m_2 = -\sqrt{3} \times \frac{1}{\sqrt{3}} = -1$

$\therefore$ The two straight lines are perpendicular.

5

[a] $\because m = 3$

$\therefore$ The equation is : $y = 3X + c$

$, \because (1, 0)$ satisfies the equation.

$\therefore 0 = 3 \times 1 + c$ $\qquad \therefore c = -3$

$\therefore$ The equation is : $y = 3X - 3$

[b] 1 $\because \overline{AD} // \overline{BC}, \overline{AE} \perp \overline{BC}, \overline{DC} \perp \overline{BC}$

$\therefore$ AECD is a rectangle

$\therefore AD = EC = 12$ cm.

$\therefore BE = 15 - 12 = 3$ cm.

In ΔAEB :

$\because m(\angle AEB) = 90°$

$\therefore (AE)^2 = (5)^2 - (3)^2 = 16$ $\qquad \therefore AE = 4$ cm

2 $\tan(\angle BAE) \times \tan(\angle ACB) = \frac{3}{4} \times \frac{4}{12} = \frac{1}{4}$

5 El-Sharkia

1

1 c 2 b 3 c 4 a 5 d 6 c

2

[a] $\because$ The slope of the given straight line $= \frac{\frac{-1}{2}}{3} = \frac{-1}{6}$

$\therefore$ The slope of the required straight line $= \frac{-1}{6}$

$\therefore$ Its equation is : $y = \frac{-1}{6} x + c$

$, \because (-6 , -1)$ satisfies the equation

$\therefore -1 = \frac{-1}{6} \times -6 + c \qquad \therefore c = -2$

$\therefore$ The equation is : $y = \frac{-1}{6} x - 2$

[b] $\because \cos x \tan x + \frac{1}{2} = 1$

$\therefore \cos x \times \frac{\sin x}{\cos x} = \frac{1}{2}$

$\therefore \sin x = \frac{1}{2} \qquad \therefore x = 30^\circ$

3

[a] $\because$ ABCD is a rectangle

$\therefore$ The two diagonals bisect each other

$\therefore$ The midpoint of $\overline{AC}$ = the midpoint of $\overline{BD}$

$\therefore \left(\frac{1+0}{2} , \frac{1-3x}{2}\right) = \left(\frac{3+x}{2} , \frac{3+y}{2}\right)$

$\therefore \frac{3+x}{2} = \frac{1}{2} \qquad \therefore 3 + x = 1 \qquad \therefore x = -2$

$, \frac{1+6}{2} = \frac{3+y}{2} \qquad \therefore 3 + y = 7 \qquad \therefore y = 4$

[b] In Δ ABC :

$\because m(\angle ABC) = 90^\circ \qquad \therefore (AC)^2 = 3^2 + 4^2 = 25$

$\therefore AC = 5$ cm.

$, \because \overline{BD} \perp \overline{AC}$

$\therefore BD = \frac{AB \times BC}{AC} = \frac{4 \times 3}{5} = 2.4$ cm.

$, \because (BC)^2 = CD \times AC \qquad \therefore 9 = CD \times 5$

$\therefore CD = \frac{9}{5} = 1.8$ cm. $\qquad \therefore AD = 5 - 1.8 = 3.2$

$\therefore \tan x \tan y + \sin A = \frac{1.8}{2.4} \times \frac{3.2}{2.4} + \frac{3}{5} = 1 + \frac{3}{5}$

$= 1\frac{3}{5}$

4

[a] $\because$ The slope of the given straight line

$= \frac{0-2}{-1-3} = \frac{1}{2}$

$\therefore$ The slope of the required straight line $= -2$

$\therefore$ Its equation is : $y = -2x + c$

$, \because (5 , -2)$ satisfies the equation.

$\therefore -2 = -2 \times 5 + c \qquad \therefore c = 8$

$\therefore$ The equation is : $y = -2x + 8$

[b] $\because AB = \sqrt{(-1-1)^2 + (-2-4)^2} = \sqrt{4+36}$

$= \sqrt{40}$ length units

$, BC = \sqrt{(2+1)^2 + (-3+2)^2} = \sqrt{9+1}$

$= \sqrt{10}$ length units

$, AC = \sqrt{(2-1)^2 + (-3-4)^2} = \sqrt{1+49}$

$= \sqrt{50}$ length units

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore \Delta$ ABC is right-angled at B

, its area $= \frac{1}{2} \times \sqrt{40} \times \sqrt{10} = 10$ square units.

5

[a] $\because \cos 60^\circ = \frac{1}{2}$ (1)

$, 2\cos^2 30^\circ - \tan 45^\circ = 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1 = \frac{1}{2}$ (2)

From (1) and (2) :

$\therefore \cos 60^\circ = 2\cos^2 30^\circ - \tan 45^\circ$

[b] Let $C(x , 0)$

$\because$ The slope of $\overleftrightarrow{AB} = \frac{5-1}{2+2} = 1$

$, \because \overline{AB} \perp \overline{BC} \qquad \therefore$ The slope of $\overleftrightarrow{BC} = -1$

$\therefore \frac{0-5}{x-2} = -1 \qquad \therefore x - 2 = 5$

$\therefore x = 7 \qquad \therefore C(7 , 0)$

$\therefore$ The slope of $\overleftrightarrow{AC} = \frac{0-1}{7+2} = \frac{-1}{9}$

$\therefore$ The equation of $\overleftrightarrow{AC}$ is : $y = \frac{-1}{9} x + c$

$, \because C(7 , 0)$ satisfies the equation.

$\therefore 0 = \frac{-1}{9} \times 7 + c \qquad \therefore c = \frac{7}{9}$

$\therefore$ The equation of $\overleftrightarrow{AC}$ is : $y = \frac{-1}{9} x + \frac{7}{9}$

6 El-Monofia

1

1 d 2 d 3 c 4 a 5 c 6 b

2

[a] $\sin 45^\circ \cos 45^\circ + \sin 30^\circ \cos 60^\circ - \cos^2 30^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{2} - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{2} + \frac{1}{4} - \frac{3}{4} = 0$

[b] $\because AB = \sqrt{(-4-1)^2 + (2+2)^2} = \sqrt{25+16}$

$= \sqrt{41}$ length units.

$, BC = \sqrt{(1+4)^2 + (6-2)^2} = \sqrt{25+16}$

$= \sqrt{41}$ length units.

$, AC = \sqrt{(1-1)^2 + (6+2)^2} = \sqrt{64} = 8$ length units.

$\therefore AB = BC$

$\therefore \Delta$ ABC is isosceles.

3

[a] In Δ ABC :

$\because m(\angle C) = 90^\circ$

$\therefore (AC)^2 = (10)^2 - (8)^2 = 36$

$\therefore AC = 6$ cm.

$\therefore \sin A \cos B + \cos A \sin B$

$= \frac{8}{10} \times \frac{8}{10} + \frac{6}{10} \times \frac{6}{10} = \frac{64}{100} + \frac{36}{100} = 1$

[b] $\because$ The slope of the given straight line $= \frac{3}{2}$

$\therefore$ The slope of the required straight line $= \frac{-2}{3}$

$\therefore$ Its equation is : $y = \frac{-2}{3} x + c$

$, \because (3 , 4)$ satisfies the equation

$\therefore 4 = \frac{-2}{3} \times 3 + c \qquad \therefore c = 6$

$\therefore$ The equation is : $y = \frac{-2}{3} x + 6$

4

[a] $\because 2\sin E = \left(\sqrt{3}\right)^2 - 2 \times 1 = 3 - 2 = 1$

$\therefore \sin E = \frac{1}{2} \qquad \therefore E = 30^\circ$

[b] $\because AB = \sqrt{(-1-1)^2 + (-2-4)^2} = \sqrt{4+36}$

$= \sqrt{40}$ length units

$, BC = \sqrt{(2+1)^2 + (-3+2)^2} = \sqrt{9+1}$

$= \sqrt{10}$ length units

$, AC = \sqrt{(2-1)^2 + (-3-4)^2} = \sqrt{1+49}$

$= \sqrt{50}$ length units

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore \Delta$ ABC is right-angled at B

, its area $= \frac{1}{2} \times \sqrt{40} \times \sqrt{10} = 10$ square units.

5

[a] $\because 3x + 2y = 6 \qquad \therefore 2y = -3x + 6$

$\therefore y = \frac{-3}{2} x + 3$

$\therefore$ The slope $= \frac{-3}{2}$ and it intercepts 3 units from the positive part of the y-axis

[b] $\because$ A , B , C are collinear.

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{AC}$

$\therefore \frac{3-1}{k-0} = \frac{5-1}{2-0} \qquad \therefore \frac{2}{k} = 2$

$\therefore k = 1$

7 El-Gharbia

1

1 b 2 c 3 c 4 b 5 a 6 c

2

[a] $\because 4x = \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} \times 1\right)^2 \qquad \therefore 4x = \frac{1}{4}$

$\therefore x = \frac{1}{16}$

[b] 1 Let E be the point of intersection of the two diagonals

$\therefore E = \left(\frac{3-1}{2} , \frac{2-2}{2}\right) = (1 , 0)$

2 $\because AC = \sqrt{(-1-3)^2 + (-2-2)^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ length units

$, BD = \sqrt{(-2-4)^2 + (3+3)^2}$

$= \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$ length units

$\therefore$ The area of the rhombus $= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

$= 24$ square units

3

[a] $\because$ The slope of $\overleftrightarrow{AB} = \frac{-7-1}{3-5} = 4$

, the slope of $\overleftrightarrow{BC} = \frac{3+7}{1-3} = -5$

$\therefore$ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

$\therefore$ A , B , C are not collinear.

[b] $3 - \tan 45^\circ \div 4 \sin 30^\circ = 3 - 1 \div \left(4 \times \frac{1}{2}\right)$

$= 3 - 1 \div 2 = 3 - \frac{1}{2} = 2\frac{1}{2}$

4

[a] $\because$ The slope of the straight line $= \frac{1+1}{1-2} = -2$

$\therefore$ Its equation : $y = -2x + c$

$, \because (1 , 1)$ satisfies the equation

$\therefore 1 = -2 \times 1 + c \qquad \therefore c = 3$

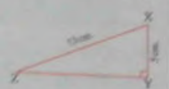
$\therefore$ The equation is : $y = -2x + 3$

[b] In $\triangle XYZ$:

$\because m(\angle Y) = 90°$

$\therefore (YZ)^2 = (13)^2 - (5)^2 = 144$

$\therefore YZ = 12$ cm.



$\therefore \tan X + \tan Z = \frac{12}{5} + \frac{5}{12} = \frac{169}{60}$

5

[a] $\because m_1 = \frac{k-1}{2-3} = 1 - k$, $m_2 = \tan 45° = 1$

, $\because L_1 \perp L_2$ $\therefore m_1 \times m_2 = -1$

$\therefore (1-k) \times 1 = -1$ $\therefore 1 - k = -1$

$\therefore k = 2$

[b] $\because$ The slope of the given straight line $= \frac{-1}{2}$

$\therefore$ The slope of the required straight line $= \frac{-1}{2}$

$\therefore$ Its equation is : $y = \frac{-1}{2} X + c$

, $\because (0 , 3)$ satisfies the equation

$\therefore 3 = \frac{-1}{2} \times 0 + c$ $\therefore c = 3$

$\therefore$ The equation is : $y = \frac{-1}{2} X + 3$

8 El-Dakahlia

1

[a] 1 b 2 a 3 c

[b] $\because (5 , 7) = \left(\frac{8 + X}{2} , \frac{y + 3}{2}\right)$

$\therefore \frac{8 + X}{2} = 5$ $\therefore 8 + X = 10$ $\therefore X = 2$

, $\frac{y+3}{2} = 7$ $\therefore y + 3 = 14$ $\therefore y = 11$

$\therefore X + y = 2 + 11 = 13$

2

[a] 1 a 2 d 3 b

[b] $\because$ ABCD is a rhombus.

$\therefore AB = BC$

$\therefore \sqrt{(6-5)^2 + (-2-3)^2} = \sqrt{(1-6)^2 + (m+2)^2}$

(squaring both sides)

$\therefore (6-5)^2 + (-2-3)^2 = (1-6)^2 + (m+2)^2$

$\therefore (m+2)^2 + 25 = 1 + 25$

$\therefore (m+2)^2 = 1$ $\therefore m + 2 = \pm 1$

$\therefore m + 2 = 1$ $\therefore m = -1$

or $m + 2 = -1$ $\therefore m = -3$

3

[a] $\because 3 \tan X - 4 \times \left(\frac{1}{2}\right)^2 = 8 \times \left(\frac{1}{2}\right)^2$

$\therefore 3 \tan X - 4 \times \frac{1}{4} = 8 \times \frac{1}{4}$

$\therefore 3 \tan X - 1 = 2$ $\therefore 3 \tan X = 3$

$\therefore \tan X = 1$ $\therefore X = 45°$

[b] 1 The distance at the beginning of the motion = 2 m.

2 $\because (0 , 2)$, $(4 , 4)$ lie on the straight line

$\therefore$ The velocity $= \frac{4-2}{4-0} = \frac{1}{2}$ m./sec.

3 The equation is $d = \frac{1}{2} t + 2$

4

[a] $\because m_1 = \frac{-3-3}{-2-4} = 1$, $m_2 = \frac{-(2k+1)}{-k} = \frac{2k+1}{k}$

, $\because L_1 \parallel L_2$ $\therefore m_1 = m_2$

$\therefore \frac{2k+1}{k} = 1$ $\therefore 2k + 1 = k$

$\therefore 2k - k = -1$ $\therefore k = -1$

[b] In $\triangle ABC$:

$\because m(\angle C) = 90°$

$\therefore \sin B = \frac{AC}{AB}$

$\therefore \sin 60° = \frac{AC}{6}$

$\therefore AC = 6 \sin 60° = 3\sqrt{3}$ m.

5

[a] In $\triangle ABC$: $\because m(\angle A) = 90°$

$\therefore (AC)^2 = (25)^2 - (7)^2 = 576$

$\therefore AC = 24$ cm. $\therefore AD = \frac{24}{2} = 12$ cm.

$\therefore \tan C + \frac{1}{\tan(\angle ABD)} = \frac{7}{24} + \frac{1}{\frac{12}{7}} = \frac{7}{24} + \frac{7}{12} = \frac{7}{8}$

[b] 1 In $\triangle ABO$:

$m(\angle AOB) = 90°$, $OA = OB$

$\therefore m(\angle A) = m(\angle B) = \frac{180° - 90°}{2} = 45°$

, $\because \sin A = \frac{OB}{AB}$ $\therefore \sin 45° = \frac{OB}{2\sqrt{2}}$

$\therefore OB = 2\sqrt{2} \times \frac{1}{\sqrt{2}} = 2$ units

$\therefore OA = OB = 2$ units.

$\therefore A(-2 , 0)$, $B(0 , 2)$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = The slope of $\overleftrightarrow{BH} = \tan 45°$

$\therefore \frac{k-2}{2-0} = 1$ $\therefore k - 2 = 2$

$\therefore k = 4$ $\therefore H(2 , 4)$

2 $\because$ The slope of $\overleftrightarrow{AB} = \tan 45° = 1$

, $\overleftrightarrow{HD} \perp \overleftrightarrow{AB}$ $\therefore$ The slope of $\overleftrightarrow{HD} = -1$

$\therefore$ The equation of $\overleftrightarrow{HD}$ is : $y = -X + c$

, $\because H(2 , 4) \in \overleftrightarrow{HD}$

$\therefore 4 = -2 + c$ $\therefore c = 6$

$\therefore$ The equation of $\overleftrightarrow{HD}$ is : $y = -X + 6$

9 Ismailia

1

1 a 2 c 3 d 4 d 5 c 6 c

2

[a] $\because 2 \sin X = \left(\sqrt{3}\right)^2 - 2 \times 1^2 = 3 - 2 = 1$

$\therefore \sin X = \frac{1}{2}$ $\therefore X = 30°$

[b] $\because m_1 = \frac{-4}{-2} = 2$, $m_2 = \frac{5-3}{2-1} = 2$

$\therefore m_1 = m_2$ $\therefore L_1 \parallel L_2$

3

[a] $\because AB = \sqrt{(2+1)^2 + (3+1)^2} = \sqrt{16+9}$

$= 5$ length units

, $BC = \sqrt{(6-2)^2 + (0-3)^2} = \sqrt{16+9}$

$= 5$ length units

, $AC = \sqrt{(6+1)^2 + (0+1)^2} = \sqrt{49+1}$

$= 5\sqrt{2}$ length units

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore \triangle ABC$ is right-angled at B

[b] $\because (4 , 2) = \left(\frac{X+6}{2} , \frac{4+y}{2}\right)$

$\therefore \frac{X+6}{2} = 4$ $\therefore X + 6 = 8$ $\therefore X = 2$

, $\frac{4+y}{2} = 2$ $\therefore 4 + y = 4$ $\therefore y = 0$

$\therefore X + y = 2 + 0 = 2$

4

[a] $\because$ The slope of the given straight line $= \frac{-2}{-1} = 2$

$\therefore$ The slope of the required straight line $= \frac{-1}{2}$

$\therefore$ Its equation is : $y = \frac{-1}{2} X + c$

, $\because (2 , -5)$ satisfies the equation

$\therefore -5 = \frac{-1}{2} \times 2 + c$ $\therefore c = -4$

$\therefore$ The equation is : $y = \frac{-1}{2} X - 4$

[b] $\because \tan 60° = \sqrt{3}$ (1)

, $\frac{2 \tan 30°}{1 - \tan^2 30°} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \sqrt{3}$ (2)

From (1) and (2) : $\therefore \tan 60° = \frac{2 \tan 30°}{1 - \tan^2 30°}$

5

[a] $\because$ The slope $= \tan 45° = 1$ and it intercepts 3 units from the positive part of the y-axis

$\therefore$ The equation is : $y = X + 3$

[b] In $\triangle ABC$:

$\because m(\angle C) = 90°$ $\therefore (AC)^2 = 5^2 - 4^2 = 9$

$\therefore AC = 3$ cm.

$\therefore \sin A \cos B + \cos A \sin B = \frac{4}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{3}{5}$

$= \frac{16}{25} + \frac{9}{25} = 1$

10 Suez

1

1 b 2 c 3 b 4 c 5 b 6 a

2

[a] In $\triangle ABC$:

$\because m(\angle B) = 90°$

$\therefore (AC)^2 = (5)^2 + (12)^2 = 169$ $\therefore AC = 13$ cm.

$\therefore \cos A \cos C = \frac{5}{13} \times \frac{12}{13} = \frac{60}{169}$ (1)

, $\sin A \sin C = \frac{12}{13} \times \frac{5}{13} = \frac{60}{169}$ (2)

From (1) and (2) : $\therefore \cos A \cos C = \sin A \sin C$

[b] $\because$ The slope $= \tan 45° = 1$

$\therefore$ The equation is : $y = X + c$

, $\because (0 , 3)$ satisfies the equation

$\therefore 3 = 0 + c$ $\therefore c = 3$

$\therefore$ The equation is : $y = X + 3$

3

[a] $\because$ The midpoint of $\overline{AC} = \left(\frac{-1+5}{2} , \frac{1+6}{2}\right) = \left(2 , \frac{7}{2}\right)$

, the midpoint of $\overline{BD} = \left(\frac{0+4}{2} , \frac{5+2}{2}\right) = \left(2 , \frac{7}{2}\right)$

$\therefore$ The midpoint of $\overline{AC}$ = the midpoint of $\overline{BD}$

$\therefore$ The two diagonals bisect each other

$\therefore$ ABCD is a parallelogram

[b] $\because 2\sin 30^\circ = 2 \times \frac{1}{2} = 1$ (1)

$, \tan^2 60^\circ - 2\tan 45^\circ = \left(\sqrt{3}\right)^2 - 2 \times 1$

$= 3 - 2 = 1$ (2)

From (1) and (2) :

$\therefore 2\sin 30^\circ = \tan^2 60^\circ - 2\tan 45^\circ$

4

[a] Let B (X , y)

$\therefore (5 , 4) = \left(\frac{3+X}{2} , \frac{-1+y}{2}\right)$

$\therefore \frac{3+X}{2} = 5 \qquad \therefore 3 + X = 10 \qquad \therefore X = 7$

$, \frac{-1+y}{2} = 4 \qquad \therefore -1 + y = 8 \qquad \therefore y = 9$

$\therefore$ B $(7 , 9)$

[b] $\because m_1 = \frac{5-4}{2+1} = \frac{1}{3} \; , \; m_2 = \frac{1}{3}$

$\therefore m_1 = m_2 \qquad \therefore L_1 \parallel L_2$

5

[a] $\because \sqrt{(0-X)^2 + (2-3)^2} = 5\sqrt{2}$ (squaring both sides)

$\therefore X^2 + 1 = 50 \qquad \therefore X^2 = 49$

$\therefore X = \pm 7$

[b] $\boxed{1}$ In Δ ABC :

$\because m(\angle B) = 90^\circ \qquad \therefore \sin(\angle ACB) = \frac{15}{25}$

$\therefore m(\angle ACB) \simeq 36^\circ\, 52'\, 12''$

$\boxed{2}$ $\because (BC)^2 = (25)^2 - (15)^2 = 400$

$\therefore$ BC = 20 cm.

$\therefore$ The area of the rectangle ABCD $= 15 \times 20$
$= 300 \text{ cm}^2$

11 Damietta

1

$\boxed{1}$ a $\quad \boxed{2}$ d $\quad \boxed{3}$ b $\quad \boxed{4}$ a $\quad \boxed{5}$ c $\quad \boxed{6}$ b

2

[a] L.H.S. $= \left(\sqrt{3}\right)^2 - 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 3 - 1$
$= 2 =$ R.H.S.

[b] $\because \frac{y-1}{X} = \frac{1}{3} \qquad \therefore y - 1 = \frac{1}{3}X$

$\therefore y = \frac{1}{3}X + 1$

$\therefore$ The slope of the given straight line $= \frac{1}{3}$

$\therefore$ The slope of the required straight line $= \frac{1}{3}$ and it intercepts 4 units from the negative part of the y-axis

$\therefore$ Its equation is : $y = \frac{1}{3}X - 4$

3

[a] $\because 3\tan X = 4 \times \left(\frac{1}{2}\right)^2 + 8 \times \left(\frac{1}{2}\right)^2 = 1 + 2 = 3$

$\therefore \tan X = 1 \qquad \therefore X = 45^\circ$

[b] $\because m_1 = \frac{k-1}{2-3} = 1 - k \; , m_2 = \tan 135^\circ = -1$

$, \because L_1 \parallel L_2 \qquad \therefore m_1 = m_2$

$\therefore 1 - k = -1 \qquad \therefore k = 2$

4

[a] $\because (4 , y) = \left(\frac{X+6}{2} , \frac{3+5}{2}\right)$

$\therefore \frac{X+6}{2} = 4 \qquad \therefore X + 6 = 8 \qquad \therefore X = 2$

$, y = \frac{3+5}{2} = 4 \qquad \therefore X + y = 2 + 4 = 6$

[b] $\because AB = \sqrt{(2-6)^2 + (0-0)^2} = \sqrt{16}$
$= 4$ length units

$, BC = \sqrt{(4-2)^2 + \left(2\sqrt{3} - 0\right)^2} = \sqrt{4+12}$
$= 4$ length units

$, AC = \sqrt{(4-6)^2 + \left(2\sqrt{3} - 0\right)^2} = \sqrt{4+12}$
$= 4$ length units

$\therefore$ AB = BC = AC

$\therefore \Delta$ ABC is equilateral.

5

[a] $\because$ The slope of the given straight line $= \frac{-1}{2}$

$\therefore$ The slope of the required straight line = 2

$\therefore$ Its equation is : $y = 2X + c$

$, \because (-2 , 3)$ satisfies the equation

$\therefore 3 = 2 \times -2 + c \qquad \therefore c = 7$

$\therefore$ The equation is : $y = 2X + 7$

[b] $\boxed{1}$ In Δ ABC : $\because m(\angle B) = 90^\circ$

$\therefore (BC)^2 = (25)^2 - (15)^2 = 400$

$\therefore$ BC = 20 cm.

$\therefore \cos(\angle ACB) = \frac{20}{25} = \frac{4}{5}$

$\boxed{2}$ The area of the rectangle ABCD $= 15 \times 20$
$= 300 \text{ cm}^2$

12 Beni Suef

1

$\boxed{1}$ a $\quad \boxed{2}$ a $\quad \boxed{3}$ d $\quad \boxed{4}$ a $\quad \boxed{5}$ b $\quad \boxed{6}$ d

2

[a] $\because AB = \sqrt{(-2-3)^2 + (4+1)^2}$
$= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$ length units

$, BC = \sqrt{(3-4)^2 + (-1-5)^2} = \sqrt{1+36}$
$= \sqrt{37}$ length units

$, AC = \sqrt{(-2-4)^2 + (4-5)^2} = \sqrt{36+1}$
$= \sqrt{37}$ length units

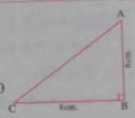
$\therefore$ BC = AC

$\therefore \Delta$ ABC is an isosceles triangle.

[b] $\because \tan X - 4 \times \frac{1}{2} \times \frac{1}{2} = 0 \qquad \therefore \tan X - 1 = 0$

$\therefore \tan X = 1 \qquad \therefore X = 45^\circ$

3



[a] $\boxed{1}$ In Δ ABC :

$\because m(\angle B) = 90^\circ$

$\therefore (AC)^2 = 6^2 + 8^2 = 100$

$\therefore$ AC = 10 cm.

$\therefore \cos A \cos C - \sin A \sin C$

$= \frac{6}{10} \times \frac{8}{10} - \frac{8}{10} \times \frac{6}{10} = 0$

$\boxed{2}$ $\because \cos C = \frac{8}{10}$

$\therefore m(\angle C) \simeq 36^\circ\, 52'\, 12''$

[b] $\because \frac{y-2}{X} = \frac{1}{2} \qquad \therefore y - 2 = \frac{1}{2}X$

$\therefore y = \frac{1}{2}X + 2$

$\therefore$ The slope $= \frac{1}{2}$ and it intercepts 2 units from the positive part of the y-axis.

4

[a] $\because \sin^2 45^\circ = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$ (1)

$, 2\cos^2 30^\circ - 1 = 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1$

$= 2 \times \frac{3}{4} - 1 = \frac{1}{2}$ (2)

From (1) and (2) :

$\therefore \sin^2 45^\circ = 2\cos^2 30^\circ - 1$

[b] $\because$ The slope of the given straight line $= \tan 45^\circ = 1$

$\therefore$ The slope of the required straight line = 1

$\therefore$ Its equation is : $y = X + c$

$, \because (3 , -5)$ satisfies the equation.

$\therefore -5 = 3 + c \qquad \therefore c = -8$

$\therefore$ The equation is : $y = X - 8$

5

[a] $\because (4 , y) = \left(\frac{6+X}{2} , \frac{5+3}{2}\right)$

$\therefore \frac{6+X}{2} = 4 \qquad \therefore 6 + X = 8 \qquad \therefore X = 2$

$, y = \frac{5+3}{2} = 4 \qquad \therefore X + y = 2 + 4 = 6$

[b] $\because m_1 = \frac{4-5}{-2+2} = \frac{-1}{0} \qquad \therefore m_1$ is undefined

$\therefore L_1 \parallel$ the y-axis.

$, \because m_2 = \frac{3-3}{5-2} = 0 \qquad \therefore L_2 \parallel$ the X-axis.

$\therefore L_1 \perp L_2$

13 Assiut

1

$\boxed{1}$ b $\quad \boxed{2}$ a $\quad \boxed{3}$ a $\quad \boxed{4}$ c $\quad \boxed{5}$ d $\quad \boxed{6}$ b

2

[a] $\sin^2 60^\circ + \cos^2 60^\circ + \tan^2 45^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + (1)^2 = \frac{3}{4} + \frac{1}{4} + 1 = 2$

[b] $\because$ The slope $= \frac{1+1}{1-2} = -2$

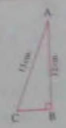
$\therefore$ The equation is : $y = -2X + c$

$, \because (1 , 1)$ satisfies the equation.

$\therefore 1 = -2 \times 1 + c \qquad \therefore c = 3$

$\therefore$ The equation is : $y = -2X + 3$

3



[a] In Δ ABC :

$\because m(\angle B) = 90^\circ$

$\therefore \sin C = \frac{12}{13}$

$\therefore m(\angle C) \simeq 67^\circ\, 22'\, 48''$

[b] $\because m_1 = \frac{3+1}{6-X} = \frac{4}{6-X} \; , m_2 = \tan 45^\circ = 1$

$, \because L_1 \perp L_2 \qquad \therefore m_1 \times m_2 = -1$

$\therefore \frac{4}{6-X} \times 1 = -1 \quad \therefore X - 6 = 4$

$\therefore X = 10$

4

[a] $\because \cos 30^\circ = \frac{\sqrt{3}}{2}$ (1)

$, \frac{\sin 30^\circ \sin 60^\circ}{\sin 45^\circ \cos 45^\circ} = \frac{\frac{1}{2} \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}} = \frac{\sqrt{3}}{2}$ (2)

From (1) and (2) : $\therefore \cos 30^\circ = \frac{\sin 30^\circ \sin 60^\circ}{\sin 45^\circ \cos 45^\circ}$

[b] $\because$ The midpoint of $\overline{AC} = \left(\frac{1+7}{2} , \frac{0+8}{2}\right) = (4 , 4)$

, the midpoint of $\overline{BD} = \left(\frac{-1+9}{2} , \frac{4+4}{2}\right) = (4 , 4)$

$\therefore$ The midpoint of $\overline{AC}$ = the midpoint of $\overline{BD}$

$\therefore$ The two diagonals bisect each other

$\therefore$ ABCD is a parallelogram.

5

[a] $\because \frac{X}{2} + \frac{y}{3} = 1$ (multiplying by 3)

$\therefore \frac{3}{2} X + y = 3$ $\qquad \therefore y = \frac{-3}{2} X + 3$

$\therefore$ The slope $= \frac{-3}{2}$ and it intercepts 3 units from the positive part of the y-axis.

[b] 1 Let A $(X , 0)$, B $(0 , y)$

$\therefore (4 , 3) = \left(\frac{X+0}{2} , \frac{0+y}{2}\right)$

$\therefore \frac{X}{2} = 4$ $\qquad \therefore X = 8$

, $\frac{y}{2} = 3$ $\qquad \therefore y = 6$

$\therefore$ A $(8 , 0)$, B $(0 , 6)$

2 $\because$ The slope of $\overleftrightarrow{AB} = \frac{6-0}{0-8} = \frac{-3}{4}$

, $\overleftrightarrow{AB}$ cuts 6 units from the positive part of the y-axis.

$\therefore$ The equation of $\overleftrightarrow{AB}$ is : $y = \frac{-3}{4} X + 6$

14 Luxor

1

1 a $\quad$ 2 b $\quad$ 3 c $\quad$ 4 c $\quad$ 5 d $\quad$ 6 a

2

[a] $\because \sin X = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

$\therefore m(\angle X) = 30°$

[b] $\because$ The slope of $\overleftrightarrow{AB} = \frac{2-1}{1-0} = 1$

, the slope of $\overleftrightarrow{BC} = \frac{3-2}{2-1} = 1$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$

$\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$

, $\because$ B is a common point between $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$

$\therefore$ A , B , C are collinear.

3

[a] $\because \tan 30° \tan 60° = \frac{1}{\sqrt{3}} \times \sqrt{3} = 1$ $\qquad$ (1)

, $\sin^2 45° + \cos^2 45° = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{2} + \frac{1}{2} = 1$ $\qquad$ (2)

From (1) and (2) :

$\therefore \tan 30° \tan 60° = \sin^2 45° + \cos^2 45°$

[b] $\because \frac{-k}{-2} = \tan 45°$ $\qquad \therefore \frac{k}{2} = 1$

$\therefore k = 2$

4

[a] $\because \sqrt{(2-6)^2 + (0-X)^2} = 5$ (squaring both sides)

$\therefore (2-6)^2 + (0-X)^2 = 25$

$\therefore 16 + X^2 = 25$ $\qquad \therefore X^2 = 9$

$\therefore X = 3$ or $X = -3$

[b] **Constr :** Draw $\overline{AD} \perp \overline{BC}$

Proof : $\because$ AB = AC , $\overline{AD} \perp \overline{BC}$

$\therefore$ BD = CD = 6 cm.

$\therefore$ In Δ ABD :

$\because m(\angle ADB) = 90°$

$\therefore \cos B = \frac{BD}{AB} = \frac{6}{10} = \frac{3}{5}$

$\therefore m(\angle B) \simeq 53° \ 7' \ 48''$

5

[a] $\because$ The slope of $\overleftrightarrow{AB} = \frac{2-4}{1+1} = -1$

$\therefore$ The slope of the axis of symmetry of $\overline{AB} = 1$

$\therefore$ Its equation is : $y = X + c$

, $\because$ The midpoint of $\overline{AB} = \left(\frac{-1+1}{2} , \frac{4+2}{2}\right)$

$= (0 , 3)$

, $\because (0 , 3)$ satisfies the equation.

$\therefore 3 = 0 + c$ $\qquad \therefore c = 3$

$\therefore$ The equation is : $y = X + 3$

[b] $\because$ ABCD is rectangle

$\therefore$ The two diagonals bisect each other

$\therefore$ The midpoint of $\overline{AC}$ = The midpoint of $\overline{BD}$

$\therefore \left(\frac{1+0}{2} , \frac{1-3X}{2}\right) = \left(\frac{3+X}{2} , \frac{3+y}{2}\right)$

$\therefore \frac{1}{2} = \frac{3+X}{2}$ $\qquad \therefore 3 + X = 1$

$\therefore X = -2$

, $\frac{1+6}{2} = \frac{3+y}{2}$ $\qquad \therefore 3 + y = 7$

$\therefore y = 4$

15 New Valley

1

1 c $\quad$ 2 d $\quad$ 3 c $\quad$ 4 a $\quad$ 5 b $\quad$ 6 c

2

[a] $\because \sin X = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

$\therefore X = 30°$

[b] $\because$ The slope of $\overleftrightarrow{AB} = \frac{2-1}{2-1} = 1$

, the slope of $\overleftrightarrow{BC} = \frac{3-2}{3-2} = 1$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$

$\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$

, $\because$ B is a common point between $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$

$\therefore$ A , B , C are collinear.

3

[a] The equation is : $y = 2X + 7$

[b] $\because AB = \sqrt{(-2-3)^2 + (4+1)^2} = \sqrt{25+25} = \sqrt{50}$

$= 5\sqrt{2}$ length units

, $BC = \sqrt{(3-4)^2 + (-1-5)^2} = \sqrt{1+36}$

$= \sqrt{37}$ length units

, $AC = \sqrt{(-2-4)^2 + (4-5)^2} = \sqrt{36+1}$

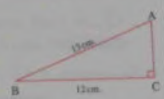
$= \sqrt{37}$ length units

$\therefore$ BC = AC

$\therefore \Delta$ ABC is an isosceles triangle.

4

[a] In Δ ABC :



$\because m(\angle C) = 90°$

$\therefore (AC)^2 = (13)^2 - (12)^2$

$= 25$

$\therefore$ AC = 5 cm.

$\therefore \sin A \cos B + \cos A \sin B$

$= \frac{12}{13} \times \frac{12}{13} + \frac{5}{13} \times \frac{5}{13} = \frac{144}{169} + \frac{25}{169} = 1$

[b] $\because$ The midpoint of $\overline{AB} = \left(\frac{1+3}{2} , \frac{-2-4}{2}\right)$

$= (2 , -3)$

$\therefore$ The slope of the straight line $= \frac{6+3}{1-2} = -9$

$\therefore$ Its equation is : $y = -9X + c$

, $\because (1 , 6)$ satisfies the equation

$\therefore 6 = -9 \times 1 + c$

$\therefore c = 15$

$\therefore$ The equation is : $y = -9X + 15$

5

[a] $\because m_1 = \frac{-2+4}{1-3} = -1$, $m_2 = \tan 45° = 1$

$\therefore m_1 m_2 = -1 \times 1 = -1$ $\qquad \therefore L_1 \perp L_2$

[b] $\because \overline{AD} \parallel \overline{BC}$, $\overline{AF} \perp \overline{BC}$

, $\overline{DE} \perp \overline{BC}$

$\therefore$ AFED is a rectangle. $\quad \therefore$ FE = AD = 5 cm.

$\therefore$ BF + EC = 6 cm. $\qquad \therefore$ BF = 3 cm.

In Δ ABF :

$\therefore \cos B = \frac{BF}{AB} = \frac{3}{5}$

$\therefore m(\angle B) \simeq 53° \ 7' \ 48''$

, $\because m(\angle AFB) = 90°$

$\therefore (AF)^2 = (AB)^2 - (BF)^2 = (5)^2 - (3)^2 = 16$

$\therefore$ AF = 4 cm.

$\therefore$ The area of the trapezium ABCD

$= \frac{1}{2}(5 + 11) \times 4 = 32 \text{ cm}^2.$

Answers of examinations on Port Said specifications of trigonometry & geometry

Exam 1 Port Said 2023

First Answers of multiple choice questions

1 (a)	2 (c)	3 (c)	4 (b)	5 (c)
6 (a)	7 (d)	8 (c)	9 (b)	10 (b)
11 (c)	12 (d)	13 (c)	14 (b)	15 (c)
16 (c)	17 (d)	18 (b)	19 (c)	20 (b)
21 (c)				

Second Answers of essay questions

22

$\because$ The slope of $\overleftrightarrow{AB} = \frac{4-0}{0-4} = -1$

$\therefore$ The equation of $\overleftrightarrow{AB}$ is : $y = -X + c$

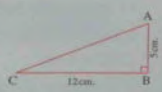
, $\because$ (0 , 4) satisfies the equation

$\therefore 4 = 0 + c \qquad \therefore c = 4$

$\therefore$ The equation of $\overleftrightarrow{AB}$ is : $y = -X + 4$

23

In Δ ABC :



$\because m(\angle B) = 90°$

$\therefore (AC)^2 = (5)^2 + (12)^2 = 169$

$\therefore$ AC = 13 cm.

$\therefore \sin^2 A + \cos^2 A = \left(\frac{12}{13}\right)^2 + \left(\frac{5}{13}\right)^2 = 1$

24

In Δ ABC :

$\because AB = \sqrt{(1-3)^2 + (5-3)^2} = \sqrt{4+4}$

$= 2\sqrt{2}$ length units.

, $BC = \sqrt{(1-1)^2 + (3-5)^2} = \sqrt{0+4}$

$= 2$ length units

, $AC = \sqrt{(1-3)^2 + (3-3)^2} = \sqrt{4+0}$

$= 2$ length units

$\therefore$ BC = AC

$\therefore \Delta$ ABC is isosceles

Exam 2 Port Said 2024

First Answers of multiple choice questions

1 (d)	2 (a)	3 (b)	4 (b)	5 (a)
6 (c)	7 (b)	8 (a)	9 (c)	10 (a)
11 (b)	12 (c)	13 (a)	14 (c)	15 (b)
16 (b)	17 (c)	18 (a)	19 (c)	20 (c)
21 (c)				

Second Answers of essay questions

22

$\because \cos H = \left(\frac{1}{\sqrt{2}}\right)^2 \times \sqrt{3} = \frac{\sqrt{3}}{2}$

$\therefore m(\angle H) = 30°$

23

$\because$ The slope of $\overleftrightarrow{AB} = \frac{5+1}{6+3} = \frac{2}{3}$

, the slope of $\overleftrightarrow{BC} = \frac{3-5}{3-6} = \frac{2}{3}$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$

$\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$

, $\because$ B is a common point

$\therefore$ A , B and C are collinear

24

$\because$ A (−4 , 0) , B (0 , 4)

$\therefore$ The slope of $\overleftrightarrow{AB} = \frac{4-0}{0+4} = 1$

, $\because \overleftrightarrow{AB}$ cuts 4 units from the positive part of the y-axis

$\therefore$ The equation of $\overleftrightarrow{AB}$ is : $y = X + 4$

Exam 3

First Answers of multiple choice questions

1 (d)	2 (b)	3 (d)	4 (a)	5 (d)
6 (c)	7 (a)	8 (b)	9 (d)	10 (a)
11 (d)	12 (a)	13 (b)	14 (b)	15 (a)
16 (a)	17 (b)	18 (b)	19 (c)	20 (c)
21 (a)				

Second Answers of essay questions

22

$\because AB = \sqrt{(5-1)^2 + (1-1)^2} = \sqrt{16} = 4$ length units

, $BC = \sqrt{(3-5)^2 + (4-1)^2} = \sqrt{4+9} = \sqrt{13}$ length units

, $AC = \sqrt{(3-1)^2 + (4-1)^2} = \sqrt{4+9} = \sqrt{13}$ length units

$\therefore$ BC = AC

$\therefore \Delta$ ABC is isosceles

23

$\because$ The slope of the given straight line $= \frac{-1}{3}$

$\therefore$ The slope of the required straight line $= \frac{-1}{3}$

$\therefore$ Its equation is : $y = \frac{-1}{3} X + c$

, $\because$ (3 , −5) satisfies the equation

$\therefore -5 = \frac{-1}{3} \times 3 + c \qquad \therefore c = -4$

$\therefore$ The equation is : $y = \frac{-1}{3} X - 4$

24

$\cos 60° \sin 30° - \sin 60° \tan 60° + \cos^2 30°$

$= \frac{1}{2} \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times \sqrt{3} + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{2} + \frac{3}{4} = -\frac{1}{2}$

Exam 4

First Answers of multiple choice questions

1 (d)	2 (b)	3 (a)	4 (c)	5 (d)
6 (a)	7 (c)	8 (b)	9 (a)	10 (d)
11 (c)	12 (d)	13 (c)	14 (c)	15 (c)
16 (b)	17 (d)	18 (d)	19 (c)	20 (b)
21 (d)				

Second Answers of essay questions

22

$\because \sin 60° = \frac{\sqrt{3}}{2}$ (1)

, $2 \sin 30° \cos 30° = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ (2)

From (1) and (2) :

$\therefore \sin 60° = 2 \sin 30° \cos 30°$

23

$\because$ The midpoint of $\overline{AC} = \left(\frac{-3+2}{2}, \frac{-1+4}{2}\right) = \left(\frac{-1}{2}, \frac{3}{2}\right)$

, the midpoint of $\overline{BD} = \left(\frac{6-7}{2}, \frac{5-2}{2}\right) = \left(\frac{-1}{2}, \frac{3}{2}\right)$

$\therefore$ The midpoint of $\overline{AC}$ = the midpoint of $\overline{BD}$

$\therefore$ The two diagonals bisect each other

$\therefore$ ABCD is a parallelogram

24

$\because$ The slope = 2

$\therefore$ The equation is : $y = 2X + c$

, $\because$ (1 , 3) satisfies the equation

$\therefore 3 = 2 \times 1 + c \qquad \therefore c = 1$

$\therefore$ The equation is : $y = 2X + 1$

Exam 5

First Answers of multiple choice questions

1 (a)	2 (b)	3 (c)	4 (a)	5 (b)
6 (a)	7 (c)	8 (c)	9 (d)	10 (a)
11 (b)	12 (b)	13 (b)	14 (a)	15 (b)
16 (c)	17 (a)	18 (d)	19 (b)	20 (b)
21 (a)				

Second Answers of essay questions

22

$\because \sin X = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

$\therefore X = 30°$

23

$\because MA = \sqrt{(3+1)^2 + (-1-2)^2} = \sqrt{16+9}$

$= 5$ length units

, $MB = \sqrt{(-4+1)^2 + (6-2)^2} = \sqrt{9+16}$

$= 5$ length units

, $MC = \sqrt{(2+1)^2 + (-2-2)^2} = \sqrt{9+16}$

$= 5$ length units

$\therefore$ MA = MB = MC

$\therefore$ A , B and C lie on the circle M

24

$\because$ The slope of the given straight line $= \frac{-1}{3}$

$\therefore$ The slope of the required straight line = 3

$\therefore$ Its equation is : $y = 3X + c$

, $\because$ (1 , 3) satisfies the equation

$\therefore 3 = 3 \times 1 + c \qquad \therefore c = 0$

$\therefore$ The equation is : $y = 3X$